Multistage MMSE PIC Space–Time Receiver With Non-Mutually Exclusive Grouping

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***Abstract*—The arrival of new data services for wireless mobile communications requires an efficient use of the available bandwidth. Interference-limited cellular systems based on code-division multiple access (CDMA) can benefit from multiuser detection (MUD) and beamforming with antenna array to reduce multiple-access interference. Group-based techniques have been proposed to reduce the complexity of space–time MUD and have been shown to provide a performance–complexity tradeoff between matched filtering and full MUD. In this paper, the intergroup interference, which is a limiting factor in group-based systems, is reduced using multistage parallel interference cancellation after group-based minimum mean square error (MMSE) linear filtering. In addition, the extra resources that are available at the receiver are exploited by sharing users among groups. The proposed receiver is shown to converge, as the number of stages increases, to the full space–time MMSE linear MUD filter. The results show that the new approach provides bit error rate (BER) performance close to the full MUD receiver at a fraction of the complexity.**

***Index Terms*—Code-division multiple access (CDMA), group-based detection, multiuser detection (MUD), space–time systems.**

I. Introduction

THE ABUNDANCE of new data services offered for wireless mobile communications creates a need for efficient use of the available bandwidth. For multiuser communications systems based on direct-spread or wideband code-division multiple access (WCDMA), such as third-generation cellular systems, the limiting factor in bandwidth efficiency consists of multiple-access interference (MAI). Several techniques for reducing MAI exist in the literature, including multiuser detection (MUD) and beamforming with antenna array [1], [2].

Optimal MUD takes the form of trellis decoding and is very complex for a large number of users or in channels with large delay spreads [1]. Several approaches for reducing the complexity of the optimal MUD have been researched, including suboptimal linear filter techniques such as minimum mean square error (MMSE) and zero forcing (see, e.g., [3] and [4]), as well as iterative approaches such as parallel interference cancellation (PIC) [5]–[8] and successive interference cancellation (SIC) [9].

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To further reduce the complexity of the MUD, it has been proposed to exploit the spatial dimension that is available when using antenna arrays at the base station [10]–[12]. With beamforming, users can be "separated" into spatial equivalence classes or groups; each group is then detected using a separate and independent multiuser detector. A similar concept for user grouping was previously studied for single-antenna systems in [13] and, more recently, in [14] and [15]. Provided the groups are well separated, this technique has the potential to significantly reduce the complexity of the MUD. However, because of the inherent group nonorthogonality, this reduced-complexity approach introduces *intergroup interference* (IGI) that degrades the performance of the receiver when compared to the full space–time MUD (STMUD) receiver, which jointly operates on all the users.

In most of the existing literature on group-based STMUD, the IGI is often disregarded based on the assumption that the spatial filtering provides sufficient attenuation (see, e.g., [10] and [11]). In practice, because the maximum number of users per group is limited by hardware constraints and because of the nonorthogonality among groups, the IGI may become an important factor in performance degradation.

In this paper, we use multistage PIC among groups to reduce the IGI. We show that under mild interference conditions, the equivalent filter of the proposed linear *multistage group-based STMUD PIC* receiver (MS-GRP-PIC) with MMSE group-based filters converges, as the number of PIC stages increases, to the full STMUD. The convergence rate depends on the residual IGI after filtering. The bit error rate (BER) performance obtained after a few stages is shown to be comparable to the full STMUD BER performance, at a fraction of the computational complexity. In addition, the extra resources that are available in the receiver may be exploited to *share* users among groups [16]. The results show that sharing has the potential to improve the detection for all users in the group at essentially no additional cost in complexity.

The paper is organized as follows: Section II introduces the system model and STMUD receivers. In Section III, we develop the proposed MS-GRP-PIC receiver with user sharing. In Section IV, we discuss the convergence of the equivalent linear filter coefficients, the error probability, and the numerical complexity of the MS-GRP-PIC receiver. The computer simulation results shown in Section V demonstrate the performance advantages of the proposed receiver structure. Finally, a brief conclusion is presented in Section VI.

All vectors are column vectors, unless indicated otherwise, and are denoted by lowercase bold characters. Bold-faced capital characters denote matrices; and $\mathbf{A}^T$, $\mathbf{A}^H$, and $\mathbf{A}^{-1}$, respectively, denote the transpose, conjugate transpose, and inverse of matrix $\mathbf{A}$. In addition, $\mathbf{I}_n$ denotes the identity matrix of dimension $n \times n$, $\mathbf{0}_{n\times m}$ denotes the zero matrix of dimension $n \times m$, $\|\cdot\|$ denotes the Euclidean vector norm, $\text{vec}(\cdot)$ is an operation that concatenates the columns of its matrix argument into a column vector of appropriate dimensions, and $\otimes$ denotes the Kronecker product operation [17]. Finally, $E[\cdot]$ denotes statistical expectation.

II. Background

A. System Model

The uplink of a synchronous WCDMA communications system with K users is considered. Blocks of N information symbols are simultaneously transmitted through a dispersive channel with finite impulse response length W (in terms of number of samples) to a common multiantenna base station receiver. Each user terminal equipment is assumed to have a single transmit antenna.

Let $\mathbf{x}^{(m)} \in \mathbb{C}^{(NL_c+W-1)\times 1}$ be the received signal vector for antenna element $m \in \{1,\ldots,M\}$, where M is the number of antennas, and L_c is the spreading factor. The complete set of observations can be represented in vector form as $\mathbf{x} = \text{vec}([\mathbf{x}^{(1)} \ \ldots \ \mathbf{x}^{(M)}]^T) \in \mathbb{C}^{M(NL_c+W-1)\times 1}$.

Similarly, the NK transmitted symbols can be represented as $\mathbf{d} = \text{vec}([\mathbf{d}^{(1)} \ \ldots \ \mathbf{d}^{(K)}]^T) \in \mathcal{A}^{NK\times 1}$, where $\mathbf{d}^{(k)} \in \mathcal{A}^{N\times 1}$ is the vector of symbols for user k, and $\mathcal{A}$ is the symbol alphabet (e.g., for BPSK $\mathcal{A} = \{\pm 1\}$). The symbols are assumed to be independent identically distributed and normalized such that $E[\mathbf{d}\mathbf{d}^H] = \mathbf{I}_{NK}$.

Let $\mathbf{v}_k \in \mathbb{C}^{M(L_c+W-1)\times 1}$ be the kth user space–time *effective signature* vector, i.e., the space–time response to a single symbol of unit value transmitted by user k (assuming a single-antenna terminal), as observed by the multiantenna receiver. The effective signature incorporates the multiaccess and scrambling codes convolved with the channel impulse response, which is assumed to be constant for the duration of a block.

Define $\mathbf{V} = [\mathbf{v}_1 \ \ldots \ \mathbf{v}_K] \in \mathbb{C}^{M(L_c+W-1)\times K}$ to be the effective signature matrix for the set of K users. In this paper, the matrix $\mathbf{V}$ is assumed to be known by the receiver with sufficient accuracy, as it is commonly presumed in the linear MUD literature [3]. Then, the total received vector may be conveniently expressed as

$$\mathbf{x} = \mathbf{T}\mathbf{d} + \mathbf{n} \tag{1}$$

where $\mathbf{T} \in \mathbb{C}^{M(NL_c+W-1)\times NK}$ is a block-Toeplitz matrix constructed from N blocks of $\mathbf{V}$ in the following configuration [3]:

$$\mathbf{T} = \begin{bmatrix} [\mathbf{V}] & & & \\ & [\mathbf{V}] & & \\ & & \ddots & \\ & & & [\mathbf{V}] \end{bmatrix} \quad (\text{block shift } ML_c;\ \text{total shift } (N-1)ML_c) \tag{2}$$

and all the other entries outside the $\mathbf{V}$ blocks take the value 0. The vector $\mathbf{n} \in \mathbb{C}^{M(NL_c+W-1)\times 1}$ in (1) contains white circular complex Gaussian noise samples with covariance matrix $E[\mathbf{n}\mathbf{n}^H] = \sigma^2\mathbf{I}_{M(NL_c+W-1)}$, where σ^2 is the noise power.

