Subspace-Based Blind Channel Estimation for MIMO-OFDM Systems With Reduced Time Averaging

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***Abstract*—Among the various approaches recently proposed for blind estimation of wideband multiple-input–multiple-output (MIMO) wireless channels, subspace-based algorithms are particularly attractive due to their good performance and simple structure. These algorithms primarily exploit the orthogonality of the noise and signal subspaces of the correlation matrix of the received signals to estimate the unknown channel coefficients. In practice, the correlation matrix is unknown and must be estimated through time averaging over multiple received samples. To this end, the unknown channel must remain time invariant through the averaging process, which may pose a serious problem in practical applications. In this paper, to relax this requirement, we propose a novel subspace-based blind channel-estimation algorithm with reduced time averaging, as obtained by exploiting the frequency correlation among adjacent subcarriers in MIMO orthogonal frequency-division multiplexing (OFDM) systems. Simulation results show that the proposed approach outperforms other previously proposed methods within a reasonable averaging time over a Third-Generation Partnership Project (3GPP) spatial channel model.**

***Index Terms*—Blind channel estimation, coherence bandwidth, correlation matrix, MIMO-OFDM, subspace approach, WiMAX, 3GPP.**

I. Introduction

Among recent studies of multiple-input–multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems, *blind* channel estimation has received a great deal of attention and has become a vital area of research. Existing blind methods can broadly be categorized as statistical or deterministic: The former methods rely on assumptions on the statistics of the input sequence [1], [2], while the latter make no such assumption [3], [4]. In the first category, i.e., statistical approaches, blind channel estimation using second-order statistics can potentially achieve superior performance for a given time averaging interval than approaches based on higher order statistics [5], [6]. The second category, i.e., deterministic methods, is generally favored when the input statistics are unknown, or there may not be sufficient time samples to estimate them. To date, several interesting deterministic methods have been developed [7], [8]; however, most of them are exclusively for single-input–single-output (SISO) or single-carrier transmissions.

Amid second-order-statistics-based blind approaches, subspace-based estimation is attractive since estimates can often be obtained in a simple form by optimizing a quadratic cost function [9]. Without employing any precoder, a subspace-based method is proposed for OFDM systems by utilizing the redundancy introduced by the cyclic prefix (CP) [10], and it is further extended for MIMO-OFDM systems in [11]. Virtual carriers (VCs) are subcarriers that are set to zero with no information being transmitted. The presence of VCs provides another useful resource that can be used for channel estimation. Such a scheme is proposed for OFDM systems [12], and it is further extended to MIMO-OFDM systems in [13].

The aforementioned approaches primarily exploit the separability of the noise and signal subspaces by applying the eigenvalue decomposition (EVD) to the correlation matrix of the received signals. In practice, the correlation matrix can only be estimated by averaging over multiple time samples, given that the wireless channel is time invariant during this averaging period. Since the quadratic cost function is constructed from the eigenvectors of the noise subspace obtained from the EVD, the accuracy of the eigenvectors obtained from the sample correlation matrix dominates the performance of the estimation. Hence, in a time-invariant environment, the more time samples are averaged, the better the estimation performance is.

Considering that radio propagation conditions can only be invariant over a limited time interval (related to fading conditions, user mobility, etc.), it is legitimate to wonder how many samples are sufficient to obtain a sample correlation matrix meeting a certain level of accuracy in the channel estimate. A basic rule is to assure that the number of time samples must be no less than the dimension of the correlation matrix to make it full rank or invertible. However, to achieve the desired estimation accuracy in the presence of noise, the required number of time samples for the CP and VC approaches may become prohibitive.

More recently, variants of the statistics-based methods have been proposed, e.g., by inserting zero padding instead of CP for each OFDM block [14] or by introducing the so-called repetition index [15] and remodulation [16] on the received signal. However, the number of required time samples is still implicitly proportional to the size of the inverse fast Fourier transform in the OFDM modulator. We also note that deterministic approaches still need to accumulate data samples to algebraically obtain channel estimates, and their performance in noise improves as the number of samples increases. Therefore, as the dimension of the parameter space is increased in the MIMO-OFDM context, the number of samples required for deterministic methods to achieve an acceptable level of performance will also inevitably be increased.

In this paper, to relax the time invariance requirement in practical MIMO-OFDM systems, we propose a novel subspace-based blind channel estimation algorithm with reduced time averaging. This is achieved by exploiting the frequency correlation among adjacent subcarriers in OFDM transmissions through subcarrier grouping [17], for which some supportive field measurements can also be found in [18]. The resulting gain in performance comes at the cost of an ambiguity matrix with larger dimensions; however, this dimension can easily be reduced to the normal one when precoding is applied [19] or when the ratio of the coherence bandwidth to the channel bandwidth is large. Through simulations over Third-Generation Partnership Project (3GPP) spatial channel model (SCM) wireless channels, the proposed approach is shown to outperform the approach from [16]. In particular, it can achieve a normalized mean square error (NMSE) of 10^{-4} on the channel estimates within only 50 time samples (when the SNR $= 15$ dB), which is also very competitive over the deterministic approaches exclusively designed for SISO and single-carrier transmissions.

In a nutshell, the contribution of this paper is not only that it shows that the proposed blind approach can work with a small number of time samples but that it may also come with improved performance and robustness over existing statistical and deterministic methods.

