Collaborative Uplink Transmit Beamforming With Robustness Against Channel Estimation Errors

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***Abstract*—We consider the uplink of collaborative wireless communication systems, where multiple relay terminals decode the signal of a nearby user and forward it to a distant single-antenna base station. We present a collaborative uplink transmit beamforming strategy that can be employed at the relay terminals to provide robustness against uncertainties in the channel state information. The proposed beamforming scheme is obtained using the available knowledge about the second-order statistics of the channel and the possibly erroneous channel state information. The beamforming weight vector is derived by minimizing the total transmitted power subject to a constraint that preserves the received signal at the base station for all the channel realizations within a prescribed uncertainty set. We present two beamforming algorithms based on different mathematical descriptions of the uncertainty set. Both algorithms can be applied to line-of-sight (LOS) propagation and flat-fading channels. In the first algorithm, the robust beamforming vector is computed at the base station using the uplink data and fed back to the cooperating relay terminals. This centralized processing scheme allows any additional convex constraint to be easily incorporated into the beamforming strategy. In the second algorithm, the beamforming vector of each terminal is locally computed using the available knowledge about the terminal's channel and a single parameter (Lagrange multiplier) that is broadcast from the base station to all the cooperating terminals. Simulation results are presented, showing the superior performance of our proposed algorithms compared with classical transmit beamforming techniques in both LOS propagation and flat-fading channels.**

***Index Terms*—Array signal processing, convex optimization, cooperative systems, distributed beamforming.**

I. Introduction

MULTIPLE antenna signaling techniques have proven to be very effective in improving the performance of wireless communication systems [1]–[3]. Transmit beamforming is one of the main approaches to exploiting the spatial characteristics of the channel, as it is capable of providing spatially matched transmission and reducing interference. The power gain that is achieved by transmit beamforming is proportional to the number of transmit antennas. Unfortunately, it is not usually practical to mount a large number of antennas on a mobile terminal due to various limitations such as size and cost. Collaborative beamforming techniques have recently been introduced to overcome this limitation and achieve multiple antenna gain in wireless communication networks [4].

Collaborative transmit beamforming can also be applied in wireless relay networks. Multihop relaying is one of the major modifications, under study, for wireless cellular networks to increase their capacity, coverage, and throughput [5]. Collaborative transmit beamforming can be used at the relay terminals to transmit the signal of a nearby user to the distant base station, hence enabling this user to achieve the high data rates that are envisioned for fourth-generation wireless systems.

The main idea behind collaborative beamforming is to consider a group of nearby terminals as forming one virtual antenna array. Therefore, with these terminals synchronously transmitting some common information, a spatial beam can be formed in the direction of the targeted base station. Collaborative beamforming techniques differ from their classical counterparts due to their distributed nature, that is, the array elements are distributed among different terminals and not located in a common processing unit. Hence, only a limited amount of information can be shared between the cooperating terminals with possible errors and delays [6]. Therefore, collaborative beamforming algorithms are required that are *robust* against channel estimation errors. In [7], a distributed beamforming architecture was proposed, where a master transmitter coordinates the synchronization of other (slave) transmitters. Matched filtering, i.e., classical beamforming, is applied by the cooperating nodes to transmit the common information signal to the targeted base station. However, the robustness of the beamforming vector against any remaining synchronization or channel estimation errors might be insufficient.[1]

Many adaptive beamforming algorithms have been recently proposed to provide robustness against various mismatches in the channel state information for line-of-sight (LOS) propagation environments, e.g., [8]–[11]. These algorithms are based on preserving all the received signals within a predefined uncertainty set centered around the channel vector estimate. A worst-case performance optimization approach was adopted in all these approaches, and the robust beamforming problem was converted to a convex optimization problem [12]. The problem of robust transmit beamforming for flat-fading channels was considered in [13]–[15]. A worst-case performance optimization approach was also adopted in [13] and [14] to provide robustness against mismatches in the channel vector and the

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[1]The beamforming algorithms that are presented in this paper can also be applied instead of matched filtering in the distributed beamforming protocols presented in [7].

channel covariance matrix, respectively. This approach has the advantage of avoiding any statistical assumptions about the channel state and/or the mismatches such as those made in [15]. However, besides the limitations on the channel model, none of the above beamforming algorithms can be directly applied to collaborative beamforming scenarios where the array elements are distributed among the cooperating terminals, with each terminal having an estimate (together with its associated uncertainty) of its channel vector only. This limits the ability of these approaches to exploit the good estimates that some terminals may have of their channels. Moreover, all the above beamforming algorithms are based on centralized processing, and therefore, they are not suitable for some collaborative transmit beamforming scenarios, where the beamforming vector has to be locally computed by each of the cooperating terminals.

In this paper, we consider the problem of robust collaborative beamforming for uplink transmission. We propose a beamforming framework based on worst-case performance optimization, which is applicable to LOS propagation and flat-fading channels. First, we present a unified signal model for both types of channels. Our signal model divides the available channel information into two parts—the first part is perfectly known by the terminals, and the second is assumed to exist within a predefined norm-bounded uncertainty set. For example, in the case of LOS propagation, each terminal has perfect knowledge of its array manifold, i.e., the location of its sensors relative to a local reference point and the direction of the targeted base station, and the uncertainty in the terminal location and/or phase synchronization errors can be bounded by a constant. Also, in the case of flat-fading channels, the channel covariance matrix of each terminal can be obtained with high accuracy using the downlink measurements [16], [17], and the norm of the error in the channel state of each terminal can be bounded by a constant that depends on the coherence time of the channel. Using this unified signal model, we formulate the robust transmit beamforming problem as minimizing the total transmitted power by the cooperating terminals subject to a constraint that preserves the received signal at the targeted base station for all the channel vectors within a predefined uncertainty set. Our uncertainty set differs from that used in [9]–[11] in that a distinct robustness parameter is used to bound the error in the channel estimate of each terminal. As a result, the proposed beamformer is capable of optimally utilizing the good estimates that some terminals might have of their channels. In contrast, earlier robust beamforming approaches use a single robustness parameter to bound the error in the channel vector of the whole array, as the array elements are assumed to be located within a single processing unit.

We develop two different transmit beamforming algorithms based on different mathematical descriptions of the channel uncertainty set. Both formulations lead to convex optimization problems that can be efficiently solved with polynomial complexity [12]. In the first algorithm, the beamforming weight vector is computed at the targeted base station by solving a second-order cone program (SOCP) [18]. The beamforming vector is then fed back to the cooperating terminals to be used in subsequent transmissions. One of the advantages of this formulation is that any additional convex constraint on the weight vector can be easily incorporated into the robust beamforming problem. Some examples of these constraints include constraining the maximum power that is transmitted by each terminal and suppressing or reducing the interference caused to nearby base stations. In the second algorithm, the base station computes only one parameter (Lagrange multiplier) using the uplink data and feeds it back to the cooperating terminals through a common broadcasting channel. Each terminal can then locally compute its beamforming vector using the local information available about its channel only. Hence, this algorithm is well suited to collaborative transmission scenarios, where a limited amount of information has to be shared among the cooperating terminals. We show that this algorithm is equivalent to an optimum power-allocation strategy among the eigen beams of the channels of the cooperating terminals based on the power of each eigen beam and the uncertainty in the channel estimate of each terminal. We present some numerical simulations showing the superior performance of our proposed algorithms compared with classical nonrobust beamforming in terms of the transmission efficiency (the ratio between the received SNR and the transmitted power) and the symbol error rate (SER).

This paper is organized as follows. In Section II, we present the unified signal model for LOS propagation and fading channels and review some classical beamforming techniques that are optimal when the channel vector is known at the transmitter. Sections III and IV contain the derivation of the proposed robust transmit beamforming algorithms. Section V presents numerical simulation results that compare the performance of the new beamformers with the existing ones for various mismatch scenarios, and Section VI contains the concluding remarks. Finally, the Appendix contains parts of the derivation of our decentralized robust beamforming algorithm.

II. Signal Model

We consider the uplink of a narrowband wireless communication system, where M terminals are collaboratively transmitting a common signal to the same base station, as shown in Fig. 1. We assume that the information signal is *a priori* shared by the cooperating terminals, and we focus our attention on the uplink collaborative beamforming problem [4].[2] The mth terminal is equipped with a k_m-element antenna array. The base station is assumed to have a single antenna.[3] Hence, the received baseband signal at the base station at the ith time instant can be written as

$$y(i) = \sum_{m=1}^{M} \boldsymbol{w}_m^H \boldsymbol{h}_m s(i) + v(i) \tag{1}$$

$$= \boldsymbol{w}^H \boldsymbol{h} s(i) + v(i) \tag{2}$$

[2]We consider a decode-and-forward cooperative relaying scenario in which the relay terminals are much closer to the source than to the base station. Their close proximity to the source allows them to detect its information symbols with a very low probability of error.

[3]For LOS propagation environments, all the algorithms that are presented in this paper can be directly extended to the case where the base station is equipped with multiple antennas. This is due to the plane wave assumption, where all the antennas of the base station are located in the direction θ_m relative to the mth terminal. Matched filtering (or adaptive beamforming in the presence of interference) can be used for reception at the base station.

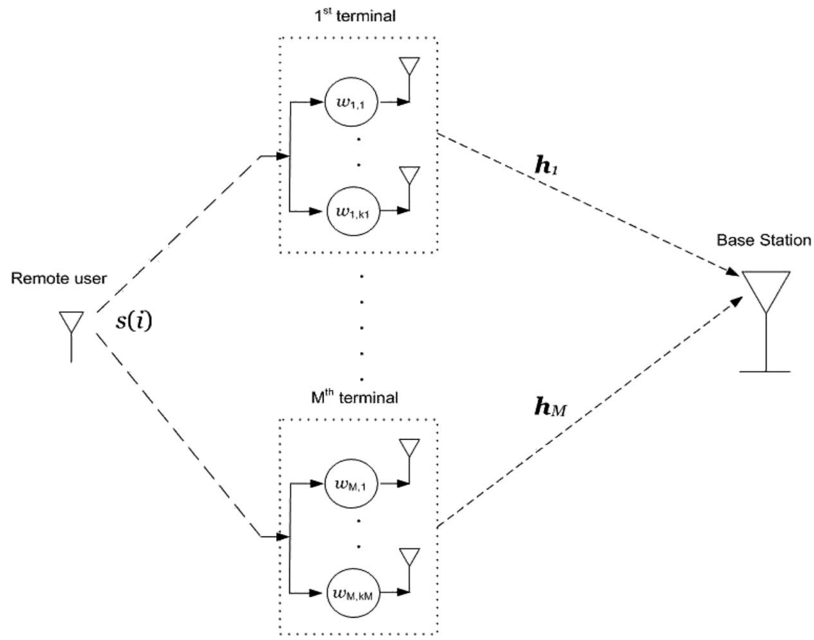

Fig. 1. System model.

where $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively, $s(i)$ is the common information signal that is transmitted by the M cooperating terminals, $\boldsymbol{h}_m$ is the $k_m \times 1$ vector containing the coefficients of the channel from the mth terminal to the base station, $\boldsymbol{w}_m$ is the $k_m \times 1$ beamforming vector of the mth terminal, and $v(i)$ is the white circular Gaussian noise with zero mean and variance σ_v^2. The $K \times 1$ stacked channel vector $\boldsymbol{h}$ is given by $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_M^T]^T$, where $K = \sum_{m=1}^{M} k_m$, and the $K \times 1$ vector $\boldsymbol{w} = [\boldsymbol{w}_1^T, \ldots, \boldsymbol{w}_M^T]^T$ is the corresponding beamforming vector.