Notice that the above model and ensuing results can be generalized to account for colored noise and to support the asynchronous case, as well as users with multiantenna terminals.

B. Full STMUD

In a linear STMUD receiver, the observation signals of all users are processed to obtain a joint estimate of the transmitted symbols. Note that for short-code WCDMA systems, such as the time-division duplexing mode of the Third Generation Partnership Project (3GPP) [18], both multiaccess and scrambling codes are fixed for the duration of a block. The MUD weights can therefore be used for several consecutive symbols intervals, as opposed to other CDMA technologies, where long codes are employed, which results in having to recompute the MUD filter at every symbol interval.

In a space–time system, the number of observation samples grows linearly with the number of antenna elements. Due to the potentially large dimension of the observation vector, it is common to apply the linear MMSE filter at the output of the matched filter [1], which is defined here as

$$\mathbf{y} \triangleq \mathbf{T}^H\mathbf{x} \tag{3}$$

where $\mathbf{T}^H$ can be interpreted as a filter matched to the space–time effective signature. In this approach, the MMSE soft symbol estimate is given by

$$\mathbf{z}_{\text{MMSE}} = \mathbf{M}_o^H\mathbf{y} \tag{4}$$

where $\mathbf{M}_o \in \mathbb{C}^{NK\times NK}$ is the optimal filter matrix that minimizes the MMSE cost function

$$J(\mathbf{M}) = E\|\mathbf{d} - \mathbf{M}^H\mathbf{y}\|^2. \tag{5}$$

The optimal weights, taking both MAI and ISI into consideration, can be expressed as (see, e.g., [1])

$$\mathbf{M}_o = (\mathbf{T}^H\mathbf{T} + \sigma^2\mathbf{I})^{-1}. \tag{6}$$

In the so-called *full STMUD* receiver, the soft symbol estimate vector $\mathbf{z}_{\text{MMSE}}$ is then quantized by a nonlinear function $\mathcal{Q}(\cdot)$ to provide the hard symbol estimates, i.e.,

$$\hat{\mathbf{d}}_{\text{MMSE}} = \mathcal{Q}(\mathbf{z}_{\text{MMSE}}) \tag{7}$$

for BPSK $\mathcal{Q}(\cdot) = \text{sgn}(\text{Re}(\cdot))$, where both the real part and sign functions operate element-wise on their respective vector argument.

C. Group-Based STMUD

In a group-based MUD receiver, the data symbols from each group are jointly detected using a dedicated multiuser detector with a reduced dimension. The grouping is based on spatiotemporal correlation; users with large cross correlation are usually assigned to the same group for better detection. In practice, the grouping has to be performed in real time and needs to be regularly recomputed. In [16], we propose an effective grouping mechanism based on the cross correlation between effective signature vectors. This approach is used here to provide the grouping, which is required for designing the filter weights.

To simplify the presentation on group-based systems, some definitions need to be introduced. First, we define a *selection matrix* as an $n \times m$ $(n \geq m)$ matrix that contains exactly one entry of value 1 in each column and no more than one such entry per row; all other entries take the value 0. Second, we define the $n \times m$ *selection matrix complement* as another selection matrix of dimension $n \times (n-m)$ such that $[\mathbf{P}\ \bar{\mathbf{P}}]$ is a permutation matrix,[1] where $\mathbf{P}$ and $\bar{\mathbf{P}}$ are the selection matrix and the selection matrix complement, respectively.

Let K_j denote the number of users in group $j \in \{1,\ldots,G\}$, where G is the number of groups; for mutually exclusive grouping, we have $K = \sum_j K_j$. The $NK \times NK_j$ selection matrix for the symbols associated with users of group j can thus be expressed as

$$\mathbf{P}_j = \mathbf{I}_N \otimes [\mathbf{e}_{g_{j,1}}, \ldots, \mathbf{e}_{g_{j,K_j}}], \tag{8}$$

where $\mathbf{e}_{g_{j,l}}$ is the elementary vector of dimension $K \times 1$ that contains zeros, except at position $g_{j,l}$, where it contains the value 1, and $g_{j,l}$ is the index of the lth user that belongs to group j. Using (8), we can express the symbols associated with group j in vector form as $\mathbf{d}_j \triangleq \mathbf{P}_j^T\mathbf{d}$ and the columns of $\mathbf{T}$ associated with users of group j as $\mathbf{T}_j \triangleq \mathbf{T}\mathbf{P}_j$. Similarly, let the $NK \times N(K-K_j)$ selection matrix complement for group j be given by

$$\bar{\mathbf{P}}_j \triangleq \mathbf{I}_N \otimes [\mathbf{e}_{\bar{g}_{j,1}}, \ldots, \mathbf{e}_{\bar{g}_{j,(K-K_j)}}], \tag{9}$$

where $\bar{g}_{j,l}$ is the index of the lth user *not* in group j. Using (9), we have that the symbols associated with users that do not belong to group j can be expressed as $\bar{\mathbf{d}}_j \triangleq \bar{\mathbf{P}}_j^T\mathbf{d}$ and the corresponding columns of $\mathbf{T}$ as $\bar{\mathbf{T}}_j \triangleq \mathbf{T}\bar{\mathbf{P}}_j$.

Using the above definitions and assuming a predetermined and fixed grouping, the optimum MMSE group-based linear filter of reduced dimensions $NK_j \times NK_j$ is given by [16]

$$\mathbf{M}_{o,j} = \left(\mathbf{R}_j\mathbf{R}_j^H + \mathbf{C}_j\mathbf{C}_j^H + \sigma^2\mathbf{R}_j\right)^{-1}\mathbf{R}_j^H \tag{10}$$

where $\mathbf{R}_j \triangleq \mathbf{T}_j^H\mathbf{T}_j$, and $\mathbf{C}_j \triangleq \mathbf{T}_j^H\bar{\mathbf{T}}_j$. The optimal group-based MMSE estimate for the symbols of group j becomes

$$\mathbf{z}_{o,j} \triangleq \mathbf{M}_{o,j}^H\mathbf{T}_j^H\mathbf{x}. \tag{11}$$

Notice that the filter in (10) takes into consideration the IGI via the term $\mathbf{C}_j\mathbf{C}_j^H$.

[1] An $n \times n$ matrix is called a *permutation matrix* if exactly one entry in each row and column is equal to 1, and all other entries are 0 (see, e.g., [19]).

III. Multistage PIC With User Sharing

In the context of multistage PIC, it is reasonable to expect that the IGI will be reduced after each step. Under this assumption, it is computationally advantageous to reduce the complexity of the group MMSE filter in (10) and introduce a MUD suboptimal filter that neglects IGI

$$\mathbf{M}_j = \left(\mathbf{T}_j^H\mathbf{T}_j + \sigma^2\mathbf{I}\right)^{-1}. \tag{12}$$

In the following, we refer to (12) as the *GRP-STMUD* filter.

In practice, the number of groups G and the maximum number of users per group $K_{\max}$ in a group-based receiver are constrained by the hardware. If the receiver is designed to support the worst-case scenario of a fully loaded WCDMA system (i.e., up to L_c simultaneous users), then, in underloaded situations, the total number of resources will necessarily exceed the total number of simultaneous users, i.e., $GK_{\max} > K$. In such cases, there are empty computing resources that are freely available, making it possible to *share* users among groups.

