Design Criteria for Omnidirectional STBC in Massive MIMO Systems

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***Abstract*—In this letter, we study the design criteria for omnidirectional space time block codes (STBCs) in downlink massive multiple-input multiple-output (MIMO) systems. To this end, we first derive a tighter upper bound on the average error probability and then use it to formulate two design criteria, namely the rank criterion and the sum determinant criterion (SDC). The rank criterion is similar to that for conventional MIMO, but it is obtained here by invoking the new upper bound for massive MIMO. The SDC is more accurate than its conventional MIMO counterpart since it takes into account the sum of the determinants of the codeword difference matrices, as opposed to the minimum of these determinants. In addition, the SDC provides a useful metric for the performance comparison and design of STBCs. The effectiveness of the proposed SDC for omnidirectional STBC is validated by simulations for massive MIMO systems.**

***Index Terms*—Massive MIMO, omnidirectional, space time block code, STBC, design criteria, diversity gain, coding gains.**

I. INTRODUCTION

MASSIVE multiple-input multiple-output (MIMO) is recognized as a key enabling technology to meet the exacting data traffic requirements for the fifth generation (5G) of cellular wireless networks. The basic idea is to equip the base station (BS) with a very large number of antennas, on the order of hundred or more, to achieve an unprecedented spectral and energy efficiency, considered unfeasible with conventional MIMO [1]. As an emerging technology for 5G, several major theoretical and practical challenges remain on the way to achieving the full potential offered by massive MIMO [2].

Especially, when establishing a radio link from a BS to multiple users, the channel state information (CSI) is not readily available at the transmitter. Hence, non-CSI transmission techniques such as space-time block codes (STBCs) are required which must be appropriately designed for large number of antennas [1]. Omnidirectional STBCs, which guarantee a strong receiving signal in any spatial direction and are therefore often used in information broadcasting scenarios, are also of great interest for massive MIMO systems. These techniques have been recently investigated in [3] where the authors adapt low-dimensional STBCs to large-scale antennas arrays under omnidirectional transmission constraints. To this end, precoding matrices are designed and combined with conventional STBCs matrices, resulting in new high dimensional codewords that better suit massive MIMO channels.

Research on the application of omnidirectional STBC in massive MIMO is still at an early stage and many issues remain unsettled. One basic question is how to design an STBC scheme for massive MIMO such that it exhibits a satisfactory performance in terms of bit error rate (BER). In this regard, it is important to investigate practical STBC criteria for large-scale antenna systems, similar to the well-known rank and minimum determinant criteria (MDC) for conventional MIMO [4]. The other issue is how to devise a quantitative metric that can consistently characterize the relative BER performance of different STBCs. For instance, in traditional MIMO, the MDC of a code achieving a given BER may exceed that of another code with better performance [5].

In this letter, we first derive a tighter upper bound on the average error probability which serves as a basis in the formulation of two STBC design criteria for omnidirectional massive MIMO. The first metric turns out to be similar to the rank criterion for the conventional MIMO [4], yet it is obtained here by invoking the new upper bound for massive MIMO. The second metric, termed sum determinant criterion (SDC), takes into account the sum of the determinants of the codeword difference matrices, rather than the minimum of these determinants as for MDC. Therefore, SDC exhibits additional accuracy compared to MDC which paves the way to the design of STBCs with better performance. Furthermore, in certain cases where MDC cannot consistently predict the STBCs' BER performance, SDC is shown to overcome this drawback. We then show that the proposed criteria hold for small-scale MIMO and thus, are valid regardless of the number of BS antennas. Finally, the theoretical results are supported by computer simulations.

II. SYSTEM MODEL

We consider a downlink multiuser massive MIMO system where a BS equipped with M antennas broadcasts information signals, encoded using a low-dimensional omnidirectional STBC to K user terminals (UTs). For simplicity, we consider single-antenna UTs, but extension to multi-antenna UTs is straightforward. Similar to [3], we assume that the BS and UTs are operating over narrow-band Rayleigh flat-fading channels, and that the CSI is available at each UT but not at the BS.