The channel vectors $\{\boldsymbol{h}_m\}_{m=1}^{M}$ depend on the deployment scenario. We will present a unified signal model that represents two common wireless communication environments, namely, free space propagation, i.e., LOS communication, and flat-fading propagation environments.

A. LOS Propagation Environment

In the case of LOS propagation, the channel vector of the mth terminal can be written as [19], [20]

$$\boldsymbol{h}_m = e^{-j\phi_m} \boldsymbol{a}_m(\theta_m) \tag{3}$$

where

$$\boldsymbol{a}_m(\theta_m) = [g_{m,1}(\theta_m), g_{m,2}(\theta_m) \exp\left(-j2\pi f_0 \tau_{m,2}(\theta_m)\right) \\ \ldots, g_{m,k_m}(\theta_m) \exp(-j2\pi f_0 \tau_{m,k_m})(\theta_m)]^T \tag{4}$$

f_0 is the carrier frequency, $g_{m,i}(\theta_m)$ is the radiation gain[4] of the ith antenna of the mth terminal toward the base station (located in the direction θ_m), $\tau_{m,i}(\theta_m)$ is the propagation delay of the signal that is transmitted from the ith antenna of the mth terminal toward the base station relative to that transmitted from the first antenna of the mth terminal, $\phi_m = 2\pi f_0 T_m$, and T_m is the propagation delay of the signal that is transmitted from the first antenna of the mth terminal relative to that transmitted from a virtual antenna that is located at a common reference point. We can write the stacked channel vector as

$$\boldsymbol{h} = \boldsymbol{V}\boldsymbol{n} \tag{5}$$

where $\boldsymbol{n} = [e^{-j\phi_1}, \ldots, e^{-j\phi_M}]^T$ is the so-called *channel realization driving vector*, the $K \times M$ matrix $\boldsymbol{V}$ is given by

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{a}_1(\theta_1) & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{a}_2(\theta_2) & \mathbf{0} & \vdots \\ \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \ldots & \mathbf{0} & \boldsymbol{a}_M(\theta_M) \end{bmatrix} \tag{6}$$

and $\mathbf{0}$ is a vector of appropriate dimension containing zeros. We can see from (5) that the channel vector can be decomposed into the product of a matrix $\boldsymbol{V}$, whose columns are the local array manifold vectors of the M terminals, and a vector $\boldsymbol{n}$ containing the relative phase offsets of these terminals.

Ochiai *et al.* [4] describe some of the practical problems that occur in collaborative networks in the case of LOS propagation environments. One of the major problems in these networks is due to the phase estimation errors between different relay terminals, which can be attributed to synchronization errors and/or uncertainty in the relative locations of the terminals. It is reasonable to assume that the array manifold of each terminal is calibrated and that the terminals can determine the direction of the targeted base station with enough accuracy, that is, the array manifold vectors of different terminals are known. Therefore, we can model the mismatch in the stacked channel

[4]The radiation gain in a certain direction is the ratio of the square root of the radiation intensity in that direction to the square root of the average intensity over all directions [20]. For omnidirectional antennas, the radiation gain is equal to unity.

vector in (5) as an error in the channel realization driving vector $\boldsymbol{n}$ [19], i.e.,

$$\boldsymbol{h} = \boldsymbol{V}(\hat{\boldsymbol{n}} + \boldsymbol{\Delta}) \tag{7}$$

where $\hat{\boldsymbol{n}} = [e^{-j\hat{\phi}_1}, \ldots, e^{-j\hat{\phi}_M}]^T$ is the estimate of $\boldsymbol{n}$, $\{\hat{\phi}_m\}$ are the presumed phase offsets of the M terminals, and $\boldsymbol{\Delta}$ is the $M \times 1$ vector containing the phase errors of the M terminals.

B. Flat-Fading Propagation Environment

In the case of multipath flat-fading channels, the channel vector of the mth terminal can be written as [21]

$$\boldsymbol{h}_m = \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{n}_m \tag{8}$$

where $\boldsymbol{R}_m = E\{\boldsymbol{h}_m \boldsymbol{h}_m^H\}$ is the covariance matrix of the channel vector of the mth terminal, $E\{\cdot\}$ denotes the statistical expectation, and $\boldsymbol{n}_m$ is a $k_m \times 1$ vector of independent zero mean, unit variance, and circular Gaussian random variables. The channel covariance matrix captures the spatial properties of the propagation environment, i.e., the mean and angular spread of the power-angle profile, and is given by [22]

$$\boldsymbol{R}_m = \frac{K_r}{K_r+1} \boldsymbol{a}_m(\theta_{0,m}) \boldsymbol{a}_m^H(\theta_{0,m}) + \frac{1}{K_r+1} \int f(\theta) \boldsymbol{a}_m(\theta) \boldsymbol{a}_m^H(\theta) d\theta \tag{9}$$

where K_r is the Rician K-factor, $\theta_{0,m}$ is the direction of the LOS component of the channel of the mth terminal, and $f(\theta)$ is the power density function (power-angle profile) of the channel. Note that we have assumed that the channel vectors of different terminals are independent, that is, the terminals are assumed to be well separated in space.

Using (8), we can model the stacked channel vector $\boldsymbol{h}$ by the same model in (5) with the $K \times K$ matrix $\boldsymbol{V}$ given by

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{R}_1^{\frac{1}{2}} & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_2^{\frac{1}{2}} & \boldsymbol{0} & \vdots \\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{R}_M^{\frac{1}{2}} \end{bmatrix} \tag{10}$$

and the $K \times 1$ channel realization driving vector $\boldsymbol{n} = [\boldsymbol{n}_1^T, \ldots, \boldsymbol{n}_M^T]^T$.

The base station can estimate the channel vector $\boldsymbol{h}$ from the uplink measurements and feed it back to the cooperating terminals. However, if the feedback delay is greater than the coherence time of the channel, the new realization of the channel vector will be independent of the feedback [22]. Nevertheless, one usually resorts to some quasi-stationarity assumption, that is, the second-order statistics of the channel stay approximately constant within a certain stationarity time [23]. Therefore, the terminals can estimate the uplink channel covariance matrix from the downlink measurements, e.g., by averaging the downlink measurements over the fast-fading time interval in time-division duplex systems. Also, for frequency-division duplex systems, the uplink covariance matrix can still be estimated by the terminals with enough accuracy by averaging the frequency-translated downlink measurements [17]. Hence, we can assume that each cooperating terminal has access to its channel covariance matrix and a possibly erroneous estimate of the channel realization. Hence, we model the actual stacked channel vector by the same model as that in (7), where the K-dimensional vector $\boldsymbol{\Delta}$ is the error in the channel realization driving vector.

Note that the signal model for the LOS propagation environment can be considered as a special case of the above model when the channel covariance matrix of each terminal is of rank 1, i.e., $\boldsymbol{h}_m = e^{-j\phi_m} \boldsymbol{a}_m(\theta_m)$, and thus, $\boldsymbol{R}_m = \boldsymbol{a}_m(\theta_m)\boldsymbol{a}_m^H(\theta_m)$. Hence, the perfect calibration of the local array manifolds of the terminals is equivalent to the perfect knowledge of the second-order statistics of their channels.

C. Optimal Uplink Transmit Beamforming

For LOS propagation and flat-fading channels, we can write the received signal at the base station as

$$y(i) = \boldsymbol{w}^H \boldsymbol{V} \boldsymbol{n} s(i) + v(i) \tag{11}$$

where $\boldsymbol{n} = \hat{\boldsymbol{n}} + \boldsymbol{\Delta}$. The received SNR at the base station is given by

$$\text{SNR} = \frac{E\left\{\left|\boldsymbol{w}^H \boldsymbol{V} \boldsymbol{n} s(i)\right|^2\right\}}{E\left\{|v(i)|^2\right\}} = \frac{1}{\sigma_v^2} |\boldsymbol{w}^H \boldsymbol{V} \boldsymbol{n}|^2 \tag{12}$$

where the constellation is assumed to have unit power, i.e., $E\{|s(i)|^2\} = 1$, and thus, $\|\boldsymbol{w}\|^2$ represents the transmitted signal power.

If the cooperating terminals have perfect knowledge of the channel vector $\boldsymbol{h} = \boldsymbol{V}\boldsymbol{n}$, the optimum transmit beamforming technique can be found by maximizing the transmission efficiency, i.e., the ratio between the received SNR at the base station and the transmitted power from the cooperating terminals, as given by

$$\eta_T \triangleq \frac{|\boldsymbol{w}^H \boldsymbol{V} \boldsymbol{n}|^2}{\sigma_v^2 \boldsymbol{w}^H \boldsymbol{w}}. \tag{13}$$

The above performance measure can be maximized by selecting the beamforming vector such that the denominator of (13) is minimized while fixing the numerator, i.e.,

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{w} \quad \text{s.t. } \boldsymbol{w}^H \boldsymbol{V} \boldsymbol{n} = 1 \tag{14}$$

and the optimal beamformer weight vector is given by $\boldsymbol{w} = \kappa \boldsymbol{V} \boldsymbol{n}$, where κ is a scalar that controls the transmitted power. However, each terminal has a possibly erroneous estimate $\hat{\boldsymbol{n}}$ of the channel realization driving vector due to channel estimation errors and/or delayed or erroneous feedback. The classical beamforming vector $\boldsymbol{w}_c = \boldsymbol{V}\hat{\boldsymbol{n}}$ uses this estimate instead of the actual one, which might result in severe degradation in the received SNR.