With user sharing, the groups are no longer mutually exclusive. We define an *extended* group as a group that contains a set of *conventional* users with an optional set of *shared* users. Each user is a conventional user of only one group but may be a shared user of up to $G-1$ groups. For each extended group, the filter output that corresponds to the set of conventional users is selected for detection; the filter output that corresponds to the set of shared users is used to improve signal detection for the conventional users.

Let K_j' be the dimension of extended group j, of which K_j are conventional users and the remaining $(K_j' - K_j)$ are shared users, and let $\mathbf{P}_j' \in \mathbb{R}^{NK \times NK_j'}$ be the selection matrix for extended group j, i.e., $\mathbf{T}_j' \triangleq \mathbf{T}\mathbf{P}_j' \in \mathbb{C}^{M(NL_c+W-1)\times NK_j'}$ contains the columns of $\mathbf{T}$, corresponding to the users of extended group j (in this paper, the "prime" superscript designates variables associated to extended groups). Let $g_{j,l}^s$ be the index of the lth shared user of extended group j. Without loss of generality, the selection matrix for the extended group can take the form

$$\mathbf{P}_j' = \left[\mathbf{P}_j \,\middle|\, \mathbf{I}_N \otimes [\mathbf{e}_{g_{j,1}^s}, \ldots, \mathbf{e}_{g_{j,(K_j'-K_j)}^s}]\right] \tag{13}$$

where $\mathbf{P}_j$ is the selection matrix for the conventional users only, and $\mathbf{e}_{g_{j,1}^s}$ is the elementary vector of dimension $K \times 1$ with value 1 at position $g_{j,l}^s$. It follows that $\mathbf{M}_j' \triangleq (\mathbf{T}_j'^H\mathbf{T}_j' + \sigma^2\mathbf{I})^{-1} \in \mathbb{C}^{NK_j'\times NK_j'}$ and $\mathbf{z}_j' \triangleq \mathbf{M}_j'^H\mathbf{T}_j'^H\mathbf{x} \in \mathbb{C}^{NK_j'\times 1}$ can be defined as the GRP-STMUD filter and soft output for the extended group j, respectively. Thus, the input of the extended GRP-STMUD filter consists of the matched filter output not only of the conventional users but also of the shared users, which, in effect, increases the observation space dimension of the linear filter.

Similarly, define $\bar{\mathbf{P}}_j' \in \mathbb{C}^{NK\times N(K-K_j')}$ so that $\bar{\mathbf{T}}_j' \triangleq \mathbf{T}\bar{\mathbf{P}}_j'$ contains the columns of $\mathbf{T}$, corresponding to the users that

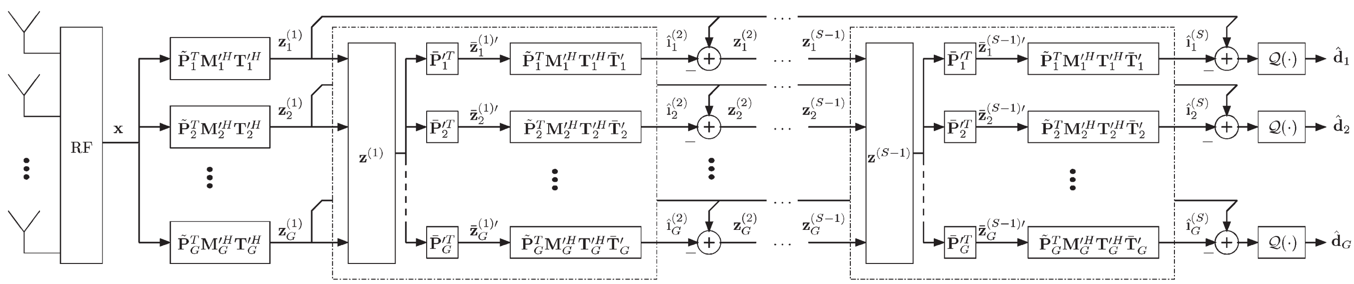


Fig. 1. Block diagram for the proposed MS-GRP-PIC receiver.

do not belong to extended group j. The selection matrix complement $\bar{\mathbf{P}}'_j$ takes the form

$$\bar{\mathbf{P}}'_j = \mathbf{I}_N \otimes \left[\mathbf{e}_{\bar{g}'_{j,1}}, \ldots, \mathbf{e}_{\bar{g}'_{j,(K-K'_j)}}\right] \tag{14}$$

where $\bar{g}'_{j,l}$ is the index of the lth user *not* in extended group j. Finally, let

$$\tilde{\mathbf{P}}_j = \begin{bmatrix} \mathbf{I}_{NKj} \\ \mathbf{0}_{N(K'_j - K_j)\times NK_j} \end{bmatrix} \tag{15}$$

so that $\mathbf{z}_j = \tilde{\mathbf{P}}_j^T \mathbf{z}'_j$ is the vector that contains the soft symbol estimates only for the conventional users of extended group j.

In the proposed MS-GRP-PIC receiver, the vector of symbol estimates is obtained ensuing a series of successive improvements to the interference estimates for the conventional users of each extended group. The proposed approach for PIC is based on the matrix algebraic approach without grouping presented in [7]. Let s represent the PIC stage index, and let $\mathbf{z}^{(s)} \in \mathbb{C}^{NK\times 1}$ be the soft symbol estimate vector at stage s. The interference estimate at stage $s \geq 1$ for the conventional users of extended group j can be expressed as

$$\hat{\imath}_j^{(s)} = \begin{cases} \mathbf{0}_{NK_j}, & s = 1 \\ \tilde{\mathbf{P}}_j^T \mathbf{M}_j'^H \mathbf{T}_j'^H \bar{\mathbf{T}}'_j \bar{\mathbf{z}}^{(s-1)\prime}, & s > 1 \end{cases} \tag{16}$$

where $\bar{\mathbf{z}}_j^{(s-1)\prime} = \bar{\mathbf{P}}_j'^T \mathbf{z}^{(s-1)}$ is the soft symbol estimate vector from the previous stage for the users outside extended group j. Notice that the term $\mathbf{T}_j'^H \bar{\mathbf{T}}'_j$ in (16) corresponds to the cross correlation matrix between the effective signatures associated with the symbols of the users that belong to extended group j and the effective signatures associated with the symbols of the users outside extended group j.

After interference cancellation, the soft symbol estimate vector for users of group j can be expressed as

$$\mathbf{z}_j^{(s)} = \tilde{\mathbf{P}}_j^T \mathbf{M}_j'^H \mathbf{T}_j'^H \mathbf{x} - \hat{\imath}_j^{(s)}, \qquad 1 \leq s \leq S, \tag{17}$$

where S is the maximum number of stages in the PIC.