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II. Problem Formulation

We consider a MIMO-OFDM system with N_T transmit and N_R receive antennas, employing N_C subcarriers. Let the OFDM symbol transmitted over the kth subcarrier be denoted as $\mathbf{x}[k] \stackrel{\text{def}}{=} [\,x_1[k]\;x_2[k]\;\cdots\;x_{N_T}[k]\,]^T$, where $x_q[k]$ is the symbol transmitted at the qth transmit antenna. Then, the OFDM symbol transmitted over the N_C subcarriers can be written as $\mathbf{x} \stackrel{\text{def}}{=} [\,\mathbf{x}[0]^T\;\mathbf{x}[1]^T\;\cdots\;\mathbf{x}[N_C-1]^T\,]^T$ and represents the *input vector*. At the receiver, let the received OFDM symbol over the kth subcarrier be denoted as $\mathbf{y}[k] \stackrel{\text{def}}{=} [\,y_1[k]\;y_2[k]\;\cdots\;y_{N_R}[k]\,]^T$, where $y_p[k]$ is the symbol received at the pth receive antenna. Then, the OFDM symbol received over N_C subcarriers can be written as $\mathbf{y} \stackrel{\text{def}}{=} [\,\mathbf{y}[0]^T\,\mathbf{y}[1]^T\cdots\mathbf{y}[N_C-1]^T\,]^T$ and represents the *observation*. In the following, we assume that 1) the length of the CP appended to each OFDM symbol is longer than the maximum excess delay of the channel; 2) the channel is time invariant at least over each OFDM symbol; and 3) the average power of the transmit symbol alphabet is normalized, i.e., $E[|x_q[k]|^2] = 1$.

Let $\mathbf{n}$ represent the *noise* defined in a similar way. Then, the input–output relation of the MIMO-OFDM system may be expressed by

$$\mathbf{y} = \boldsymbol{\mathcal{H}}\mathbf{x} + \mathbf{n} \tag{1}$$

where matrix $\mathcal{H} \stackrel{\text{def}}{=} \text{diag}(\mathbf{H}[0]\;\;\cdots\;\;\mathbf{H}[N_C-1])$ has size $(N_RN_C) \times (N_TN_C)$, with diagonal blocks defined as

$$\mathbf{H}[k] = \begin{bmatrix} h_{1,1}[k] & h_{1,2}[k] & \cdots & h_{1,N_T}[k] \\ h_{2,1}[k] & h_{2,2}[k] & \cdots & h_{2,N_T}[k] \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R,1}[k] & h_{N_R,2}[k] & \cdots & h_{N_R,N_T}[k] \end{bmatrix} \tag{2}$$

and $h_{p,q}[k]$ denotes the frequency response from the qth transmit to the pth receive antenna over the kth subcarrier.

In this paper, our interest lies in the blind second-order-statistics-based estimation of the channel coefficients, i.e., $\{h_{p,q}[k]\}$, directly from the observation $\mathbf{y}$.

III. Subspace-Based Blind Estimation

In the case of blind approaches based on second-order statistics, the main concern is to estimate the correlation matrix meeting a certain level of confidence over a time averaging interval as short as possible. We first briefly comment on the time averaging requirement in the *traditional* approaches: For subspace-based algorithms that apply channel estimation in the time domain and assuming that $N_R \geq N_T$ (see, e.g., [11] and [13]), the channel matrix is block Toeplitz and can be written as $\mathcal{H}_{td} = \sum_{l=0}^{L-1} \mathbf{B}^l \otimes \mathbf{H}(l)$, where L represents the channel order, $\mathbf{B}$ is an $[(N_C + N_{\text{cp}})N_F - L] \times (N_C + N_{\text{cp}})N_F$ *backward shift* matrix [20], with N_F denoting the number of OFDM symbols in each input vector and observation, N_{cp} denoting the length of the CP, and $\mathbf{H}(l) \stackrel{\text{def}}{=} (1/N_C)\sum_{k=0}^{N_C-1} \mathbf{H}[k]\exp(j2\pi kl/N_C)$.

Then, the dimension of the correlation matrix of the observations is $[(N_C + N_{\text{cp}})N_F - L]N_R$, which can be approximated by $N_CN_FN_R$ if $N_C \gg N_{\text{cp}}$ and $N_CN_F \gg L$. On the basis of these considerations, we can conclude that, in the context of MIMO-OFDM, we need to choose the number of time samples $T_{\text{av}} \geq N_CN_FN_R$ to achieve acceptable performance for these time-domain approaches. As N_C is normally chosen between 128 and 2048 to alleviate the adverse effects from the frequency-selective channels, we can see that these algorithms require an extremely large T_{av} for obtaining an acceptable time-averaged correlation matrix. Below, we develop an improved procedure that exploits correlation over the frequency domain to relax such a requirement.

A. Proposed Approach

In the context of MIMO-OFDM, although a pilot-based subspace method in the frequency domain was proposed in [21], to our knowledge, a blind one constructed directly from (1) has seldom been considered, mainly because there are a large number $N_TN_RN_C \geq N_TN_RL$ of unknowns to be estimated (recall that L represents the channel order). Nevertheless, the number of unknowns can be reduced by exploiting the frequency correlation among adjacent OFDM subcarriers with some loss in the estimation performance. In return, the dimension of the correlation matrix and, hence, the number of time samples required for time averaging can significantly be reduced. The details are given below for the case $N_R > N_T$; however, if oversampling is used at the receiver, the case $N_R \leq N_T$ is also possible [13].