Nevertheless, we can assume that matrix $\boldsymbol{V}$ in (6) and (10), which corresponds to the second-order statistics of the channel, is known with high accuracy and does not change during the feedback time. Hence, the terminals can compute (or the base station computes and feeds back) a beamforming weight vector that is robust against uncertainties in the channel realization driving vector $\boldsymbol{n}$. In Sections III and IV, we will present two algorithms for computing the robust beamforming weight vector based on different mathematical formulations of the uncertainty set associated with the channel realization driving vector. In both algorithms, we use a worst-case performance optimization approach that maximizes the worst-case transmission efficiency (corresponding to the worst-case channel error) at the targeted base station. Each formulation is converted to a convex optimization problem that can be efficiently solved in polynomial time.

III. Centralized Robust Transmit Beamforming

In this section, we present a collaborative uplink transmit beamforming algorithm that is robust against errors in the channel-realization driving vector. This algorithm is centralized in the sense that the base station or a local processing center computes the beamforming coefficients using the uplink measurements. These coefficients are then fed back to the cooperating terminals to be used for beamforming. We start by defining the uncertainty set $\mathcal{A}_1$, which is associated with the estimate $\hat{\boldsymbol{n}}$ of the channel realization driving vector as

$$\mathcal{A}_1 \triangleq \left\{ \tilde{\boldsymbol{n}} = \left[\hat{\boldsymbol{n}}_1^T + \boldsymbol{\Delta}_1^T, \ldots, \hat{\boldsymbol{n}}_M^T + \boldsymbol{\Delta}_M^T \right]^T \Big| \|\boldsymbol{\Delta}_m\| \leq \varepsilon_m \right\}. \tag{15}$$

In the case of LOS propagation environments, the uncertainty vectors $\{\boldsymbol{\Delta}_m\}_{m=1}^M$ reduce to scalar quantities that reflect the magnitude of the error in the phase offset of each terminal. Hence, ε_m can be estimated as $\varepsilon_m = |e^{-j\tilde{\phi}_m} - e^{-j\hat{\phi}_m}|$, where $\tilde{\phi}_m$ is the phase offset causing the worst-case phase error. This error corresponds to the maximum uncertainty in the location of the mth terminal and the maximum phase error due to its local oscillator imperfections [24]. In the case of fading channels, if the channel estimates are obtained via feedback from the base station, then parameter ε_m is a function of the estimation accuracy, the feedback delay, and the coherence time of the channel of the mth terminal.

Note that the error in the channel estimate of each relay terminal is bounded by a distinct parameter. In contrast, the uncertainty set defined in earlier robust beamforming approaches, e.g., [11], uses a single parameter to bound the error of the stacked channel vector, which does not allow the exploitation of the good estimates that some terminals may have of their channels. Furthermore, it can be easily shown that direct application of the algorithm in [11] to the collaborative transmit beamforming problem will yield a weight vector that is equivalent to the classical beamforming vector, regardless of the value of the robustness parameter.

To provide robustness against errors in the channel vector, we will use the ideas that are presented in [10] and [11]. We will modify the constraint in (14) such that a high gain is provided not only for the presumed channel realization driving vector $\hat{\boldsymbol{n}}$ but also for all the vectors in $\mathcal{A}_1$. Therefore, we can write the robust uplink beamforming weight vector as the solution of the following optimization problem:

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{w} \quad \text{s.t. } |\boldsymbol{w}^H \boldsymbol{V} \tilde{\boldsymbol{n}}| \geq 1 \; \forall \tilde{\boldsymbol{n}} \in \mathcal{A}_1. \tag{16}$$

The constraint in the above optimization problem will be satisfied for all vectors $\tilde{\boldsymbol{n}} \in \mathcal{A}_1$ if it is satisfied for the worst-case (mismatched) channel realization driving vector in set $\mathcal{A}_1$. Hence, we can write (16) as

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{w} \quad \text{s.t. } \min_{\tilde{\boldsymbol{n}} \in \mathcal{A}_1} |\boldsymbol{w}^H \boldsymbol{V} \tilde{\boldsymbol{n}}| \geq 1. \tag{17}$$

Below, we will use an approach that is similar to that in [11] to find the minimum of $|\boldsymbol{w}^H \boldsymbol{V} \tilde{\boldsymbol{n}}|$ over $\mathcal{A}_1$. Invoking the triangle inequality, we can write

$$|\boldsymbol{w}^H \boldsymbol{V} \tilde{\boldsymbol{n}}| \geq |\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}}| - |\boldsymbol{w}^H \boldsymbol{V} \boldsymbol{\Delta}| \tag{18}$$

where $\boldsymbol{\Delta} = [\boldsymbol{\Delta}_1^T, \ldots, \boldsymbol{\Delta}_M^T]^T$.

For the case of flat-fading channels, we have

$$\begin{aligned} |\boldsymbol{w}^H \boldsymbol{V} \boldsymbol{\Delta}| &= \left| \sum_{m=1}^M \boldsymbol{w}_m^H \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{\Delta}_m \right| \leq \sum_{m=1}^M \left| \boldsymbol{w}_m^H \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{\Delta}_m \right| \\ &\leq \sum_{m=1}^M \varepsilon_m \left\| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \right\| \end{aligned} \tag{19}$$

with equality in (18) and (19) if and only if

$$\boldsymbol{\Delta}_m = -\varepsilon_m e^{j\psi} \frac{\boldsymbol{R}_m^{\frac{H}{2}} \boldsymbol{w}_m}{\left\| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \right\|}, \quad \text{where } \psi = \arg\{\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}}\}. \tag{20}$$

Note that $\boldsymbol{w}_m$ is the beamforming vector of the mth terminal. Combining (18) and (19), we can write

$$\min_{\tilde{\boldsymbol{n}} \in \mathcal{A}_1} |\boldsymbol{w}^H \boldsymbol{V} \tilde{\boldsymbol{n}}| = |\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}}| - \sum_{m=1}^M \varepsilon_m \left\| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \right\|. \tag{21}$$

Therefore, the robust beamformer weight vector can be found by solving the following optimization problem:

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{w} \quad \text{s.t. } |\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}}| - \sum_{m=1}^M \varepsilon_m \left\| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \right\| \geq 1. \tag{22}$$

The above optimization problem is nonconvex due to the absolute value operator in the constraint. However, we can always phase-rotate the weight vector $\boldsymbol{w}$ such that $\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}}$ is real without changing the value of the cost function [11]. Hence, the robust beamforming problem can be converted into the following standard SOCP [25]:

$$\begin{aligned} \min_{\boldsymbol{w}, \gamma, \alpha_i} \quad & \gamma \\ \text{s.t.} \quad & \|\boldsymbol{w}\| \leq \gamma \\ & \mathrm{Im}\{\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}}\} = 0 \\ & \boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} - \sum_{m=1}^M \varepsilon_m \alpha_m \geq 1 \\ & \left\| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \right\| \leq \alpha_m, \qquad \forall m = 1, \ldots, M. \end{aligned} \tag{23}$$

Similarly, for the case of the LOS propagation environment, we can write

$$|\boldsymbol{w}^H \boldsymbol{V}\tilde{\boldsymbol{n}}| \geq |\boldsymbol{w}^H \boldsymbol{V}\hat{\boldsymbol{n}}| - \sum_{m=1}^{M} \varepsilon_m \left|\boldsymbol{a}_m^H \boldsymbol{w}_m\right| \tag{24}$$

with equality in (18) and (24) if and only if

$$\Delta_m = -\varepsilon_m e^{j\psi} \frac{\boldsymbol{a}_m^H \boldsymbol{w}_m}{|\boldsymbol{a}_m^H \boldsymbol{w}_m|}. \tag{25}$$

Therefore, we can convert the robust beamforming problem in (16) to the following standard SOCP [25]:

$$\begin{aligned} \min_{\boldsymbol{w},\gamma,\alpha_i} \quad & \gamma \\ \text{s.t.} \quad & \|\boldsymbol{w}\| \leq \gamma \\ & \operatorname{Im}\{\boldsymbol{w}^H \boldsymbol{V}\hat{\boldsymbol{n}}\} = 0 \\ & \boldsymbol{w}^H \boldsymbol{V}\hat{\boldsymbol{n}} - \sum_{m=1}^{M} \varepsilon_m \alpha_m \geq 1 \\ & \left|\boldsymbol{a}_m^H \boldsymbol{w}_m\right| \leq \alpha_m, \qquad \forall m = 1, \ldots, M. \end{aligned} \tag{26}$$

The above SOCPs in (23) and (26) can be efficiently solved using interior point optimization methods [25]. The computational complexity that is associated with solving an SOCP can be calculated as follows [18]. The number of iterations that are required to solve an SOCP problem using interior point methods is bounded by the square root of the number of constraints. The computational complexity that is associated with each iteration is of $\mathcal{O}(n_v^2 \sum_i q_i)$, where $n_v = 2K + M + 1$ is the number of design parameters, and q_i is the dimension of the ith constraint. Therefore, the worst-case computational load of each of (23) and (26) is of $\mathcal{O}(\sqrt{M}K(M+K)^2)$.

Based on the propagation model, and given matrix $\boldsymbol{V}$ and the presumed channel realization driving vector $\hat{\boldsymbol{n}}$, the base station can compute the robust uplink beamforming vector for each of the M terminals by solving the SOCP optimization problem in (23) or (26). Instead of feeding back the channel coefficients, i.e., the classical beamforming vector, the base station feeds back the robust beamforming vector $\boldsymbol{w}_m$ to the mth terminal. Hence, no channel estimation is required at the terminals, as all the processing is done by the base station. Note that if the relay terminals are located in close vicinity of each other, and if they can estimate their uplink channels, e.g., using the channel reciprocity [16], they can feed forward their uplink channel estimates together with their associated uncertainty to a local processing center that calculates the beamforming vector. The processing center then feeds back the beamforming coefficients to the relay terminals.

One of the major advantages of using interior point methods to solve the optimization problems in (23) and (26) is that any additional convex constraint can be easily incorporated into the robust beamforming problem. We provide below some possible examples of these constraints.

1) *Maximum power constraints*. Due to physical considerations, the maximum transmission power of each terminal might be limited. One possible way of satisfying this constraint is to constrain the norm of the beamforming vector of each terminal, i.e., for the mth terminal

$$\|\boldsymbol{w}_m\| \leq \sqrt{P_m} \tag{27}$$

which is a convex second-order cone (SOC) constraint of $2k_m + 1$ real dimensions.