Fig. 1 illustrates the proposed MS-GRP-PIC receiver in block diagram form. The S-stages MS-GRP-PIC receiver can be summarized by the following equations: the soft symbol estimation update equation, given here for $s \geq 1$ by

$$\mathbf{z}^{(s)} = \mathbf{F}'\mathbf{y} - \mathbf{G}'\mathbf{z}^{(s-1)} \tag{18}$$

where

$$\mathbf{F}' \triangleq \begin{bmatrix} \tilde{\mathbf{P}}_1^T \mathbf{M}_1'^H \mathbf{P}_1'^T \\ \vdots \\ \tilde{\mathbf{P}}_G^T \mathbf{M}_G'^H \mathbf{P}_G'^T \end{bmatrix} \tag{19}$$

$$\mathbf{G}' \triangleq \begin{bmatrix} \tilde{\mathbf{P}}_1^T \mathbf{M}_1'^H \mathbf{T}_1'^H \bar{\mathbf{T}}'_1 \bar{\mathbf{P}}_1'^T \\ \vdots \\ \tilde{\mathbf{P}}_G^T \mathbf{M}_G'^H \mathbf{T}_G'^H \bar{\mathbf{T}}'_G \bar{\mathbf{P}}_G'^T \end{bmatrix} \tag{20}$$

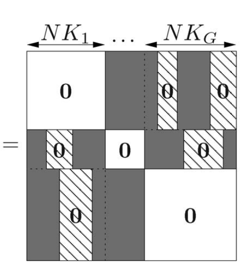
where $\mathbf{y}$ is the matched filter output defined in (3), and $\mathbf{z}^{(0)} = 0$, and the decision equation, which can be expressed as

$$\hat{\mathbf{d}} = \mathcal{Q}(\mathbf{z}^{(S)}). \tag{21}$$

The number of stages S can be determined in real time, for instance, by some metrics based on convergence, but in practice, it is usually constrained by the hardware to a small value.

Notice that the term $\tilde{\mathbf{P}}_j^T \mathbf{M}_j'^H \mathbf{T}_j'^H \bar{\mathbf{T}}'_j \bar{\mathbf{z}}^{(s-1)}$ in (16) consists of the GRP-STMUD filter output response for the conventional users of extended group j to the excitation caused by the users outside the group. Thus, the term $\tilde{\mathbf{P}}_j^T \mathbf{M}_j'^H \mathbf{T}_j'^H \bar{\mathbf{T}}'_j$ in each block row of (20) integrates the corresponding effects of the channel, matched filter, and MUD linear filter into a single matrix. The right matrix product by $\bar{\mathbf{P}}_j'^T$ in (20) causes columns of zeros to be inserted at the location associated with the symbols of the users of extended group j. It effectively transforms the $NK_j \times N(K - K'_j)$ matrix into an $NK_j \times NK$ matrix. Assume, for illustration purposes and without loss of generality, that the user indices are arranged according to their respective grouping so that users $k \in \{1, \ldots, K_1\}$ belong to conventional group 1, users $k \in \{K_1 + 1, \ldots, K_1 + K_2\}$ belong to conventional group 2, and so on. Then, the structure of the matrix $\mathbf{G}'$ used in (18) and defined in (20) can be shown to take the form

$\mathbf{G}' =$



, (22)

where for each block row, the white areas and the dashed areas correspond to the columns of conventional and shared users, respectively. These columns are filled with zeros since they correspond to symbols that do not interfere with the detection for that group.

The system of equations in (16)–(20) represent the general case where groups may contain shared users. In the traditional group-based PIC receiver, there is no user sharing among groups. In that case, the system of equations can be obtained by removing the "prime" superscript in (16)–(18) and noting that since $K'_j = K_j$, we have $\tilde{\mathbf{P}}_j = \mathbf{I}_{NK_j}$ [see (15)]. Furthermore, the structure of $\mathbf{G}$ is similar to what is illustrated in (22) but with no off-diagonal blocks of zeros [20].

IV. Analysis

A. Convergence

Using the above matrix-based model, it can be shown that at the final stage S, the group-based soft symbol estimate vector in (18) can be expressed as

$$\mathbf{z}^{(S)} = \sum_{s=1}^{S} (-\mathbf{G}')^{s-1}\mathbf{F}'\mathbf{y} \tag{23}$$

$$= \left(\mathbf{I} - (-\mathbf{G}')^S\right)(\mathbf{I}+\mathbf{G}')^{-1}\mathbf{F}'\mathbf{y} \tag{24}$$

where (24) is obtained from (23) by using the expression for the convergence of the geometric matrix sum [17]. Assuming for now that the inverse in (24) exists, it can be observed that the convergence properties of the soft symbol estimate is essentially determined by the eigenvalues of $\mathbf{G}'$. The necessary and sufficient condition for convergence as $S \to \infty$ is therefore $\lambda'_{\max} < 1$, with

$$\lambda'_{\max} = \arg\max_{\forall n} |\lambda'_n|, \tag{25}$$

where λ'_n is the nth eigenvalue of $\mathbf{G}'$, $n = 1, \ldots, NK$. Note that since $\mathbf{G}'$ is not Hermitian, its eigenvalues are not necessarily real valued. According to the Geršgorin disc theorem (see, e.g., [19]), each eigenvalue of $\mathbf{G}'$ satisfies at least one of the inequalities

$$\left|\lambda'_n - g'_{pp}\right| \le r'_p, \qquad \text{where } r'_p = \sum_{\substack{q=1\\ q\neq p}}^{NK} \left|g'_{pq}\right| \tag{26}$$

where g'_{pq} is the element at position (p,q) of $\mathbf{G}'$ for p, $q \in \{1, \ldots, NK\}$. Because of the structure of $\mathbf{G}'$ in (22), $g'_{pp} = 0$, $\forall p$. Thus, to guarantee the convergence of (23), we require

$$r'_{\max} < 1, \qquad \text{where } r'_{\max} \triangleq \max_{\forall p} r'_p. \tag{27}$$

It is possible to interpret r'_p as the sum of the absolute values of the residual IGI after filtering, as can be observed from (20) and (26). The actual value taken by each r'_p, therefore, depends on the MAI, on the grouping, and on how much IGI reduction is provided by the GRP-STMUD linear filters.

In general, it is difficult to guarantee convergence, but in practical interference scenarios, the conditions for convergence can be satisfied by using a combination of intelligent grouping algorithms and resource allocation mechanisms. For example, by grouping users with strong mutual interference together, their contribution to r'_p may be reduced. Alternatively, in a typical cellular system, problematic users can be reallocated to a different time slot/frequency. In addition, note that larger groups are, in general, preferable over (more numerous) smaller groups.

Provided the conditions for convergence are met, it can be seen that the sum in (23) converges as $S \to \infty$ to

$$\mathbf{z}^{(\infty)} = \underbrace{(\mathbf{I}+\mathbf{G}')^{-1}\mathbf{F}'}_{\mathbf{M}^H_{(\infty)}}\mathbf{y} \tag{28}$$

where $\mathbf{M}_{(\infty)}$ is defined here as the *total* linear filter.

Proposition 1: The total linear filter $\mathbf{M}_{(\infty)}$ in (28) minimizes the MMSE cost function in (5) and is, thus, equivalent to the full STMUD filter in (6).

Proof: Define $\mathbf{M}' \triangleq \mathrm{diag}(\mathbf{M}'_1, \ldots, \mathbf{M}'_G)$, $\mathbf{D}' = \mathrm{diag}(\mathbf{T}'^H_1\mathbf{T}'_1, \ldots, \mathbf{T}'^H_G\mathbf{T}'_G)$, $\tilde{\mathbf{P}}' = \mathrm{diag}(\tilde{\mathbf{P}}'_1, \ldots, \tilde{\mathbf{P}}'_G)$, and $\mathbf{P}' = [\mathbf{P}'_1, \ldots, \mathbf{P}'_G]$. By using these definitions and observing that $\mathbf{F}' = \tilde{\mathbf{P}}^T\mathbf{M}'^H\mathbf{P}'^T$, the matrix $(\mathbf{I}+\mathbf{G}')$ to be inverted in (28) can then be expressed in the form

$$(\mathbf{I}+\mathbf{G}') = \mathbf{I} + \tilde{\mathbf{P}}'^T\mathbf{M}'^H(\mathbf{P}'^T\mathbf{T}^H\mathbf{T} - \mathbf{D}'\mathbf{P}'^T) \tag{29}$$

$$= \mathbf{I} + \tilde{\mathbf{P}}'^T\mathbf{M}'^H\mathbf{P}'^T(\mathbf{T}^H\mathbf{T}+\sigma^2\mathbf{I}) - \tilde{\mathbf{P}}'^T\mathbf{M}'^H(\mathbf{D}'+\sigma^2\mathbf{I})\mathbf{P}'^T \tag{30}$$

$$= \mathbf{F}'(\mathbf{T}^H\mathbf{T}+\sigma^2\mathbf{I}) \tag{31}$$

where the fact that $\tilde{\mathbf{P}}^T\mathbf{P}'^T = \mathbf{I}$ has been used in (31). If there are no identical extended groups, which is always the case in a practical system, it can be shown that $\mathbf{F}'$ is full rank, and thus, its inverse exists.