Let the frequency span of P adjacent subcarriers reside inside the coherence bandwidth of the wireless channel, which is defined here as the range of frequencies over which the frequency response matrix of the MIMO channel does not appreciably change [22]. Let $\Omega \stackrel{\text{def}}{=} \{0, 1, \ldots, N_C - 1\}$, i.e., the index set of the N_C subcarriers, be partitioned into P disjoint subsets (assuming that $(N_C/P) = \zeta \in \mathbb{Z}^+$), with each subset denoted as $\Omega_p \stackrel{\text{def}}{=} \{\omega_{p,1}, \omega_{p,2}, \ldots, \omega_{p,\zeta}\}$, where $\omega_{p,i} \stackrel{\text{def}}{=} p - 1 + (i-1)P$, $i = 1, 2, \ldots, \zeta$, for $p = 1, 2, \ldots, P$. Note that $\Omega_1 \cup \Omega_2 \cup \cdots \cup \Omega_P = \Omega$ and $\Omega_i \cap \Omega_j = \emptyset$, where $\emptyset$ denotes the empty set. Define

$$\mathbf{x}_p \stackrel{\text{def}}{=} \{\mathbf{x}[k] | k \in \Omega_p\} = [\,\mathbf{x}[\omega_{p,1}]^T\;\;\mathbf{x}[\omega_{p,2}]^T\;\;\cdots\;\;\mathbf{x}[\omega_{p,\zeta}]^T\,]^T \tag{3}$$

and let $\mathbf{y}_p$ and $\mathbf{n}_p$ be defined in a similar way. Then, (1) can be rewritten for the pth subset as $\mathbf{y}_p = \boldsymbol{\mathcal{H}}_p\mathbf{x}_p + \mathbf{n}_p$, $p = 1, 2, \ldots, P$, where $\mathcal{H}_p \stackrel{\text{def}}{=} \text{diag}(\mathbf{H}[\omega_{p,1}]\;\;\cdots\;\;\mathbf{H}[\omega_{p,\zeta}])$ is assumed to be of full rank with size $(N_R\zeta) \times (N_T\zeta)$. The identification of $\mathcal{H}_p$ can then be achieved based on $\mathbf{R}_{\mathbf{y}_p} = E[\mathbf{y}_p\mathbf{y}_p^H]$, which can be rewritten as

$$\mathbf{R}_{\mathbf{y}_p} = \mathcal{H}_p\mathbf{R}_{\mathbf{x}_p}\mathcal{H}_p^H + \mathbf{R}_{\mathbf{n}_p} \tag{4}$$

where $\mathbf{R}_{\mathbf{x}_p} \stackrel{\text{def}}{=} E[\mathbf{x}_p\mathbf{x}_p^H]$ is assumed to be of full rank, and $\mathbf{R}_{\mathbf{n}_p} \stackrel{\text{def}}{=} E[\mathbf{n}_p\mathbf{n}_p^H] = \sigma_n^2\mathbf{I}$. Since the P adjacent subcarriers are assumed to reside inside the coherence bandwidth, the subchannel matrices $\mathcal{H}_p$, $p = 1, 2, \ldots, P$ can be *approximated*[1] by denoting $\bar{\mathcal{H}} \stackrel{\text{def}}{=} \mathcal{H}_1 = \mathcal{H}_2 = \cdots = \mathcal{H}_P$. Hence, an estimate of the correlation matrix in (4) can be obtained by

$$\hat{\mathbf{R}}_{\bar{\mathbf{y}}} = \frac{1}{PT_{\text{av}}}\sum_{j=1}^{T_{\text{av}}}\sum_{p=1}^{P}\mathbf{y}_{p(j)}\mathbf{y}_{p(j)}^H \tag{5}$$

where $\mathbf{y}_{p(j)} \in \mathbb{C}^{(N_R\zeta)\times 1}$ denotes the jth observation of $\mathbf{y}_p$ at some physical time t_j. Therefore, the number of the time samples T_{av} required can significantly be reduced since the dimension of the correlation matrix is reduced by a factor of P, and an averaging over P subsets, which is equivalent to the frequency averaging, is applied at each time epoch.

[1] In practice, there are always small variations of the subchannel matrices over the assumed coherence bandwidth. The effects of such small variations on the estimation performance are considered and analyzed in [23].

By applying the EVD to $\mathbf{R}_{\mathbf{y}_p}$, we can express (4) by $\mathbf{R}_{\mathbf{y}_p} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{U}$ is a matrix whose columns are the orthonormal eigenvectors of $\mathbf{R}_{\mathbf{y}_p}$, and which can be partitioned as $\mathbf{U} = [\mathbf{U}_s|\mathbf{U}_n] = [\mathbf{u}_1 \ \cdots \ \mathbf{u}_{d_s}|\mathbf{u}_{d_s+1} \ \cdots \ \mathbf{u}_{d_s+d_n}]$. The signal subspace can thus be denoted as $\Re(\mathbf{U}_s)$, while its orthogonal complement, i.e., the noise subspace, can be denoted as $\Re(\mathbf{U}_n)$, with $d_s \stackrel{\text{def}}{=} \text{rank}(\bar{\mathcal{H}}) = N_T\zeta$ and $d_n \stackrel{\text{def}}{=} (N_R - N_T)\zeta$. $\mathbf{\Lambda}$ is a diagonal matrix consisting of the corresponding eigenvalues of $\mathbf{R}_{\mathbf{y}_p}$ and is denoted as $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{d_s+d_n})$, with $\lambda_{\max} = \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{d_s+d_n} = \lambda_{\min} \geq 0$. Since $\bar{\mathcal{H}}$ and $\mathbf{U}_s$ share the same range space and are orthogonal to the range space of $\mathbf{U}_n$, we can arrive at $\mathbf{u}_j^H\bar{\mathcal{H}} = \mathbf{0}$, $j = d_s+1, \ldots, d_s+d_n$. Although $\bar{\mathcal{H}}$ can be solved from the set of homogeneous linear equations, due to finite time averaging intervals, only an estimate of the noise subspace $\mathbf{U}_n$ is available in practice. In this case, by denoting $\hat{\mathbf{u}}_j$ as the perturbed version of $\mathbf{u}_j$, we may obtain the channel estimate by minimizing a quadratic cost function given by $C(\bar{\mathcal{H}}) = \sum_{j=d_s+1}^{d_s+d_n} \|\hat{\mathbf{u}}_j^H\bar{\mathcal{H}}\|_2^2$ and avoiding the trivial solution $\bar{\mathcal{H}} = \mathbf{0}$ by introducing a suitable constraint as discussed below.