2) *Interference suppression*. Another possible constraint is to suppress the interference that is caused at nearby base stations due to the cooperative transmission. This constraint can be written as

$$\boldsymbol{w}^H \boldsymbol{h}^{(v)} = 0 \tag{28}$$

where $\boldsymbol{h}^{(v)}$ is the stacked channel vector from the M relay terminals to the vth nearby base station. This constraint is a linear constraint that can be easily incorporated in the beamforming problem, e.g., by substituting $\boldsymbol{w} = \boldsymbol{N}_{\boldsymbol{h}^{(v)}}^{\perp} \boldsymbol{v}$, where $\boldsymbol{N}_{\boldsymbol{h}^{(v)}}^{\perp}$ is the $K \times (K-1)$ matrix spanning the subspace that is orthogonal to the vector $\boldsymbol{h}^{(v)}$, and $\boldsymbol{v}$ is the $(K-1)$-dimensional vector containing the new optimization variables.

3) *Interference reduction*. Each interference suppression constraint with the form of (28) reduces one of the degrees of freedom available for beamforming and, hence, reduces the received signal power at the desired base station. One possible solution that does not consume as much degrees of freedom is imposing soft nulls in the directions of other base stations, i.e., limiting the transmitted interference power in those directions. This constraint can be written as

$$\left|\boldsymbol{w}^H \boldsymbol{h}^{(v)}\right| \leq \zeta^{(v)} \tag{29}$$

where $\zeta^{(v)}$ is a design parameter that controls the maximum admissible interference power received at the vth nontargeted base station due to the collaborative transmission. This constraint is an SOC constraint of dimension 3 that can be easily incorporated in the robust beamforming problem.

4) *Robust interference reduction*. In practical operating environments, the above interference reduction constraints can be inefficient due to errors in estimating channel vector $\boldsymbol{h}^{(v)}$. Using the same signal model and the notation discussed in Section II [with the superscript $(\cdot)^{(v)}$ referring to the vth base station channel], we can write the actual channel vector $\boldsymbol{h}^{(v)}$ as

$$\boldsymbol{h}^{(v)} = \boldsymbol{V}^{(v)} \boldsymbol{n}^{(v)} = \boldsymbol{V}^{(v)} \left(\hat{\boldsymbol{n}}^{(v)} + \boldsymbol{\Delta}\right) \tag{30}$$

where the error vector $\boldsymbol{\Delta}$ belongs to the uncertainty set

$$\mathcal{A}^{(v)} \triangleq \left\{ \boldsymbol{\Delta} = \left[\boldsymbol{\Delta}_1^T, \ldots, \boldsymbol{\Delta}_M^T\right]^T \middle| \|\boldsymbol{\Delta}_m\| \leq \varepsilon_m^{(v)} \right\}. \tag{31}$$

Therefore, we can use the robust interference reduction constraint

$$\max_{\boldsymbol{\Delta} \in \mathcal{A}^{(v)}} \left|\boldsymbol{w}^H \boldsymbol{V}^{(v)} \left(\hat{\boldsymbol{n}}^{(v)} + \boldsymbol{\Delta}\right)\right| \leq \zeta^{(v)} \tag{32}$$

instead of the constraint in (29). Using the triangle inequality, we can write

$$\left|\boldsymbol{w}^H\boldsymbol{V}^{(v)}\left(\hat{\boldsymbol{n}}^{(v)}+\boldsymbol{\Delta}\right)\right| \leq \left|\boldsymbol{w}^H\boldsymbol{V}^{(v)}\hat{\boldsymbol{n}}^{(v)}\right| + \left|\boldsymbol{w}^H\boldsymbol{V}^{(v)}\boldsymbol{\Delta}\right|. \tag{33}$$

Using (19), we can write the constraint in (32) in the case of fading channels as

$$\left|\boldsymbol{w}^H\boldsymbol{V}^{(v)}\hat{\boldsymbol{n}}^{(v)}\right| + \sum_{m=1}^{M}\varepsilon_m^{(v)}\left\|\boldsymbol{R}_m^{(v)\frac{1}{2}}\boldsymbol{w}_m\right\| \leq \zeta^{(v)} \tag{34}$$

which can be written as the following group of SOC constraints:

$$\left|\boldsymbol{w}^H\boldsymbol{V}^{(v)}\hat{\boldsymbol{n}}^{(v)}\right| \leq 1-\sum_{m=1}^{M}\varepsilon_m^{(v)}\alpha_m^{(v)} \tag{35}$$

$$\left\|\boldsymbol{R}_m^{(v)\frac{1}{2}}\boldsymbol{w}_m\right\| \leq \alpha_m^{(v)}, \qquad \forall m=1,\ldots,M. \tag{36}$$

Note that (35) is an SOC constraint of dimension 3, and the mth constraint in (36) is also an SOC constraint of dimension $2k_m+1$.

Similarly, in the case of LOS communication, we can write (32) as

$$\left|\boldsymbol{w}^H\boldsymbol{V}^{(v)}\hat{\boldsymbol{n}}^{(v)}\right| + \sum_{m=1}^{M}\varepsilon_m^{(v)}\left\|\boldsymbol{a}_m^{(v)H}\boldsymbol{w}_m\right\| \leq \zeta^{(v)} \tag{37}$$

which is equivalent to a group of $M+1$ SOC constraints, each of dimension 3.

IV. Decentralized Robust Transmit Beamforming

In this section, we provide an alternate formulation of the robust beamforming problem that allows the computation of the beamforming vector locally at each terminal with minimum feedback from the base station. We define the ellipsoidal uncertainty set $\mathcal{A}_2$ as

$$\mathcal{A}_2 \triangleq \left\{\tilde{\boldsymbol{n}}=\hat{\boldsymbol{n}}+\boldsymbol{\Delta}\middle|\boldsymbol{\Delta}^H\boldsymbol{K}_\varepsilon^{-1}\boldsymbol{\Delta}\leq 1\right\} \tag{38}$$

where the diagonal matrix $\boldsymbol{K}_\varepsilon$ determines the lengths of the axes of the hyperellipsoid. In the case of LOS propagation, the $M\times M$ matrix $\boldsymbol{K}_\varepsilon$ is given by

$$\boldsymbol{K}_\varepsilon = \operatorname{diag}\left\{\varepsilon_1^2,\varepsilon_2^2,\ldots,\varepsilon_M^2\right\} \tag{39}$$

whereas for fading channels, we define the $K\times K$ matrix $\boldsymbol{K}_\varepsilon$ as

$$\boldsymbol{K}_\varepsilon = \operatorname{diag}\left\{\left[\varepsilon_1^2\mathbf{1}_{k_1}^T,\varepsilon_2^2\mathbf{1}_{k_2}^T,\ldots,\varepsilon_M^2\mathbf{1}_{k_M}^T\right]^T\right\} \tag{40}$$

where $\mathbf{1}_k$ is the $k\times 1$ vector containing all ones. We can also write an equivalent description of the set $\mathcal{A}_2$ as

$$\mathcal{A}_2 = \left\{\tilde{\boldsymbol{n}}=\hat{\boldsymbol{n}}+\boldsymbol{D}\boldsymbol{u}\middle|\|\boldsymbol{u}\|\leq 1\right\} \tag{41}$$

where $\boldsymbol{D}=\boldsymbol{K}_\varepsilon^{1/2}$.

Following the same steps as those of the derivation in the previous section, we modify the constraint in (14) to prevent performance degradation for all the mismatched channel vectors in the set $\mathcal{A}_2$. Hence, we can write the robust transmit beamforming problem as

$$\min_{\boldsymbol{w}}\boldsymbol{w}^H\boldsymbol{w} \qquad \text{s.t.} \min_{\boldsymbol{u},\|\boldsymbol{u}\|\leq 1}\left|\boldsymbol{w}^H\boldsymbol{V}(\hat{\boldsymbol{n}}+\boldsymbol{D}\boldsymbol{u})\right|\geq 1. \tag{42}$$

Note that given the same values of $\{\varepsilon_m\}_{m=1}^M$ for the two sets $\mathcal{A}_1$ and $\mathcal{A}_2$ and under (39) or (40), $\mathcal{A}_2\subset\mathcal{A}_1$, and hence, the above formulation in (42) will provide less robustness against mismatches in the channel realization driving vector than the centralized formulation in (17).

The minimum of $|\boldsymbol{w}^H\boldsymbol{V}(\hat{\boldsymbol{n}}+\boldsymbol{D}\boldsymbol{u})|$ over set $\mathcal{A}_2$ can be found by observing that

$$\left|\boldsymbol{w}^H\boldsymbol{V}(\hat{\boldsymbol{n}}+\boldsymbol{D}\boldsymbol{u})\right| \geq |\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}|-|\boldsymbol{w}^H\boldsymbol{V}\boldsymbol{D}\boldsymbol{u}| \tag{43}$$

$$\geq |\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}|-\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\| \tag{44}$$

where (43) and (44) were derived using the triangle and Cauchy–Schwartz inequalities, respectively. The worst-case error $\boldsymbol{\Delta}=\boldsymbol{D}\boldsymbol{u}$ that satisfies (44) with equality is given by

$$\boldsymbol{\Delta}=-e^{j\psi}\frac{\boldsymbol{D}^2\boldsymbol{V}^H\boldsymbol{w}}{\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\|}. \tag{45}$$

Substituting with (44) in (42), and phase-rotating vector $\boldsymbol{w}$ so that $\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}$ is real, we can formulate the robust beamforming problem in (42) as the following SOCP:

$$\begin{aligned}\min_{\boldsymbol{w}} \quad & \boldsymbol{w}^H\boldsymbol{w}\\ \text{s.t.} \quad & \operatorname{Im}\{\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}\}=0\\ & \boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}-\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\|\geq 1.\end{aligned} \tag{46}$$

We note that the last constraint of (46) has to be satisfied with equality by the optimal beamforming weight vector, or else, we can always scale down the solution to further minimize the cost function while still satisfying the constraints. Also, the second constraint in (46) is redundant, as it is implied by the third constraint when it is satisfied with equality. Therefore, we can write (46) as

$$\min_{\boldsymbol{w}}\boldsymbol{w}^H\boldsymbol{w} \qquad \text{s.t. } \boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}-\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\|=1. \tag{47}$$

Proposition 1: The optimum beamforming vector that solves (47) is given by

$$\boldsymbol{w}=-\lambda\boldsymbol{V}\left(\boldsymbol{I}-\left(\frac{1}{\lambda}(\boldsymbol{D}^2-\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H)^{-1}+\boldsymbol{V}^H\boldsymbol{V}\right)^{-1}\boldsymbol{V}^H\boldsymbol{V}\right)\hat{\boldsymbol{n}} \tag{48}$$

where λ is the Lagrange multiplier that is associated with the constraint in (47).

Proof: See the Appendix.