According to the principle of orthogonality (e.g., [21]), the necessary and sufficient condition to minimize the MSE is given by $\boldsymbol{\xi} \equiv E[\mathbf{y}\mathbf{e}^H] = \mathbf{0}$, where $\mathbf{e} = (\mathbf{d} - (\mathbf{I}+\mathbf{G}')^{-1}\mathbf{F}'\mathbf{y})$ is the estimation error vector associated with (28). Substituting (31) for $(\mathbf{I}+\mathbf{G}')$ in the expression of the estimation error vector, we obtain

$$\begin{aligned}\boldsymbol{\xi} &= E\left[\mathbf{y}\left(\mathbf{d}^H - \mathbf{y}^H\mathbf{F}'^H(\mathbf{I}+\mathbf{G}')^{-H}\right)\right]\\ &= \mathbf{T}^H\mathbf{T}\left(\mathbf{I} - (\mathbf{T}^H\mathbf{T}+\sigma^2\mathbf{I})\mathbf{F}'^H(\mathbf{I}+\mathbf{G}')^{-H}\right)\\ &= \mathbf{T}^H\mathbf{T}\left(\mathbf{I} - (\mathbf{T}^H\mathbf{T}+\sigma^2\mathbf{I})\mathbf{F}'^H(\mathbf{F}')^{-H}(\mathbf{T}^H\mathbf{T}+\sigma^2\mathbf{I})^{-H}\right)\\ &= 0\end{aligned} \tag{32}$$

where the fact that the inverse of $\mathbf{F}'$ exists is used, resulting in the cancellation of the inner terms. We conclude that $\mathbf{z}^{(\infty)}$ in (28) is the MMSE soft symbol estimate, and thus, $\mathbf{z}^{(\infty)} \equiv \mathbf{z}_{\mathrm{MMSE}}$. ■

It can be observed in (31) that $(\mathbf{I}+\mathbf{G}')$ is full rank, and the inverse in (28) exists. Therefore, (24) is a valid expression for the corresponding geometric sum.

The error between the soft estimate after S stages in (24) and the MMSE soft estimate can be expressed as

$$\mathbf{z}^{(S)} - \mathbf{z}_{\mathrm{MMSE}} = \left(\mathbf{I} - (\mathbf{G}')^S\right)(\mathbf{I}+\mathbf{G}')^{-1}\mathbf{F}')\mathbf{y} - \mathbf{M}^H_o\mathbf{y} \tag{33}$$

$$= \left(\mathbf{I} - (-\mathbf{G}')^S\right)\mathbf{M}^H_o\mathbf{y} - \mathbf{M}^H_o\mathbf{y} \tag{34}$$

$$= (-\mathbf{G}')^S\mathbf{z}_{\mathrm{MMSE}} \tag{35}$$

TABLE I
NUMERICAL COMPLEXITY IN CFLOPS: NUMBER OF STAGES $S = 2$; $N = 1$

Receiver		Complexity	Cost for $(K, G, K_{\max})$ (16,4,4)	(64,8,8)	(256,16,16)
Full STMUD	Cholesky	$\frac{10}{3}NK^3$	1.37×10^4	8.74×10^5	5.59×10^7
	Back sub.	$6NK^2$	1.54×10^3	2.46×10^4	3.93×10^5
		Total (C_A)	1.52×10^4	8.98×10^5	5.63×10^7
MS-GRP-PIC	Cholesky	$\frac{10}{3}NGK_{\max}^3$	8.53×10^2	1.37×10^4	2.18×10^5
	Back sub.	$6NGK_{\max}^2$	3.84×10^2	3.07×10^3	2.46×10^4
	PIC	$6(S-1)NGK_{\max}K$	1.54×10^3	2.46×10^4	3.93×10^5
		Total (C_B)	2.77×10^3	4.13×10^4	6.36×10^5
Ratio of complexity		$\Gamma_c = C_B/C_A$	0.18	0.05	0.01
Largest N_c		See eq. (43)	33	280	2267

where $\mathbf{z}_{\mathrm{MMSE}}$ is given by (4). The normalized norm error can be bounded by

$$\frac{\|\mathbf{z}^{(S)} - \mathbf{z}_{\mathrm{MMSE}}\|}{\|\mathbf{z}_{\mathrm{MMSE}}\|} \leq \left\|(-\mathbf{G}')^S\right\|_2 \leq (\lambda'_{\max})^S \qquad (36)$$

where the inequalities come from properties of the matrix norm [19]. Since from (26), we have $\lambda'_{\max} \leq r'_{\max}$, the convergence rate essentially depends on the residual IGI after filtering. This information can be used together with the observations of Section III to improve the speed of convergence via better grouping.

Finally, it can be concluded that as the number of stages increases, the soft symbol estimate provided by the proposed receiver structure converges in norm to the MMSE symbol estimate provided by the full STMUD receiver. A similar conclusion has recently been obtained in [22] for a SIC group-based receiver structure.

B. MS-GRP-PIC With Weighting

It is shown in [23] that the convergence of linear PIC for CDMA systems can be guaranteed by using well-known generalizations of the Jacobi iteration method. As with traditional PIC, weighting is also necessary to guarantee the convergence of the MS-GRP-PIC receiver. Incorporating the first-order iterative method in [24] to the MS-GRP-PIC receiver, the weighted iterative equation that solves the linear system in (4) can be shown to take the form

$$\mathbf{z}_\tau^{(s)} = \begin{cases} \mathbf{0}, & s = 0 \\ \tau_s \mathbf{F}'\mathbf{y} + ((1-\tau_s)\mathbf{I} - \tau_s \mathbf{G}')\,\mathbf{z}_\tau^{(s-1)}, & 1 \leq s \leq S \end{cases} \qquad (37)$$

where $\mathbf{z}_\tau^{(s)}$ is the vector of soft symbol estimate at stage s for the weighted iterative method, and τ_s is the iterative weighting factor at stage s. Notice that when $\tau_s = 1\ \forall s$, the weighted equation in (37) is equivalent to (18).

Several approaches for selecting the set of weighting factors exist. In practice, since the maximum number of stages is likely to be known due to hardware limitations, it may be advantageous to choose the weighting factors to minimize the norm of the error after S stages, i.e., $\|\mathbf{z}_\tau^{(S)} - \mathbf{z}_{\mathrm{MMSE}}\|$. This can be achieved by using the Chebyshev iterative method, in which case, the set of weighting factors can be expressed for $1 \leq s \leq S$ as [24]

$$\frac{1}{\tau_s} = \frac{\alpha'_{\max} - \alpha'_{\min}}{2}\cos\theta_s + \frac{\alpha'_{\max} + \alpha'_{\min}}{2} \qquad (38)$$

$$\theta_s \triangleq \frac{2(s-1)+1}{2S}\pi, \qquad (39)$$

where $\alpha'_{\max}$ and $\alpha'_{\min}$ are parameters that are related to the maximum and minimum eigenvalues of $\mathbf{G}'$, respectively.