Let $\bar{\mathcal{H}}' \stackrel{\text{def}}{=} \bar{\mathcal{H}}'_\rho = [\mathbf{h}_1^\rho \ \ \mathbf{h}_2^\rho \ \ \cdots \ \ \mathbf{h}_{N_T}^\rho]$ for a fixed but arbitrarily selected integer $\rho \in [1, P]$, with $\mathbf{h}_q^\rho$ given as $\mathbf{h}_q^\rho = [\,h_{1,q}[\omega_{\rho,1}] \cdots h_{N_R,q}[\omega_{\rho,1}] \cdots h_{1,q}[\omega_{\rho,\zeta}] \cdots h_{N_R,q}[\omega_{\rho,\zeta}]\,]^T$ for $q = 1, 2, \ldots, N_T$. Then, minimizing the aforementioned quadratic cost function is equivalent to minimizing

$$C'(\bar{\mathcal{H}}') = \sum_{j=d_s+1}^{d_s+d_n} \left\|\bar{\mathcal{H}}'^T\hat{\mathbf{u}}_j^*\right\|_F^2 = \text{tr}(\bar{\mathcal{H}}'^T\hat{\mathbf{\Psi}}\bar{\mathcal{H}}'^*) \tag{6}$$

where $\hat{\mathbf{\Psi}} \stackrel{\text{def}}{=} \sum_{j=d_s+1}^{d_s+d_n} \hat{\mathbf{u}}_j^*\hat{\mathbf{u}}_j^T \in \mathbb{C}^{(N_R\zeta)\times(N_R\zeta)}$. Let the eigenvalues of $\hat{\mathbf{\Psi}}$ be ordered as $\gamma_{\min} = \gamma_1(\hat{\mathbf{\Psi}}) \leq \gamma_2(\hat{\mathbf{\Psi}}) \leq \cdots \leq \gamma_{(N_R\zeta)}(\hat{\mathbf{\Psi}}) = \gamma_{\max}$. Then, from the Rayleigh–Ritz theorem [20], we know that, for all $\mathbf{Q} \in \mathbb{C}^{(N_R\zeta)\times r}$

$$\gamma_1(\hat{\mathbf{\Psi}}) + \cdots + \gamma_r(\hat{\mathbf{\Psi}}) = \min_{\mathbf{Q}^H\mathbf{Q}=\mathbf{I}} \text{tr}(\mathbf{Q}^H\hat{\mathbf{\Psi}}\mathbf{Q}) \tag{7}$$

where r is a given integer with $1 \leq r \leq N_R\zeta$. The optimal solution $\hat{\mathbf{Q}}_o \in \mathbb{C}^{(N_R\zeta)\times r}$ is a matrix whose columns are chosen to be orthonormal eigenvectors corresponding to the r smallest eigenvalues of $\hat{\mathbf{\Psi}}$. Therefore, we can use (7) to obtain $\hat{\mathbf{Q}}_o$ and obtain the desired solution of (6) by $\bar{\mathcal{H}}'_o = \hat{\mathbf{Q}}_o^*\mathbf{A}$, where $\mathbf{A} \in \mathbb{C}^{r\times N_T}$ can be seen as an *ambiguity matrix*. To ensure that enough basis functions are available for the adequate representation of the unknown channel matrix, r should be chosen so that $r = \dim[\Re(\hat{\mathbf{Q}}_o^*)] \geq \dim[\Re(\bar{\mathcal{H}}_o)] = N_T\zeta$; in our case, we simply choose $r = N_T\zeta$.

B. Further Comments on the Proposed Approach

The computational complexity for the proposed algorithm is summarized in Table I in terms of the number of required (complex) flops. To meet the minimum requirement of time averaging in connection with (5), i.e., avoid rank deficiency, we need $PT_{\text{av}} \geq N_R\zeta$ or, equivalently, $T_{\text{av}} \geq N_RN_C/P^2$. Therefore, the reduction in the averaging time T_{av} is proportional to the square of the number of subsets, i.e., P. Assuming that $PT_{\text{av}} = N_R\zeta$ is chosen (i.e., the dimension of the correlation matrix), the total computational complexity of the proposed algorithm is given as $\mathcal{O}((N_R\zeta)^3)$, including the two EVD operations. Although the steps of matrix computations are similar to those of the traditional approaches, the complexity of the first EVD operation is generally much lower. A reduction by $P^3 \approx 10^{4.5}$ flops in the EVD operation can be expected for a typical value of $P = 32$.