For the case of LOS propagation, $\boldsymbol{V}^H\boldsymbol{V}=\boldsymbol{\Upsilon}$, where $\boldsymbol{\Upsilon}=\operatorname{diag}\{\|\boldsymbol{a}_1(\theta_1)\|^2,\ldots,\|\boldsymbol{a}_M(\theta_M)\|^2\}$, and hence, using the matrix inversion lemma, we can further simplify (48) to

$$\begin{aligned}\boldsymbol{w} =& -\lambda \boldsymbol{V}\left(\boldsymbol{I}_M - \left(\frac{1}{\lambda}\boldsymbol{\Upsilon}^{-1}(\boldsymbol{D}^2 - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H)^{-1} + \boldsymbol{I}_M\right)^{-1}\right)\hat{\boldsymbol{n}} \\ =& \boldsymbol{V}\boldsymbol{\Upsilon}^{-1}\left(\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H - \left(\boldsymbol{D}^2 + \frac{1}{\lambda}\boldsymbol{\Upsilon}^{-1}\right)\right)^{-1}\hat{\boldsymbol{n}} \\ =& \boldsymbol{V}\boldsymbol{\Upsilon}^{-1}\left(\frac{\boldsymbol{T}^{-1}\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H\boldsymbol{T}^{-1}}{\hat{\boldsymbol{n}}^H\boldsymbol{T}^{-1}\hat{\boldsymbol{n}}^H - 1} - \boldsymbol{T}^{-1}\right)\hat{\boldsymbol{n}} \\ =& \frac{\boldsymbol{V}\boldsymbol{\Upsilon}^{-1}\boldsymbol{T}^{-1}\hat{\boldsymbol{n}}}{\hat{\boldsymbol{n}}^H\boldsymbol{T}^{-1}\hat{\boldsymbol{n}} - 1} \end{aligned} \tag{49}$$

where the diagonal matrix $\boldsymbol{T} = \boldsymbol{D}^2 + (1/\lambda)\boldsymbol{\Upsilon}^{-1}$.

As stated in Section II-C, our main objective is to maximize the transmission efficiency at the targeted base station. Since changing the norm of the beamformer weight vector affects only the total transmitted power and not the transmission efficiency, we can drop the denominator of (49) and use the equivalent robust beamformer weight vector, which is given by

$$\boldsymbol{w} \equiv \boldsymbol{V}\left(\boldsymbol{D}^2\boldsymbol{\Upsilon} + \frac{1}{\lambda}\boldsymbol{I}_M\right)^{-1}\hat{\boldsymbol{n}}. \tag{50}$$

Hence, the beamforming vector for the mth terminal can be obtained by weighting its classical beamforming vector, i.e., $e^{j2\pi f\hat{T}_i(\theta)}\boldsymbol{a}_m(\theta)$, by $(\varepsilon_m^2|\boldsymbol{a}_m(\theta_m)\|^2 + 1/\lambda)^{-1}$. This can be viewed as an optimal power allocation strategy for different terminals based on the uncertainty in their phase offsets. Therefore, using the uplink training data, the base station can compute the optimal value of the Lagrange multiplier and feed it back to all the cooperating terminals through a common broadcasting channel. Each terminal then adjusts its transmitted power using the information about its phase reference uncertainty and the Lagrange multiplier. This process can be repeated to track any changes in the operating environment.

In the case of fading channels, we can write the robust beamformer weight vector in (48) as

$$\boldsymbol{w} = \frac{1}{\beta}\boldsymbol{V}\left(\boldsymbol{D}^2\boldsymbol{R} + \frac{1}{\lambda}\boldsymbol{I}_K\right)^{-1}\hat{\boldsymbol{n}} \tag{51}$$

where $\beta = \hat{\boldsymbol{n}}^H(\boldsymbol{D}^2 + (1/\lambda)\boldsymbol{R}^{-1})^{-1}\hat{\boldsymbol{n}} - 1$, and the $M \times M$ block-diagonal matrix $\boldsymbol{R} = \boldsymbol{V}^H\boldsymbol{V}$. Therefore, normalizing the transmitted power, we can write the equivalent beamforming vector of the mth terminal that maximizes the transmission efficiency as

$$\boldsymbol{w}_m \equiv \boldsymbol{R}_m^{\frac{1}{2}}\left(\varepsilon_m^2\boldsymbol{R}_m + \frac{1}{\lambda}\boldsymbol{I}_{k_m}\right)^{-1}\hat{\boldsymbol{n}}_m. \tag{52}$$

We can see that the robust beamformer weight vector of the mth terminal can be obtained by replacing the estimate $\hat{\boldsymbol{n}}_m$ of the channel realization driving vector by its robust version $(\varepsilon_m^2\boldsymbol{R}_m + (1/\lambda)\boldsymbol{I}_{k_m})^{-1}\hat{\boldsymbol{n}}_m$.[5] This linear transformation modifies the channel realization vector based on the uncertainty in the channel vector estimate and the covariance matrix of the channel that includes information about the channel strength.

[5]Hence, the computational complexity at the mth terminal is of $\mathcal{O}(k_m^3)$ due to the inversion of matrix $\varepsilon_m^2\boldsymbol{R}_m + (1/\lambda)\boldsymbol{I}_{k_m}$.

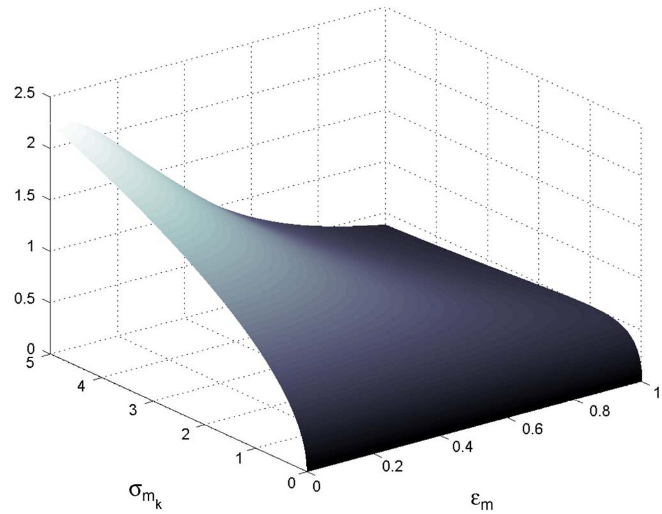

Fig. 2. Beamforming gain versus σ_{m_k} and ε_m.

In [11], Vorobyov *et al.* showed that their robust beamformer can be considered as a minimum variance distortionless response beamformer with optimal diagonal loading that matches the amount of uncertainty in the stacked channel vector. In contrast, we will show that our beamformer in (52) can be viewed as an optimum power-allocation strategy along the eigen beams of the channel covariance matrix. To see this more clearly, let the eigen decomposition of matrix $\boldsymbol{R}_m$ be given by

$$\boldsymbol{R}_m = \boldsymbol{E}_m\Sigma_m\boldsymbol{E}_m^H. \tag{53}$$

Therefore, we can write (52) as

$$\begin{aligned}\boldsymbol{w}_m &\equiv \boldsymbol{E}_m \mathrm{diag}\left\{\left[\frac{\lambda\sqrt{\sigma_{m_1}}}{1+\lambda\varepsilon_m^2\sigma_{m_1}}, \ldots, \frac{\lambda\sqrt{\sigma_{m_{k_m}}}}{1+\lambda\varepsilon_m^2\sigma_{m_{k_m}}}\right]^T\right\}\boldsymbol{E}_m^H\hat{\boldsymbol{n}}_m \\ &= \sum_{k=1}^{k_m}\frac{\lambda\sqrt{\sigma_{m_k}}\boldsymbol{e}_{m_k}^H\hat{\boldsymbol{n}}_m}{1+\lambda\varepsilon_m^2\sigma_{m_k}}\boldsymbol{e}_{m_k}\end{aligned} \tag{54}$$

where $\boldsymbol{e}_{m_k}$ is the kth eigenvector of $\boldsymbol{R}_m$, and σ_{m_k} is its associated eigenvalue. We can also write the classical beamforming vector of the mth terminal, i.e., $\boldsymbol{w}_{c_m} = \boldsymbol{R}_m^{1/2}\hat{\boldsymbol{n}}_m$, as

$$\boldsymbol{w}_{c_m} = \sum_{k=1}^{k_m}\sqrt{\sigma_{m_k}}\left(\boldsymbol{e}_{m_k}^H\hat{\boldsymbol{n}}_m\right)\boldsymbol{e}_{m_k}. \tag{55}$$

Comparing (54) and (55), we can see that our robust beamforming technique can be viewed as a power-allocation strategy along the eigen beams of the channel based on the uncertainty in the channel vector realization and the power that is associated with each eigen mode. Fig. 2 shows the power allocation for one eigen beam, i.e., $\lambda\sqrt{\sigma_{m_k}}/(\lambda\varepsilon_m^2\sigma_{m_k} + 1)$ versus different values of σ_{m_k} and ε_m for the case of $\lambda = 1$. We can see from this figure that when the channel coefficient is known perfectly, i.e., $\varepsilon_m = 0$, the beamforming power increases as the eigen beam strength increases. On the other hand, when there is any uncertainty in the channel coefficient, the beamforming power does not necessarily increase as the eigen beam

TABLE I
COMPARISON BETWEEN DIFFERENT BEAMFORMING ALGORITHMS

	Classical beamformer	Centralized robust beamformer	Decentralized robust beamformer
Number of feed-back parameters	K	K	1
Complexity at base station	None	$\mathcal{O}\left(\sqrt{M}K(M+K)^2\right)$	$\mathcal{O}\left(K^3\right)$
Complexity at mth terminal	None	None	$\mathcal{O}\left(k_m^3\right)$

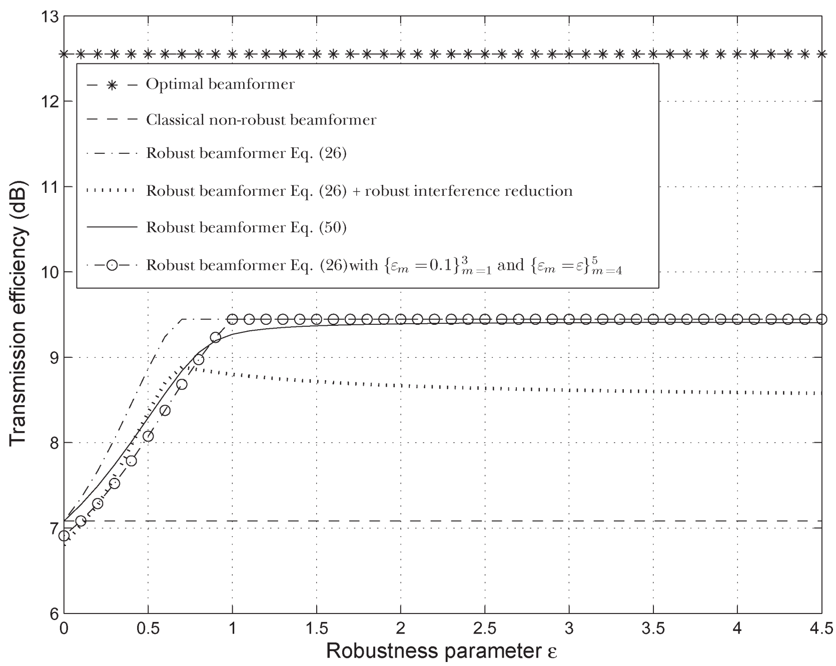

Fig. 3. Average transmission efficiency versus the parameter ε.

strength increases. In fact, if σ_{m_k} increases beyond $1/(\lambda\varepsilon_m^2)$, the beamforming power decreases due to the high uncertainty in the estimate of the channel.