Estimation of the eigenvalues of $\mathbf{G}'$ is a computationally intensive task. Fortunately, the Chebyshev iterative method is less sensitive to eigenvalue estimation errors than other iterative methods. As a result, in this paper, a computationally simple set of approximations is proposed, where the estimates for $\alpha'_{\max}$ and $\alpha'_{\min}$ are given, respectively, by

$$\hat{\alpha}'_{\max} = a + b\|\mathbf{G}'\|_F, \qquad (40)$$

$$\hat{\alpha}'_{\min} = 1 \qquad (41)$$

where $\|\cdot\|_F$ is the Frobenius norm [25], and the values for a and b are obtained from empirical data. The computer simulation results obtained in Section V demonstrate that the convergence of the MS-GRP-PIC receiver with weighting does not significantly suffer from using these estimates when compared to using the exact values of $\alpha'_{\max}$ and $\alpha'_{\min}$.

C. Complexity

The complexity characteristics of the proposed receiver can be advantageous, particularly under a time-varying channel with a large number of users, which is typical of mobile communications. Table I shows the complexity in terms of complex floating-point operations (CFLOPS) of the different parts of the computation of the solution for both full STMUD and MS-GRP-PIC receivers for the practical case of $S = 2$ stages and blocks $N = 1$. The common operations, such as matched filtering, are not listed in the table. In a practical hardware implementation, the design must take into consideration the maximum number of users; in computing the complexity, it is assumed here that $K = L_c$.

As shown in the table, the complexity gain varies from one scenario to the next, and it is independent of the block size N. If we consider the case $G = K_{\max} = \sqrt{K}$, then it can be shown that the ratio of the MS-GRP-PIC complexity over the full STMUD complexity approaches

$$\Gamma_c = \frac{1 + 9/5(S-1)}{K} \tag{42}$$

for large K. The table shows that the reduction in complexity from using the proposed approach can be significant; for the smallest value of K considered, the MS-GRP-PIC has approximately one fifth the complexity of the full STMUD. The complexity reduction is even more significant for larger values of K.

It can also be observed in Table I that most of the savings occur when computing the Cholesky factors. Since in a time-varying channel those factors need to be regularly recomputed, the proposed approach is computationally advantageous. Indeed, it can be shown for large K that it is always computationally advantageous to use the proposed approach when

$$\dot{N}_c < \frac{5K^2}{9\left(\sqrt{K} + (S-2)K\right)}, \qquad S \geq 2 \tag{43}$$

where N_c is the filter *reuse factor*, i.e., the number of consecutive blocks for which the channel and, thus, the Cholesky factors remain constant. The maximum N_c for each considered scenario is shown in Table I.

Finally, note that the cost associated to the grouping algorithms, in practice, represents a negligible portion of the MS-GRP-PIC total cost. Indeed, it can be shown that the proposed algorithms for mutually exclusive and non-mutually exclusive grouping in [16] require sorting of $K/2$ elements. Assuming the case $G = K_{\max} = \sqrt{K}$, the ratio of grouping cost to MS-GRP-PIC receiver cost shown in Table I can be approximated for large K as

$$\Gamma_g \approx \frac{K^2}{24NN_c(S-1)}. \tag{44}$$

Assuming a typical block size of $N = 100$, $S = 2$ MS-GRP-PIC iterations, and a small reuse factor of $N_c = 1$, the complexity associated with the grouping algorithm is approximately 6% of the MS-GRP-PIC cost for $K = 16$ users.

In practice, the sorting search space dimension can be significantly reduced from the worst case of $K/2$ elements. This can be achieved, for example, by reusing the calculations and/or groupings from the previous frames. Alternatively, a two-stage approach can be used. First, a preliminary and partial grouping based on thresholding is first conducted, and then, the grouping algorithms in [16] are used to complete the grouping, taking advantage of the smaller number of elements to sort. Therefore, in practice, the cost associated with the grouping algorithms can be considered negligible.

TABLE II
CHANNEL POWER-DELAY PROFILE

Tap #	Rel. power (dB)
1	0.0
2	-1.0
4	-9.0
5	-10.0
8	-15.0
11	-20.0

V. COMPUTER EXPERIMENTS

A. *Methodology*

For the computer experiments, we consider the received signal model of (1) for the uplink of a generic WCDMA system with short codes. The users have orthogonal spreading codes of length $L_c = 16$. The frequency-selective channel, which destroys code orthogonality, consists of six multipaths with a power-delay profile following the vehicular channel type A [26]. For a transmission rate of $1/T_c = 3.84$ MHz, the total channel spread in terms of T_c is $W = 11$. Table II shows the channel taps and their relative power. For each channel realization, the direction of arrival (DOA) of the different paths are randomly selected; the main path has DOA θ_0 that is uniformly distributed within the sector width of 120°, and the DOAs for the other paths are uniformly distributed within $[\theta_0 + \Delta\theta, \theta_0 - \Delta\theta]$, with $\Delta\theta = 15°$. Ideal power control is assumed so that $\|\mathbf{v}_k\|^2 = 1$, $\forall k$, and the SNR for each user becomes SNR $= 1/\sigma^2$.

For illustration purposes, two main configurations for grouping are considered for the proposed MS-GRP-PIC receiver. In grouping configuration 1, the receiver has a maximum number of groups $G = 4$ and a maximum number of users per group of $K_{\max} = 4$, and in grouping configuration 2, the receiver has $G = 16$ and $K_{\max} = 1$. Thus, for the latter case, there is actually no MUD.

Each experiment is repeated over 30 different *scenarios*, which consist of a fixed channel realization for each user and the corresponding grouping, which is also fixed, supplied by the algorithm proposed in [16]. In the cases where random grouping (RG) is considered, the grouping is randomly generated using a uniform distribution. For each scenario, the exact BER is calculated using (46), i.e., the expression for the error probability described in the Appendix, for all the considered users and receiver structures.

To compute the signal-to-interference ratio (SIR) for user k, it is assumed that the signal contributions from the users within its extended group improve the detection (because of MUD), while the signal contributions from the users outside the group interfere. The SIR is measured at the output of the matched filter to take into consideration the effect of the CDMA codes and channel coefficients. Neglecting the ISI in the SIR calculation, the signal and interference powers for user k are computed from the projection of each contribution onto the space spanned by $\mathbf{v}_k$. Let $\mathcal{G}'_j$ be the set of indices that correspond to the users of extended group j, and let $\bar{\mathcal{G}}'_j$ be the complement of $\mathcal{G}'_j$, so that elements of $\bar{\mathcal{G}}'_j$ correspond to the indices of the users not in extended group j. Without loss of generality, let user k be

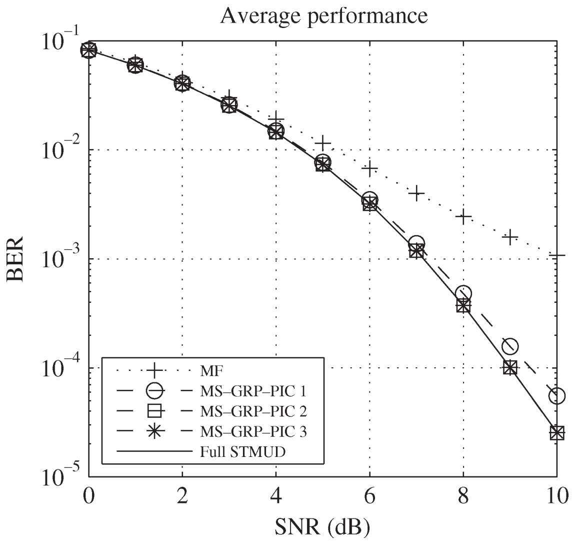


Fig. 2. BER performance of the receiver structures for moderate interference conditions averaged over 30 scenarios with $G = 4$, $K_{\max} = 4$, $LF = 12/16$, $M = 4$, and grouping configuration 1.