TABLE I
COMPUTATIONAL COMPLEXITY OF THE PROPOSED ALGORITHM

Main Step	Complexity (flops)
1. Compute $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$.	$\frac{3}{2}(PT_{av})(N_R\zeta)^2$
2. Given $d_s = N_T\zeta$ and $d_n = (N_R - N_T)\zeta$, find eigenvectors $\hat{\mathbf{u}}_j$, $j = d_s+1, \cdots, d_s+d_n$, which correspond to the d_n smallest eigenvalues of $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$.	$\mathcal{O}((N_R\zeta)^3)$
3. Form the matrix $\hat{\mathbf{\Psi}}$ from the $\hat{\mathbf{u}}_j$'s.	$2d_n(N_R\zeta)^2$
4. Find $\hat{\mathbf{Q}}_o$, whose columns are the eigenvectors of the $N_T\zeta$ smallest eigenvalues of $\hat{\mathbf{\Psi}}$.	$\mathcal{O}((N_R\zeta)^3)$
5. Obtain channel estimate $\bar{\mathcal{H}}'_o$.	$2(N_T\zeta)(N_R\zeta)N_T$

A sufficient condition for $\bar{\mathcal{H}}'$ to be uniquely determined up to an ambiguity matrix is $d_s \geq N_R$ or, equivalently $\zeta \geq N_R/N_T$ (see the Appendix). Specifically, the aforementioned algorithm and identifiability conditions were derived for the case of single-symbol processing, i.e., one OFDM input symbol at a time. However, our results can easily be extended to the general case where the OFDM system operates on blocks of multiple OFDM symbols at a time, as is necessary with many traditional approaches [11], [13].

IV. NUMERICAL RESULTS

In this section, the performance of the proposed algorithm is evaluated via numerical simulations. We explore practical scenarios where both the WiMAX specification and the 3GPP-SCM are considered. In each case, we compute the NMSE[2] on the channel estimates for different values of relevant system parameters. The performance of a recent algorithm [16] is also given for comparison, along with the minimum constrained Cramer–Rao bound (CRB) [24] for the specific estimation scenario under consideration.

We consider a MIMO-OFDM system with two transmit ($N_T = 2$) and three receive antennas ($N_R = 3$). The number of subcarriers used in the OFDM system is 256 ($N_C = 256$). For each time epoch, the incoming symbol streams are independent identically distributed quadrature phase shift keying (QPSK) symbols. The SNR is defined as the ratio of the signal power to the noise power on a subcarrier basis. All simulation results are obtained by averaging over 200 independent Monte Carlo runs. In addition, the wireless channel is assumed to remain stationary over the time-averaging intervals. In this paper, to evaluate the performance of the proposed algorithm, we employ $\mathbf{A} = (\hat{\mathbf{Q}}_o^*)^\dagger\bar{\mathcal{H}}'$, assuming that $\bar{\mathcal{H}}'$ is known (e.g., [13]).

To determine the maximum achievable P in practical scenarios, we consider adapting part of the Mobile WiMAX OFDMA-PHY for our OFDM system and to simulate it over the 3GPP-SCM. Given that the OFDM useful symbol duration is 91.4 μs and CP length is 11.4 μs, the subcarrier spacing is 10.94 kHz. Since we consider $N_C = 256$, the channel bandwidth is approximately 2.5 MHz. For each time epoch, the incoming QPSK symbols are chosen only to span over one OFDM symbol. In the 3GPP-SCM setup, the carrier frequency is 2.5 GHz, the base station antenna spacing is 10λ, and the mobile station antenna spacing is $\lambda/2$, where λ is the wavelength at the carrier frequency. Channel coefficients of each 3GPP-SCM scenario are generated according to [25]. We also present the asymptotic performance[3] of the approach given in [16], which is tailored into our system setup for comparison.

[2]The NMSE of the channel estimate is defined by $E[|h_{i,j}[k] - \hat{h}_{i,j}[k]|^2]/E[|h_{i,j}[k]|^2]$.

[3]See [16, eq. (49)], which provides a lower bound on the attainable mean square error with this specific algorithm, based on a first-order perturbation analysis.

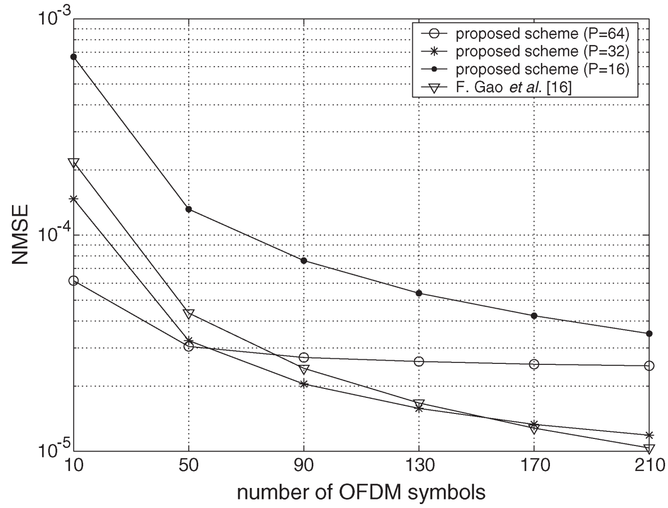

Fig. 1. NMSE versus number of OFDM symbols over 3GPP Urban Macro (SNR $= 20$ dB).