Therefore, similar to the LOS propagation environment, the base station computes and feeds back parameter λ to the cooperating terminals. Each terminal then adjusts the power that is transmitted along the eigen modes of its channel based on the knowledge of its channel realization vector estimate and covariance matrix only. Thus, the proposed beamforming scheme meets the requirements of collaborative beamforming mentioned in [4], specifically, robustness against channel estimation errors and minimum amount of information sharing between the terminals. Table I compares our centralized and decentralized beamforming algorithms with classical beamforming in terms of the number of parameters that are fed back from the base station and the computational complexity at the base station and the terminals.

V. NUMERICAL SIMULATIONS

A. LOS Propagation Environment

We consider the uplink of a wireless communication system with $M = 5$ cooperating terminals. Each terminal is equipped with an antenna array of $k_1 = 4$, $k_2 = 3$, $k_3 = 2$, $k_4 = 4$, and $k_5 = 5$ elements with half-wavelength spacing. The antenna arrays of the first, third, and fourth terminals are located parallel to the x-axis, with the center of the arrays presumed to be at $[50.75\lambda, 25\lambda]$, $[75.25\lambda, 0]$, and $[60.75\lambda, -15\lambda]$, respectively. The arrays of the second and fifth terminals are located parallel to the y-axis, with the center of the arrays presumed to be at $[75\lambda, 25.5\lambda]$ and $[90\lambda, \lambda]$, respectively.[6] The actual location of the mth terminal is displaced along the x- and y-axes from its nominal location by independent random displacements that are uniformly distributed between $[-0.5\lambda\delta_m, 0.5\lambda\delta_m]$, where $\delta_1 = 0.1$, $\delta_2 = 0.1$, $\delta_3 = 0.1$, $\delta_4 = 1.5$, and $\delta_5 = 1.5$. The desired base station is located in the far field of the arrays along $\theta = 0^\circ$, where θ is measured relative to the x-axis, and the wave propagation model is assumed planar. A second base station is located along $\theta = 50^\circ$, and the signal that is received by this base station due to the collaborative transmission is considered as interference. All the beamforming vectors are normalized to have a unit norm. Simulation results are averaged over 10^3 Monte Carlo runs.[7]

Fig. 3 shows the average transmission efficiency that is obtained using the robust beamformers in (26) and (50), with the additional robust interference reduction constraints in (37),

[6]This setup can possibly represent an indoor relaying scheme for a wireless local area network.

[7]Each simulation run corresponds to a random realization of the displacements of the terminals from their presumed locations.

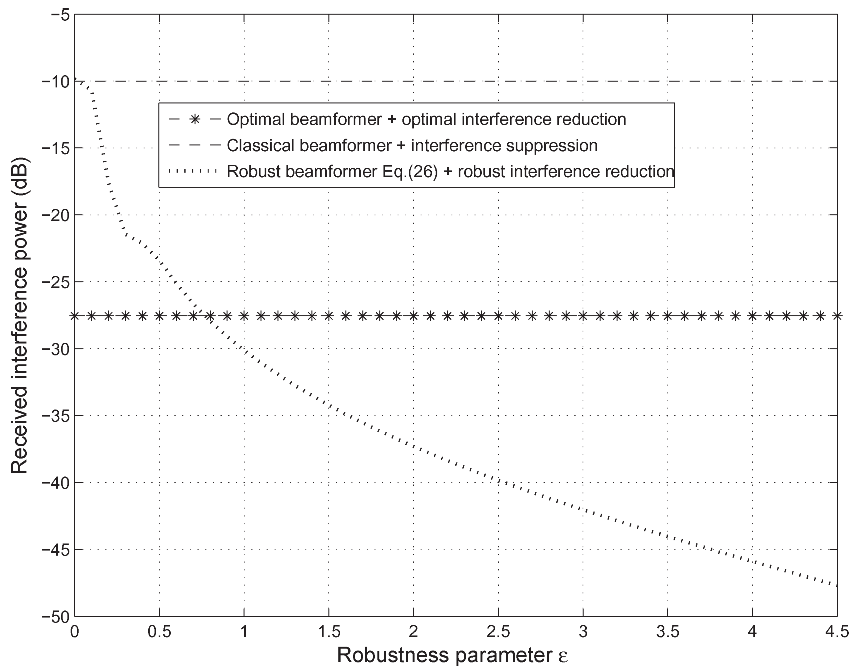

Fig. 4. Average received interference power versus the parameter ε.

where $\zeta^{(v)}$ is chosen to be equal to 10^{-2}. The uncertainty sets $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}^{(v)}$ are formed using the values $\{\varepsilon_m = \varepsilon_m^{(v)} = \varepsilon\delta_m\}$. The performance of the three beamformers is compared for different choices of the parameter ε that correspond to different sizes of the robustness sets $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}^{(v)}$. Fig. 3 also shows the average transmission efficiency that is obtained using the classical nonrobust weight vector estimate, and the maximum efficiency that can be obtained using the optimal beamformer, i.e., when the array manifold is perfectly known by the cooperating terminals. We can clearly see the performance improvements (more than 2 dB) achieved by the proposed beamforming technique compared with the classical beamformer. Moreover, our beamforming technique is not very sensitive to the exact size of the uncertainty set and performs well over a wide range of the parameter ε. We can also see that the additional robust interference reduction constraints do not significantly degrade the received signal power at the desired base station even for very large values of ε that correspond to very large sizes of $\mathcal{A}^{(v)}$. Nevertheless, these constraints prove to be efficient in reducing the interference that is received by the nontargeted base station, as will be discussed later.

We also investigate the sensitivity of our beamformer toward errors in the prior information about the magnitude of the uncertainty in the channel estimate of each terminal. We form the robustness set $\mathcal{A}_1$ using the true channel error parameters of the first three subarrays, i.e., $\{\varepsilon_m = \delta_m\}_{m=1}^{3}$, and select $\varepsilon_4 = \varepsilon_5 = \varepsilon$. The average transmission efficiency for different values of the parameter ε is also displayed in Fig. 3. We can see that our beamformer is also robust against mismatches in the prior information about the magnitude of the error of the channel vectors of the relay terminals.

Next, we compare the interference reduction capability of the robust beamformer in (26) with the additional robust interference-reduction constraints in (37) against that of the classical nonrobust beamformer with the additional interference suppression constraint $\boldsymbol{w}^H\boldsymbol{V}^{(v)}\hat{\boldsymbol{n}}^{(v)} = 0$ and the optimal beamformer (that has perfect knowledge of the channel vectors) with the optimal interference reduction constraint $|\boldsymbol{w}^H\boldsymbol{V}^{(v)}\boldsymbol{n}^{(v)}| \le \zeta^{(v)}$. Fig. 4 shows the received interference power at the base station that is located at $\theta = 50^\circ$ versus the robustness parameter ε. We can see that the robust interference reduction constraint can effectively reduce the interference power that is transmitted by the cooperating terminals (i.e., increase the null depth in the direction of the nontargeted base station) in spite of the mismatches in the channel vector. This can be explained as follows. As the size of set $\mathcal{A}^{(v)}$ increases, the null width around the direction $\theta = 50^\circ$ increases so that the constraint in (37) is satisfied for the whole set $\mathcal{A}^{(v)}$. This also leads to increasing the null depth at the direction $\theta = 50^\circ$, and hence, the interference power that is received due to our beamformer is decreased even below that produced by the optimal beamformer. On the other hand, the null imposed on the classical beamformer at $\theta = 50^\circ$ has almost no effect in reducing the interference power due to errors in the estimated stacked channel vector.

Fig. 5 shows the beampattern versus the angle of transmission, i.e., the received power at different directions. We compare the performance of the robust beamformer in (26) (with $\varepsilon = 1$) with the robust interference reduction constraint, the classical nonrobust beamformer with the interference suppression constraint $\boldsymbol{w}^H\boldsymbol{V}^{(v)}\hat{\boldsymbol{n}}^{(v)} = 0$, and the optimal beamformer with the optimal interference reduction constraint $|\boldsymbol{w}^H\boldsymbol{V}^{(v)}\boldsymbol{n}^{(v)}| \le \zeta^{(v)}$. We can see the effect of the robustness constraint that is imposed at the direction of the desired base station in providing high gain at $\theta = 0^\circ$ compared with the classical beamformer. We can also see the effect of the robust interference reduction constraint in widening and deepening the null in the direction of the nontargeted base station.

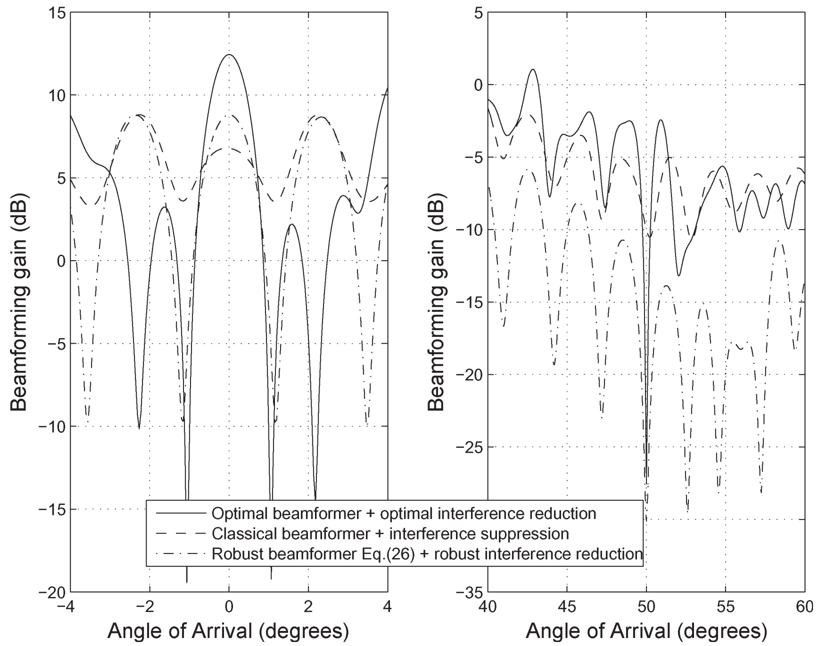

Fig. 5. Average beampattern.