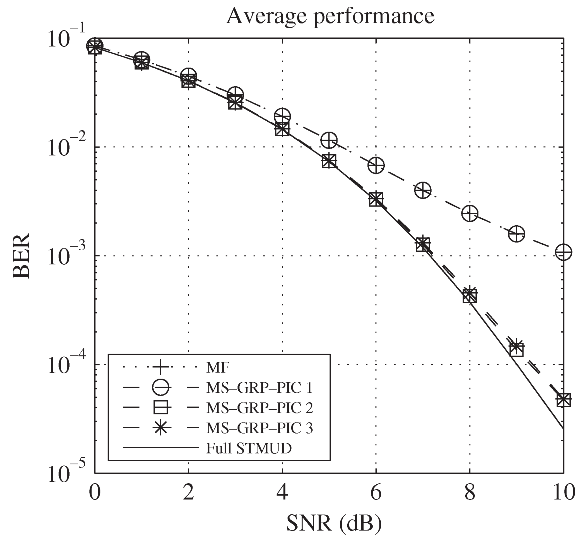


Fig. 3. BER performance of the receiver structures for moderate interference conditions averaged over 30 scenarios with $G = 4$, $K_{\max} = 4$, $LF = 12/16$, $M = 4$, and grouping configuration 2.

a conventional user of group j so that $k \in \mathcal{G}'_j$. Then, it can be shown that the SIR for user k can be expressed as

$$\mathrm{SIR}_k \triangleq \frac{\|\mathbf{v}_k\|^4}{\sum_{l \in \bar{\mathcal{G}}'_j} |\mathbf{v}_k^H \mathbf{v}_l|^2}. \quad (45)$$

The simulations are performed for different sets of parameters of interest, including, in particular, the number of antenna elements M, the *loading factor* $LF = K/K_{\max}$, and the grouping scenario.

B. Results and Discussion

Fig. 2 shows the BER performance averaged over the different scenarios and over all users for the matched filter receiver; the proposed MS-GRP-PIC receiver with $S = 1,\ 2,\ \text{and } 3$; and the full STMUD receiver defined by (6). The receiver hardware is assumed to have $M = 4$ antenna elements and uses grouping configuration 1, with $G = 4$ and $K_{\max} = 4$, for a total of $K_{\max}G = 16$ receiver resources to support, in this case, a loading factor of $LF = 12/16$ (i.e., $K = 12$ users). As expected, the MF yields the poorest performance, whereas the full STMUD provides the best BER. It can be observed that, on average, the proposed receiver requires, in this case, only two stages to provide essentially the same BER performance as the full STMUD. Note that with only $S = 1$, the BER performance is very close to that of the full STMUD, particularly at a lower SNR, where the interference is no longer the dominant source of signal degradation.

In Fig. 3, the MS-GRP-PIC receiver uses grouping configuration 2, with $G = 16$, and $K_{\max} = 1$. Since there is no MUD in this configuration, the MS-GRP-PIC with $S = 1$ is identical to the MF receiver. It can also be observed that at a high SNR, even with $S = 3$, the BER performance of the MS-GRP-PIC with no MUD does not reach the BER performance provided by the full STMUD.

TABLE III
AVERAGE SIR (IN DECIBELS)

Grouping configuration	LF	M = 1	2	4	8
Grouping config. #1:	11/16	19.7	23.4	28.1	33.9
Max. 4 groups of 4	12/16	18.2	22.1	27.0	31.9
($G = 4$, $K_{\max} = 4$)	13/16	17.5	21.7	26.4	31.7
Grouping config. #2:	11/16	13.9	17.7	21.8	26.4
Max. 16 groups of 1	12/16	12.3	16.5	20.7	25.4
($G = 16$, $K_{\max} = 1$)	13/16	12.2	16.5	20.7	25.2

Next, we consider the convergence rate of the proposed receiver for the two grouping configurations. Each configuration leads to a different SIR for different loading factors LF and numbers of antenna elements M.

Table III shows the average SIR for all users in the system for the different grouping configurations. The results clearly indicate that the grouping configuration and the number of antenna elements have a significant effect on the SIR. There is a difference of up to 13 dB in SIR between the cases with one and eight antenna elements: an improvement that is essentially achieved by increasing the observation signal dimension. The improvement in SIR between grouping configurations 1 and 2 is approximately 6 dB, on average. The loading factor also affects the SIR; a smaller LF leads to a higher SIR. From the values shown in Table III, it can be observed that the difference in the SIRs for $LF = 11/16$ and $LF = 13/16$ varies from 1.1 to 2.2 dB, depending on M.

For low-SIR cases, such as when $M = 1$, the MS-GRP-PIC may not converge, and weighting is required, as discussed in Section IV-B. Fig. 4 shows the BER convergence for the MS-GRP-PIC receiver with weighting, averaged over the different scenarios and over all users for different weighting strategies. As can be observed, the MS-GRP-PIC receiver with weighting converges to the full STMUD solution. It can also be noted that the estimated Chebyshev weights obtained using (40) and (41) perform very well when compared to the ideal case.

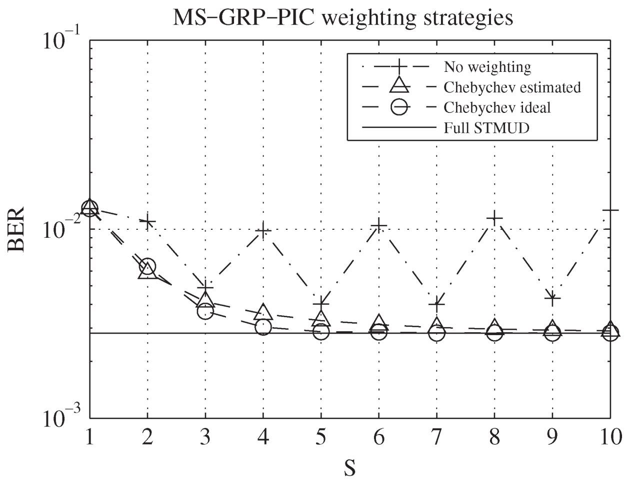


Fig. 4. MS-GRP-PIC weighting strategies for $M = 1$, $LF = 12/16$, SNR $= 8$ dB, and grouping configuration 2.

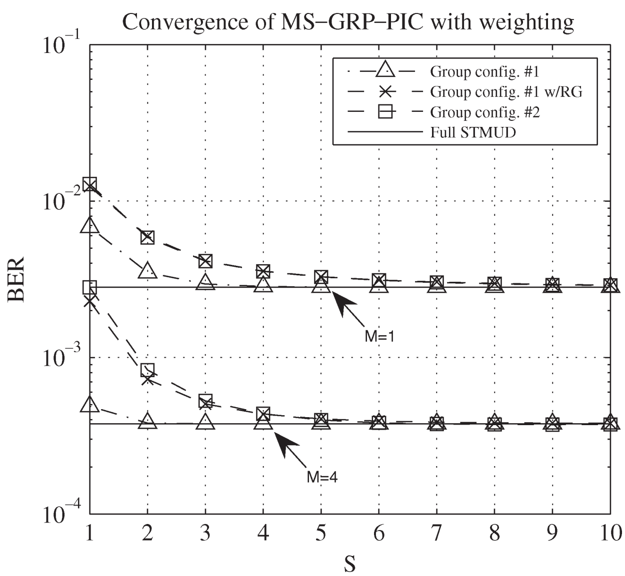


Fig. 5. BER convergence of the MS-GRP-PIC receiver with weighting for different grouping configuration at SNR $= 8$ dB with $LF = 12/16$.