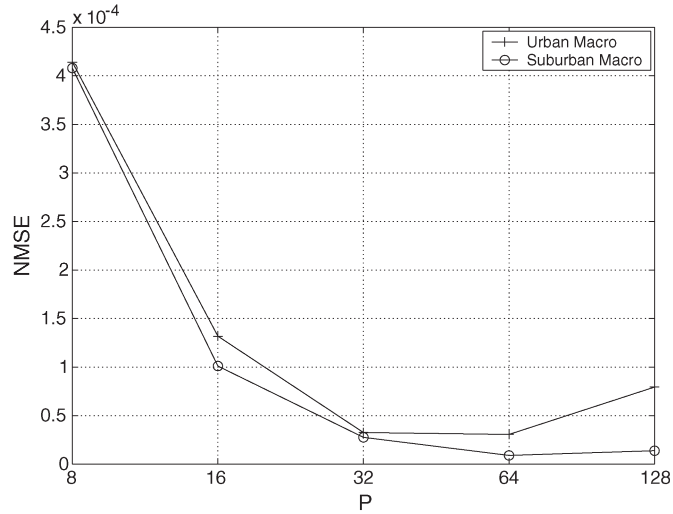

Fig. 3. NMSE versus P (when the number of OFDM symbols $T_{\rm av} = 50$).

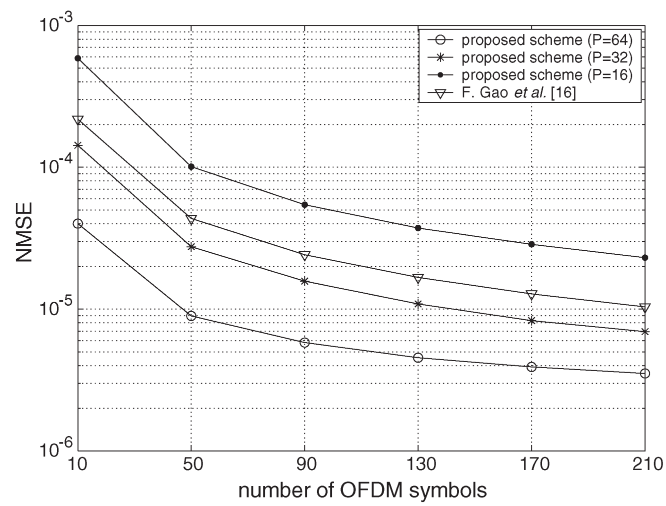

Fig. 2. NMSE versus number of OFDM symbols over 3GPP Suburban Macro (SNR $= 20$ dB).

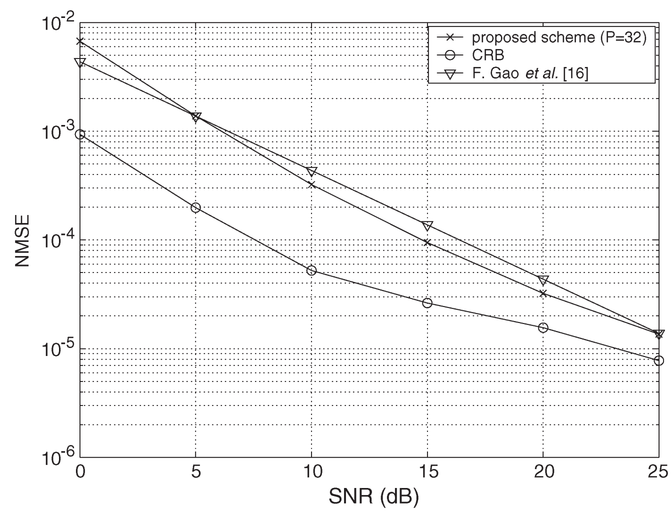

Fig. 4. Performance of proposed scheme over 3GPP Urban Macro ($T_{\rm av} = 50$) as a function of the SNR.

Figs. 1 and 2 show the NMSE versus the number of OFDM symbols over the 3GPP Urban Macro and Suburban Macro models, respectively (SNR $= 20$ dB is considered for both the scenarios). We observe that the proposed algorithm can reach NMSE $\leq 3.0 \times 10^{-5}$ in all the scenarios within 50 OFDM symbols, and the proposed algorithm is particularly effective in the 3GPP Suburban Macro scenario. We also present the NMSE versus the choice of P over the aforementioned channel models in Fig. 3, and we can observe that the best choice of P should fall between 32 and 64.

Figs. 4 and 5 show the NMSE and the corresponding CRB over 3GPP Urban Macro and Suburban Macro models, respectively. We can observe that the asymptotic performance of the referenced algorithm [16] does not show advantages over our approach. This may be explained by the fact that the dimension of its correlation matrix is approximately given as $N_R N_C = 768$, as compared with $N_R \zeta \leq 24$ in our approach. Therefore, the eigenvectors obtained from the sample correlation matrix must lead to less satisfactory estimation performance when time samples are less than 100. It should be noted that, unlike the traditional approaches that require explicit channel-order information for estimating the channel matrix, the proposed

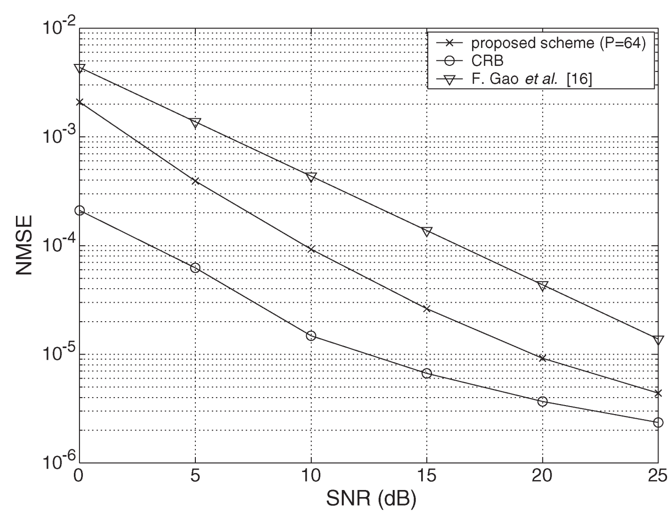

Fig. 5. Performance of the proposed scheme over 3GPP Suburban Macro ($T_{\rm av} = 50$) as a function of the SNR.