B. Flat-Fading Environment

We consider the same collaborative transmission scenario as that in Section V-A. The propagation environment for each of the five terminals is modeled as a Ricean flat-fading channel with Ricean K-factor equal to 0.1 and random LOS arrival angles uniformly distributed between [0, 2π]. The scattered component of the received signal due to each of the five terminals has a Laplacian power-angle profile, with a random mean angle of arrival that is uniformly distributed between [0, 2π] and angular spread 8°, 3°, 2°, 2°, and 10° for the first to fifth terminals, respectively [26].

We generate 100 independent channel realizations. For each channel realization, the estimate of the channel realization vector of the mth terminal is obtained as

$$\hat{\boldsymbol{n}}_m = \boldsymbol{n}_m + \frac{\delta_m}{\|\boldsymbol{\Delta}_m\|}\Delta_m \tag{56}$$

where $\boldsymbol{\Delta}_m$ is modeled as a standard circular Gaussian vector with independent components, and δ_m is the relative magnitude of the error in the channel vector estimate. The values of δ_m are given by 0.2, 3, 2, 4, and 0.1 for $m = 1$ to $m = 5$, respectively. The uncertainty sets $\mathcal{A}_1$ and $\mathcal{A}_2$ are formed using the values $\{\varepsilon_m = \varepsilon\delta_m\}_{m=1}^{M}$. Simulation results are averaged over 50 independent realizations of $\{\boldsymbol{\Delta}_m\}_{m=1}^{5}$ generated for each of the 100 independent channel realizations.

Fig. 6 shows the average transmission efficiency versus different values of the parameter ε for different beamforming techniques. From this figure, we can see that the proposed robust beamforming technique can improve the received signal power by more than 1 dB compared with the classical nonrobust beamformer. We can also notice that the transmission efficiency does not severely degrade over a wide range of the size of the robustness sets.

Fig. 7 shows the average SER versus the normalized transmission power, i.e., $\|\boldsymbol{w}\|^2/\sigma_v^2$, for different beamformers for the QPSK and quadrature-amplitude modulation 16 (QAM-16) constellations. For the robust beamformers, we have selected the value of ε that yields the highest transmission efficiency. It is clear from Fig. 7 that the power gain that is offered by the proposed beamforming technique is translated into a corresponding gain in the average SER.

Next, we consider the delayed feedback scenario that was used in [13] and [15]. In this scenario, it is assumed that the channel coefficients are independent zero-mean complex Gaussian and that they slowly change with time according to Jakes' model [22], [26]. Thus, the channel estimate of the mth terminal available at the transmitter and the actual channel vector are drawn from the same distribution and have a correlation coefficient ρ_m that depends on the channel coherence time and the feedback delay. Therefore, the estimate of the channel realization driving vector of the mth terminal is modeled as

$$\hat{\boldsymbol{n}}_m = \rho_m\boldsymbol{n}_m + \sqrt{1-\rho_m^2}\boldsymbol{v}_m \tag{57}$$

where $\boldsymbol{v}_m$ is a $k_m \times 1$ Gaussian vector with zero mean and covariance $\boldsymbol{I}$. The values of the correlation coefficients are selected as $\rho_1 = 0.95$, $\rho_2 = 0.2$, $\rho_3 = 0.2$, $\rho_4 = 0.2$, and $\rho_5 = 0.98$, and the corresponding values of the robustness parameter are $\varepsilon_m = \sqrt{1-\rho_m^2}$ for $m = 1$ to $m = 5$. Fig. 8 shows the average SER over 100 Monte Carlo simulations versus the normalized transmission power for different beamformers for the QPSK and QAM-16 constellations. We can see from Fig. 8 the performance gain that is achieved by the robust beamformers compared with classical nonrobust beamforming.

VI. Conclusion

In this paper, we have presented a framework for collaborative transmit beamforming with robustness against mismatches

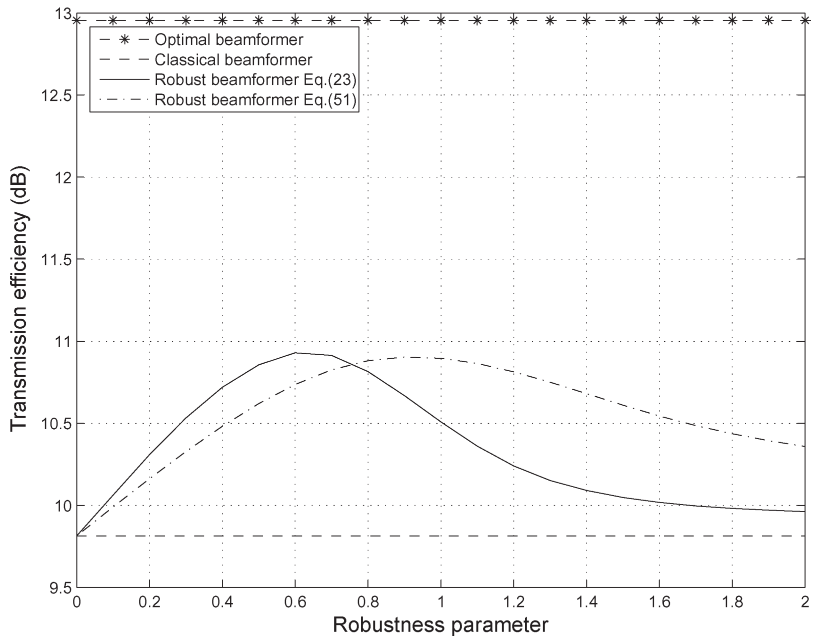

Fig. 6. Average transmission efficiency versus the parameter ε.

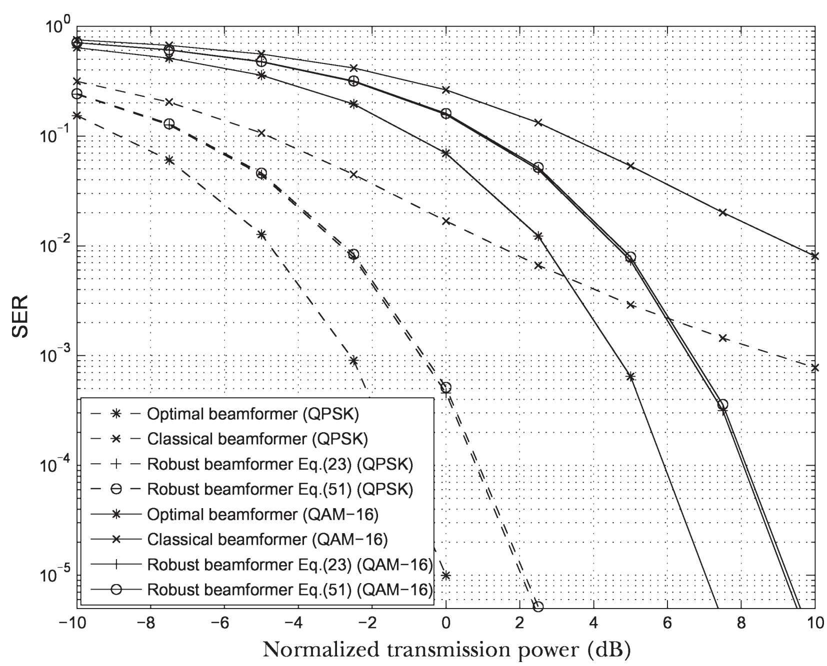

Fig. 7. SER versus normalized transmission power.

in the channel state information. Our technique is applicable to LOS propagation and fading environments. It exploits the available knowledge about the channel covariance matrix and a possibly erroneous estimate of the channel realization vector. The beamforming vector is derived by maximizing the transmission efficiency for a predefined set of channel realizations centered around the current estimate. Using two different formulations of the channel vector uncertainty set, we have developed two algorithms for robust transmit beamforming—a centralized and a decentralized algorithm. In the first algorithm, the base station calculates the beamforming coefficients using the uplink measurements by solving an SOCP optimization problem. These coefficients are then fed back to the collaborating terminals to be used in uplink transmit beamforming. In the second algorithm, each terminal can compute its beamforming coefficients using the local knowledge that is available about its channel and a single parameter that is broadcast from the base station to all the cooperating terminals. We have also shown that this algorithm is equivalent to an optimal power-allocation strategy across the eigen beams of the channel covariance matrix based on the strength of each beam and the uncertainty in the channel realization. Simulation results have been presented showing the improved performance of the proposed algorithms compared with classical beamforming techniques.

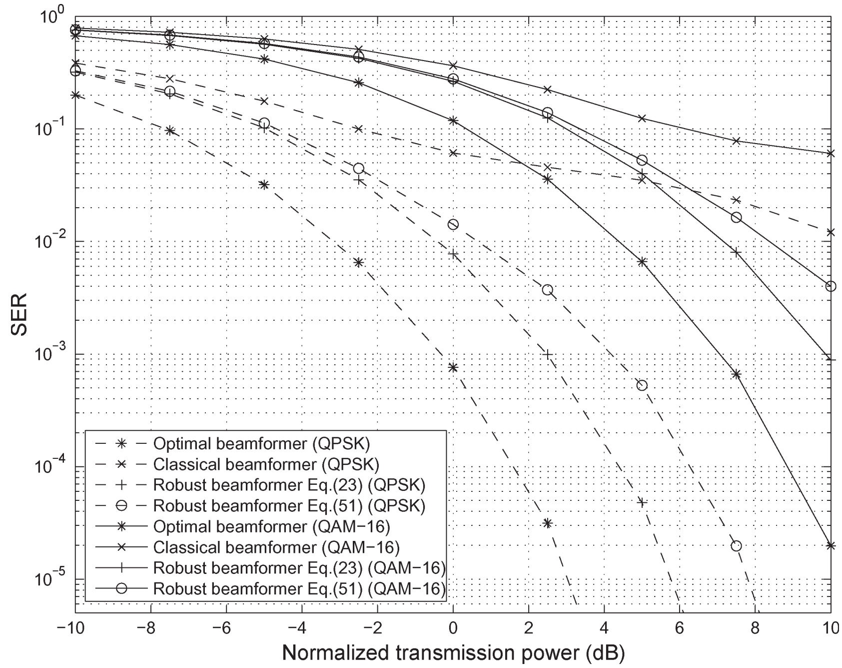

Fig. 8. SER versus normalized transmission power.