Fig. 5 shows the convergence in the BER of the MS-GRP-PIC receiver with weighting, also averaged over the different scenarios and over all users, as the number of stages S is increased for different grouping configurations for both $M = 1$ and $M = 4$ at a fixed SNR of 8 dB. For a moderate average SIR of 27.2 dB with $M = 4$, the convergence is relatively fast i.e., approximately two stages for the proposed MS-GRP-PIC receiver with weighting for grouping configuration 1 and five stages for grouping configuration 2. Note that similar results have been obtained for $M = 8$, but for conciseness, they are not shown here. For the stronger interference case (i.e., average SIR $=$ 20.7 dB) with $M = 1$, the proposed approach with grouping configuration 1 converges in three stages.

It is interesting to note in Fig. 5 that for both $M = 1$ and $M = 4$, the convergence when using an RG is significantly slower than when using the grouping algorithm in [16]. In both cases, the convergence of grouping configuration 1 with random grouping approaches the convergence of grouping configuration 2, which essentially does not benefit from grouping at all. The parameters for estimating the Chebyshev weighting coefficients using (40) for the results shown in Figs. 4 and 5 are given in Table IV.

TABLE IV
CHEBYSHEV WEIGHTING COEFFICIENTS ESTIMATION PARAMETERS FOR $LF = 12/16$ AND SNR $= 8$ dB

Group config.	Param.	M = 1	M = 4
#1	a	0.2130	0.0701
	b	0.6606	0.6327
#2	a	0.0305	0.0710
	b	0.6625	0.6412

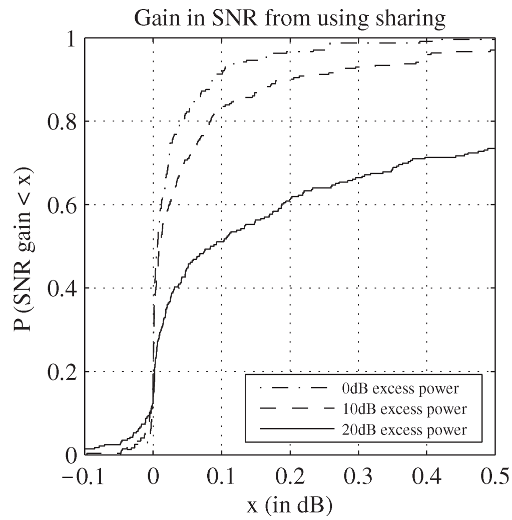


Fig. 6. CDF of SNR gain at BER $= 10^{-3}$ when using sharing for $G = 4$, $K_{\max} = 4$, $LF = 11/16$, $M = 4$.

Sharing usually benefits only a few users, depending on the scenario, and this is mainly due to the low percentage of users that are actually being shared [16]. To illustrate, consider grouping configuration 1 with $LF = 11/16$ (corresponding to $K = 11$ users). Because there are five extra resources that are available here $(GK_{\max} - K)$ for sharing, and since the maximum group size is $K_{\max} = 4$, the conventional grouping algorithm (with no sharing) will usually assign two full groups and one group with three users. This leaves space to essentially one shared user in that group and none for the others. Therefore, in general, only a few users can actually benefit from user sharing.

Fig. 6 shows the empirical cumulative distribution function (CDF) of the SNR gain, which is measured at BER $= 10^{-3}$, from exploiting user sharing under the presence of 20% of the users with excess power of 0, 10, and 20 dB. Only the users that can benefit from sharing, i.e., the users that belong to an extended group, are considered in Fig. 6. To obtain a relatively smooth CDF, the data are accumulated this time over 100 different scenarios. As can be observed, a large number of those users significantly benefit from sharing, particularly in the presence of strong interferers: At 20 dB of excess power,

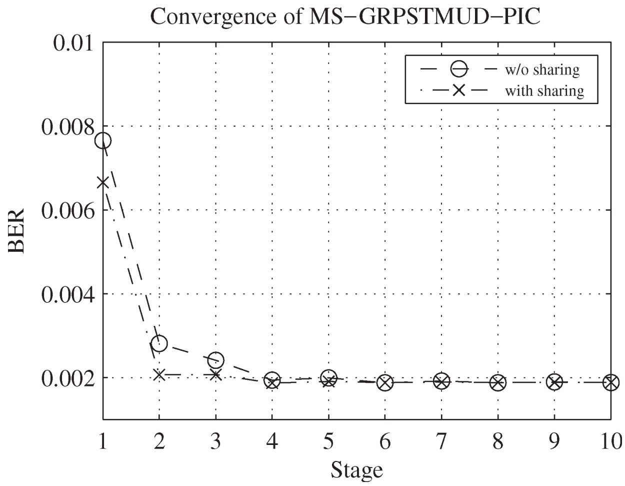


Fig.7. BER convergence for MS-GRP-PIC for users in extended groups (approximately 2% of users) for $G = 4$, $K_{\max} = 4$, $LF = 12/16$, and $M = 1$ at SNR = 8 dB.

approximately 25% of the users that are considered obtain an improvement over 0.5 dB in SNR.

User sharing can also help the convergence of the users in extended groups. Fig. 7 shows the BER convergence for the MS-GRP-PIC with and without user sharing. The average here is performed over the BER that corresponds to users that belong to extended groups only, which, in this case, corresponds to approximately 2% of the total users that were considered over the 100 different scenarios. The same experiment is repeated, with no user sharing, for comparison purposes. The considered case consists of a relatively strong interference scenario, with $M = 1$, $LF = 12/16$, and up to $G = 4$ groups of $K_{\max} = 4$. The results show the advantage of using sharing when possible. It takes, on the average, four stages to converge when user sharing is enabled; it takes over six stages otherwise.

VI. CONCLUSION

In this paper, we have proposed a multistage approach to group-based MUD to reduce the IGI. The proposed structure is shown to converge to the full MMSE receiver and to be computationally advantageous when the filter coefficients need to be regularly recomputed, as would be the case in a time-varying channel. In moderate interference scenarios, the proposed MS-GRP-PIC receiver rapidly converges, and the BER performance is close to that of the full STMUD receiver. Furthermore, we show that user sharing can improve the convergence of the MS-GRP-PIC receiver and can also provide a significant gain in the presence of strong interferers.

APPENDIX

The performance of the proposed receiver is evaluated using BER. For each block, N symbols are transmitted per user, with a possibly different probability of error for each. Consider the general case where $z_{k,n} \triangleq \mathbf{m}_{k,n}^H \mathbf{y}$ is the soft estimate for the nth symbol of user k and where $\mathbf{m}_{k,n}$ is the associated total linear filter vector. Then, the error probability for BPSK transmission can be expressed as

$$P_e^{(k,n)} = P\left(\mathrm{Re}(z_{k,n}) < 0 | b_{k,n} = 1\right) \tag{46}$$

$$= \frac{1}{2^{NK}} \sum_{\substack{\forall b \\ b_{k,n}=1}} \mathrm{erfc}\left(\frac{\mathrm{Re}(z_{k,n})}{\sigma\sqrt{2\mathbf{m}_{k,n}^H \mathbf{T}^H \mathbf{T} \mathbf{m}_{k,n}}}\right) \tag{47}$$

where $b_{k,n}$ is the nth symbol that is transmitted by user k, and $\mathrm{erfc}(\cdot)$ is the complementary error function [27]. The BER expression in (47) can be used for any linear receiver by replacing the total linear filter $\mathbf{m}_{k,n}$ with the appropriate vector [e.g., columns of $\mathbf{M}_{(\infty)}$ for the receiver described by (28)].

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