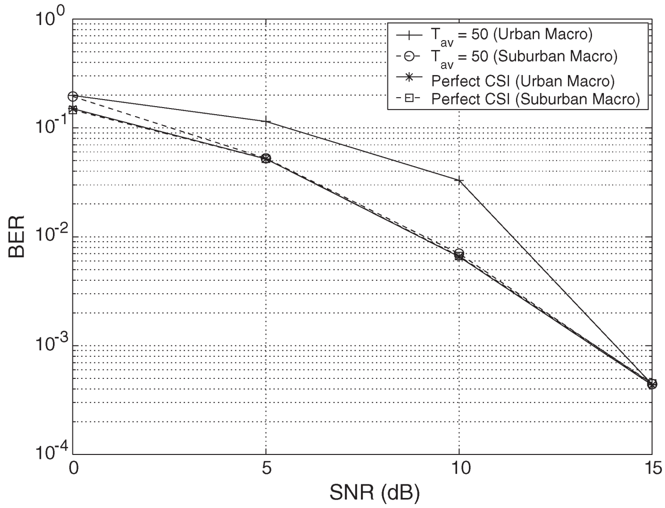

Fig. 6. BER of the proposed scheme over 3GPP Urban Macro and Suburban Macro.

algorithm requires only an upper bound on the channel order to determine the range of the parameter P. Therefore, the proposed algorithm is less sensitive to the channel modeling errors. Finally, we present the bit error rate (BER) curves of the proposed algorithm over Urban and Suburban Macro models in Fig. 6. For SNR $\geq$ 15 dB, we observe that the proposed algorithm employing only 50 OFDM symbols can reach the same performance as if perfect channel-state information (CSI) is known at the receiver.

V. Conclusion

The main contribution of this paper is in developing and evaluating a new scheme to overcome some fundamental limitation of the subspace-based approach when applied to MIMO-OFDM transmission over time-varying channels. Specifically, when considering the time invariance requirement of a practical MIMO-OFDM system with a large number of OFDM subcarriers, e.g., 128 or more, the traditional subspace-based methods require an extremely large number of time samples to obtain a good time-averaged correlation matrix, making them impractical. In this paper, by exploiting the frequency correlation among adjacent subcarriers (i.e., within the coherence bandwidth) through the concept of subcarrier grouping, we have proposed a novel subspace-based estimation method that requires a significantly smaller number of time samples. The numerical results of the proposed method in a realistic wireless channel environment have shown that it could achieve a better estimation accuracy than other previously proposed methods within a reasonable time averaging interval.

Appendix

The orthogonality relationship $\mathbf{U}_n^H \bar{\mathcal{H}} = \mathbf{0}$ can be viewed as a homogeneous linear system whose solution is $\bar{\mathcal{H}} = \mathbf{P}_{\mathfrak{R}(\mathbf{U}_n^{H\dagger})^{\perp}} \mathbf{Y}$ for some arbitrary matrix $\mathbf{Y}$, with $\mathbf{P}_{\mathfrak{R}(\mathbf{U}_n^{H\dagger})^{\perp}} \overset{\text{def}}{=} \mathbf{I} - \mathbf{U}_n^{H\dagger}\mathbf{U}_n^H$ [26]. Let $\bar{\mathcal{H}}' = \mathbf{Q}_o^* \mathbf{A}$ denote a solution to (7) constructed from the exact $\mathbf{Q}_o$ with the corresponding $\bar{\mathcal{H}}$. In addition, let $\bar{\mathcal{H}}_1' = \bar{\mathcal{H}}'\mathbf{B} = (\mathbf{Q}_o^*\mathbf{A})\mathbf{B} = \mathbf{Q}_o^*\mathbf{A}_1$, where $\mathbf{B}$ is a square matrix of dimension N_T. Then, it can be verified that $\bar{\mathcal{H}}_1 = \bar{\mathcal{H}}(\mathbf{I}_\zeta \otimes \mathbf{B})$, which is also of the form $\mathbf{P}_{\mathfrak{R}(\mathbf{U}_n^{H\dagger})^{\perp}} \mathbf{Y}_1$, with $\mathbf{Y}_1 = \mathbf{Y}(\mathbf{I}_\zeta \otimes \mathbf{B})$. This shows that $\bar{\mathcal{H}}'$ is uniquely determined up to the ambiguity matrix $\mathbf{A}$ ($\overset{\text{def}}{=} \mathbf{A}_1\mathbf{B}^{-1}$). Recall that the dimension of the solution space of a homogeneous linear system $\mathbf{A}_{m\times n}\mathbf{X}_{n\times p} = \mathbf{0}_{m\times p}$ equals $p \cdot \dim[\mathfrak{N}(A)]$; the dimension of the solution space (in $\bar{\mathcal{H}}$) in our case is then given as $d_s \cdot \dim[\mathfrak{N}(\mathbf{U}_n^H)]$. Since $\bar{\mathcal{H}}$ is a block diagonal matrix with $d_s N_R$ nonzeros entries, a sufficient (but not necessary) condition in terms of the dimension of the solution space can thus be written as $d_s \dim[\mathfrak{N}(\mathbf{U}_n^H)] \geq d_s N_R$ or simply $d_s \geq N_R$. We can rearrange the aforementioned inequality and arrive at $\zeta \geq (N_R/N_T)$.