APPENDIX
PROOF OF PROPOSITION 1

In what follows, we will derive a closed-form solution of the optimization problem in (47) using the method of Lagrange multipliers and following the guidelines in [10]. First, we impose the additional constraint $\boldsymbol{w}^H \boldsymbol{V}\hat{\boldsymbol{n}} - 1 \geq 0$. Since $\boldsymbol{w}^H \boldsymbol{V}\hat{\boldsymbol{n}}$ is real valued, we can write (47) as

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{w} \quad \text{s.t. } \|\boldsymbol{D}\boldsymbol{V}^H \boldsymbol{w}\|^2 = |\boldsymbol{w}^H \boldsymbol{V}\hat{\boldsymbol{n}} - 1|^2. \tag{A1}$$

The Lagrangian that is associated with (A1) is given by

$$\begin{aligned} L(\boldsymbol{w}, \lambda) &= \boldsymbol{w}^H \boldsymbol{w} + \lambda \left(\|\boldsymbol{D}\boldsymbol{V}^H \boldsymbol{w}\|^2 - |\boldsymbol{w}^H \boldsymbol{V}\hat{\boldsymbol{n}} - 1|^2 \right) \\ &= \boldsymbol{w}^H (\boldsymbol{I}_K + \lambda \boldsymbol{Q})\boldsymbol{w} + \lambda \hat{\boldsymbol{h}}^H \boldsymbol{w} + \lambda \boldsymbol{w}^H \hat{\boldsymbol{h}} - \lambda \end{aligned} \tag{A2}$$

where $\boldsymbol{I}_K$ denotes the $K \times K$ identity matrix, the $K \times 1$ vector $\hat{\boldsymbol{h}} = \boldsymbol{V}\hat{\boldsymbol{n}}$, and the $K \times K$ Hermitian matrix $\boldsymbol{Q} = \boldsymbol{V}\boldsymbol{D}^2 \boldsymbol{V}^H - \hat{\boldsymbol{h}}\hat{\boldsymbol{h}}^H$. By equating the complex gradient of (A2) to zero [27], we can write the optimal solution of (A1) as

$$\boldsymbol{w} = -\lambda (\boldsymbol{I}_K + \lambda \boldsymbol{Q})^{-1} \hat{\boldsymbol{h}} \tag{A3}$$

where the optimal value of the Lagrange multiplier λ satisfies the constraint in (A1), i.e.,

$$\begin{aligned} 0 &= \boldsymbol{w}^H \boldsymbol{Q}\boldsymbol{w} + \hat{\boldsymbol{h}}^H \boldsymbol{w} + \boldsymbol{w}^H \hat{\boldsymbol{h}} - 1. \\ &= \lambda^2 \hat{\boldsymbol{h}}^H (\boldsymbol{I}_K + \lambda \boldsymbol{Q})^{-1} \boldsymbol{Q} (\boldsymbol{I}_K + \lambda \boldsymbol{Q})^{-1} \hat{\boldsymbol{h}} \\ &\quad - 2\lambda \hat{\boldsymbol{h}}^H (\boldsymbol{I}_K + \lambda \boldsymbol{Q})^{-1} \hat{\boldsymbol{h}} - 1. \end{aligned} \tag{A4}$$

To solve the above equation, we define the eigen decomposition of $\boldsymbol{Q}$ as

$$\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{\Gamma}\boldsymbol{U}^H \tag{A5}$$

where $\boldsymbol{\Gamma}$ is the diagonal $K \times K$ matrix containing the eigenvalues of matrix $\boldsymbol{Q}$ arranged in nonincreasing order, and $\boldsymbol{U}$ is the $K \times K$ matrix containing the corresponding eigenvectors.[8] If we define the vector $\boldsymbol{c} = \boldsymbol{U}^H \hat{\boldsymbol{h}}$, we can write the solution of (A4) as the root of the function

$$f(\lambda) = \lambda^2 \sum_{i=1}^{K} \frac{|c_i|^2 \gamma_i}{(1 + \lambda \gamma_i)^2} - 2\lambda \sum_{i=1}^{K} \frac{|c_i|^2}{(1 + \lambda \gamma_i)} - 1 \tag{A6}$$

where γ_i is the ith eigenvalue of matrix $\boldsymbol{Q}$, and c_i is the ith entry of vector $\boldsymbol{c}$.

The value of λ can be evaluated by solving for all the roots of (A6) and selecting the root that yields the minimum value of the cost function in (A1), while satisfying the additional constraint $\boldsymbol{w}^H \boldsymbol{V}\hat{\boldsymbol{n}} - 1 \geq 0$. However, it was shown in [10] that the additional constraint is satisfied for all values of λ that are greater than a threshold $\lambda_{\min}$ and that there exists only one root of (A6) that satisfies $\lambda > \lambda_{\min}$, where

$$\lambda_{\min} = \frac{-1 - |c_K| \left(\gamma_K + |c_K|^2 \right)^{-\frac{1}{2}}}{\gamma_K} \tag{A7}$$

γ_K is the single negative eigenvalue of matrix $\boldsymbol{Q}$, and c_K is the corresponding entry of vector $\boldsymbol{c}$. Therefore, the Newton–Raphson method can be used to solve for the value of the optimal λ in (A4) [28], where the iterations are initialized with $\lambda_{\min}$, and the value of the derivative of (A6) with respect to λ is given by

$$f'(\lambda) = -2 \sum_{i=1}^{K} \frac{|c_i|^2}{(1 + \lambda \gamma_i)^3}. \tag{A8}$$

[8]The computation of the eigen decomposition of matrix $\boldsymbol{Q}$ is the computational bottleneck of evaluating the value of λ. It requires $\mathcal{O}(K^3)$ multiplications.

Substituting in (A3) with the optimal value of the Lagrange multiplier, the robust beamforming weight vector can be written as

$$\boldsymbol{w} = -\lambda\left(\boldsymbol{I}_K + \lambda \boldsymbol{V}(\boldsymbol{D}^2 - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H)\boldsymbol{V}^H\right)^{-1}\boldsymbol{V}\hat{\boldsymbol{n}}. \tag{A9}$$

$$= -\lambda \boldsymbol{V}\left(\boldsymbol{I} - \left(\frac{1}{\lambda}(\boldsymbol{D}^2 - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H)^{-1} + \boldsymbol{V}^H\boldsymbol{V}\right)^{-1}\boldsymbol{V}^H\boldsymbol{V}\right)\hat{\boldsymbol{n}} \tag{A10}$$

where we have used the matrix inversion lemma to simplify (A9) into (A10).

REFERENCES

[1] F. Rashid-Farrokhi, K. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1437–1450, Oct. 1998.

[2] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Process.*, vol. 51, no. 5, pp. 1051–1064, May 2002.

[3] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.

[4] H. Ochiai, P. Mitran, H. V. Poor, and V. Tarokh, "Collaborative beamforming for distributed wireless ad hoc sensor networks," *IEEE Trans. Signal Process.*, vol. 53, no. 11, pp. 4110–4124, Nov. 2005.

[5] R. Pabst, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80–89, Sep. 2004.

[6] H. Ochiai, P. Mitran, H. V. Poor, and V. Tarokh, "On the effects of phase estimation errors on collaborative beamforming in wireless ad hoc networks," in *Proc. IEEE ICASSP*, Philadelphia, PA, Mar. 2005, vol. 3, pp. 657–660.

[7] R. Mudumbai, G. Barriac, and U. Madhow, "On the feasibility of distributed beamforming in wireless networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1754–1763, May 2007.

[8] A. El-Keyi, T. Kirubarajan, and A. B. Gershman, "Robust adaptive beamforming based on the Kalman filter," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3032–3041, Aug. 2005.

[9] J. Li, P. Stoica, and Z. Wang, "On robust Capon beamforming and diagonal loading," *IEEE Trans. Signal Process.*, vol. 51, no. 7, pp. 1702–1715, Jul. 2003.

[10] R. Lorenz and S. P. Boyd, "Robust minimum variance beamforming," *IEEE Trans. Signal Process.*, vol. 53, no. 5, pp. 1684–1696, May 2005.

[11] S. A. Vorobyov, A. B. Gershman, and Z. Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 313–324, Feb. 2003.

[12] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[13] A. Abdel-Samad, T. N. Davidson, and A. B. Gershman, "Robust transmit eigen beamforming based on imperfect channel state information," *IEEE Trans. Signal Process.*, vol. 54, no. 5, pp. 1596–1609, May 2006.

[14] A. Abdel-Samad and A. B. Gershman, "Robust transmit eigen-beamforming with imperfect knowledge of channel correlations," in *Proc. IEEE ICC*, Seoul, Korea, May 2005, pp. 2292–2296.

[15] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2599–2613, Oct. 2002.

[16] G. Barriac and U. Madhow, "Space-time communication for OFDM with implicit channel feedback," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3111–3129, Dec. 2004.

[17] M. Bengtsson and B. Ottersten, "Optimum and suboptimum transmit beamforming," in *Handbook of Antennas in Wireless Communications*, L. C. Godara, Ed. Boca Raton, FL: CRC, 2002, ch. 18, pp. 18-1–18-33.

[18] M. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, "Applications of second-order cone programming," *Linear Algebra Appl.*, vol. 284, pp. 193–228, 1998.

[19] C. M. S. See and A. B. Gershman, "Direction-of-arrival estimation in partly calibrated subarray-based sensor arrays," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 329–338, Feb. 2004.

[20] H. L. Van Trees, *Optimum Array Processing*. New York: Wiley, 2002.

[21] Y. S. Choi and S. M. Alamouti, "Complementary beamforming: New approaches," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 41–50, Jan. 2006.

[22] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2005.

[23] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: Information-theoretic and communications aspects," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2619–2692, Oct. 1998.

[24] G. Barriac, R. Mudumbai, and U. Madhow, "Distributed beamforming for information transfer in sensor networks," in *Proc. Int. Symp. IPSN*, Berkeley, CA, Apr. 2004, pp. 81–88.

[25] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optim. Methods Soft.*, vol. 11/12, pp. 625–653, Aug. 1999.

[26] A. Stephenne and B. Champagne, "Effective multi-path vector channel simulator for antenna array systems," *IEEE Trans. Veh. Technol.*, vol. 49, no. 6, pp. 2370–2381, Nov. 2000.

[27] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice–Hall, 1993.

[28] G. Dahlquist and A. Björck, *Numerical Methods*, ser. Automatic Computation. Englewood Cliffs, NJ: Prentice–Hall, 1974.

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