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Performance Analysis of Fixed-Gain Amplify-and-Forward Relaying With MRC

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***Abstract*—Relay transmission has recently attracted much attention since it can offer spatial diversity with single antenna terminals. This paper addresses the performance of a multiple-relay system with fixed-gain amplify-and-forward (AF) relaying in Nakagami-m fading. A tight upper bound on the average symbol error probability (SEP) is obtained for a system with K relays and when the maximal ratio combining (MRC) is used at the destination. Based on the obtained bound, a maximum diversity order of $m(K+1)$, where m is the fading parameter, is shown. Moreover, the problem of power allocation (PA) to minimize the SEP upper bound is investigated. Numerical results illustrate significant gains provided by the proposed PA over equal PA (EPA) under various channel conditions.**

***Index Terms*—Amplify-and-forward (AF) protocol, diversity order, maximum ratio combining (MRC), Nakagami-m fading, performance analysis, power allocation (PA), relay communications.**

I. Introduction

Recently, relay communication has attracted a lot of research interest due to its ability to offer spatial diversity while still satisfying the size and power constraints of mobile devices [1]–[9]. The benefit comes from the cooperation of relays in a network to assist transmission from the source to the destination. This is because, with the assistance of relays, the transmission from the source to the destination can be performed over a virtual antenna array [3], [4].

The most popular signal processing methods at relays are decode-and-forward (DF) and amplify-and-forward (AF). For DF, cooperative relays first try to decode the received information and then regenerate a new version to transmit to the destination [3], [10]–[12]. On the other hand, for AF, the relays retransmit scaled versions of the received information to the destination without decoding them. Therefore, AF needs no sophisticated processing at the relays or the destination [3], [5], [13]. To limit the transmit power at the relays, the received signal at each relay can be amplified with a varying or fixed gain. The varying-gain relaying scheme maintains the constant transmit power at the relays at all times, but it requires knowledge of the instantaneous channel gains of all the source–relay links. To reduce the complexity at the relays, the fixed-gain relaying scheme has been proposed, which maintains the long-term average transmit power at each relay [6].

For AF relaying, optimal maximal ratio combining (MRC) has been considered in [4], [6], and [14]–[16]. In [14] and [15], the performance of a single-relay system with a multiple-antenna destination is considered. In [14], under Rayleigh fading, the maximum diversity orders with varying and fixed gains are shown to be $2N$ and $N+1$, respectively, where N is the number of antennas at the destination. For Nakagami-m fading, the exact MRC performance for the fixed-gain relay system is derived with the help of Kampe de Feriet's function in [15]. For multiple-relay systems with a single antenna in both transmitter and receiver, the work in [4] investigates the symbol error probability (SEP) for AF with the varying gains at relays and with the varying MRC at the destination. In [16], the performance of relay systems with fixed-gain relays over generalized fading channels is determined by using the geometric-mean (G-M) bound on the instantaneous signal-to-noise ratio (SNR). This bound is then used to evaluate the outage and error probabilities by using the well-known moment generating function (MGF) approach. Using the same technique as in [4], the authors in [6] consider the outage probability behaviors of the relay systems with both varying and fixed gains in Nakagami-m fading at the high SNR region. The work shows that when the fading severity difference between the source-to-relay and the relay-to-destination channels exists, both strategies can achieve the same diversity gain, whereas the fixed-gain AF relaying strictly loses some coding gains. However, the limitation of the method used in [4] and [6] is that it can only predict the asymptotic performance of the systems at the high SNR region, whereas in some applications, the performance at low and medium SNRs might be more important [17].

It should be pointed out that the foregoing works on AF relay systems assume that the total transmit power is uniformly allocated over the source and the relays, i.e., equal power allocation (EPA). Obviously, the performance of such AF systems with EPA is inferior to that with optimal power allocation (OPA) among the source and all the cooperating relays [18]. Such OPA schemes have recently been considered in [18]–[22]. However, all the previously proposed OPA schemes are restricted to Rayleigh fading channels. It is well known that the Nakagami-m distribution provides a much better fitting for the fading channel distributions than the Rayleigh distribution in many scenarios [23]. In fact, it includes the Rayleigh distribution $(m=1)$ as a special case [23].

Motivated by the foregoing observations, this paper first obtains a tight upper bound on the SEP for fixed-gain AF multiple-relay systems with MRC at the destination. It is illustrated that the obtained bound is effective over a wide range of channel settings. It is also shown that the MRC can achieve the maximum diversity order of $m(K+1)$. Based on the minimization of the obtained bound on the SEP performance, a novel power allocation (PA) scheme is proposed. In particular, under the assumption that all the channel statistics are available at the destination, the proposed PA coefficients can easily be calculated at the destination and then fed back to the source and relays during a startup phase. The proposed PA demonstrates significant performance gains over EPA.

The rest of this paper is organized as follows. Section II describes the system model under consideration. The upper bound on the overall

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