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Let $\mathbf{y}_k \in \mathbb{C}^{1\times T}$ for $k \in \{1, 2, \ldots, K\}$ denotes the vector of symbols received by the *k*th user during *T* consecutive time slots. This vector can be expressed as

$$\mathbf{y}_k = \mathbf{h}_k \mathbf{P}\mathbf{C} + \mathbf{w}_k, \tag{1}$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times M}$ is the channel vector between the BS antennas and the *k*th user, $\mathbf{P} \in \mathbb{C}^{M\times N}$ is the precoding matrix, $\mathbf{C} \in \mathbb{C}^{N\times T}$ is the STBC matrix with $N \ll M$ and $N \leq T$ where the last condition is needed to achieve diversity order *N* [4]. The entries of STBC matrix $\mathbf{C}$ are given by $c_{i,j} = \mathscr{F}_{i,j}(s_1, s_2, \ldots, s_Q)$ where $\mathscr{F}_{i,j}$ is a complex-valued function and $s_q \in \mathbb{C}$ are the information symbols, taken from a constellation such as PSK or QAM. The parameter *Q* is the number of embedded symbols in $\mathbf{C}$ and can be expressed as $Q = RT$ where *R* is the rate of the STBC. The set of all possible matrices $\mathbf{C}$ (or codewords) is referred to as the STBC codebook; we assume that any codeword can be transmitted with equal probability. In (1), $\mathbf{w}_k \in \mathbb{C}^{1\times T}$ is an additive white Gaussian noise (AWGN) noise with distribution $\mathcal{CN}(\mathbf{0}_{1\times T}, \rho^{-1}\mathbf{I}_T)$, where under proper normalization (see below), $\rho \in \mathbb{R}$ is the signal-to-noise (SNR) at each UT.

In our simulation work, we consider the one-ring scattering model under a far-field assumption. Specifically, the channel vector $\mathbf{h}_k$ in (1) is distributed as $\mathcal{CN}(\mathbf{0}_M, \mathbf{R}_k)$ where $\mathbf{R}_k = \int_{-\pi/2}^{\pi/2} \mathbf{a}^H(\theta)\mathbf{a}(\theta)S_k(\theta)d\theta \in \mathbb{R}^{M\times M}$ is the channel covariance matrix with $\mathbf{a}(\theta) \in \mathbb{C}^{1\times M}$ and $S_k(\theta) \in \mathbb{R}_+$ being the array manifold and the angular power spectrum (APS) at the angle θ, respectively [3]. In the case of a uniform linear array (ULA), it is shown in [6], that for any fixed positive integers *p* and *q*, we have

$$\lim_{M\to\infty} \Big[\mathbf{R}_k - \mathbf{U}_M^H \mathbf{D}_k \mathbf{U}_M\Big]_{p,q} = 0, \tag{2}$$

where $\mathbf{U}_M \in \mathbb{C}^{M\times M}$ is the unitary *M* point discrete Fourier transform (DFT) matrix, and $\mathbf{D}_k$ is a diagonal matrix with entries $[\mathbf{D}_k]_{i,i} = \beta_k M S_k(f(\psi_{i-1}))[f(\psi_i) - f(\psi_{i-1})]$, where in the last expression β_k is the large scale fading gain, $\psi_0 = 0$, $\psi_i = i/M$, and $f(\psi) = \arcsin(2\psi - 1)$.

In order to achieve a spatial diversity order of *N*, the STBC size must be selected such that $N \leq \mathrm{rank}(\mathbf{R}_k)$. The precoding matrix $\mathbf{P}$, which adapts the small dimensional STBC for transmission over a massive MIMO channel must be of rank *N* and can be designed based on the constant-amplitude zero autocorrelation sequences under the constraints of equal power on the transmitting antennas and omnidirectional transmission [3]. For power normalization purposes, it is assumed that $\mathrm{Tr}(\mathbf{P}\mathbf{P}^H) = 1$, $\mathbb{E}(\mathrm{Tr}(\mathbf{C}\mathbf{C}^H)) = T$, and $\mathrm{Tr}(\mathbf{R}_k) = M$.

III. STBC Design Criteria for Massive MIMO

Let $P_e \equiv P_e(\rho)$ denote the probability of a decoding error as a function of SNR ρ and assume that there exists an upper bound $P_e \leq P_e^{\mathrm{ub}}$, which in turn can be written as $P_e^{\mathrm{ub}} \triangleq \zeta(C_g^{\mathrm{ub}}\rho)^{-D_g^{\mathrm{ub}}}$ where ζ is a constant. The parameters D_g^{ub} and C_g^{ub} are then called the diversity gain and coding gain associated to the upper bound, respectively [3], [4]. We define an asymptotic upper bound P_e^{ub} as an upper bound on $\lim_{M\to\infty} P_e$, and the corresponding parameters D_g^{ub} and C_g^{ub} are referred to as the asymptotic diversity and coding gain, respectively. In this section, we first derive the designing criteria of omnidirectional STBC in massive MIMO systems according to the conventional notions in [4]. We then propose a tighter upper bound on the error probability that allows us to formulate more accurate design criteria.

A. Conventional Design Criteria

Let $\hat{\mathbf{C}}_{\mathrm{ML}}^{(k)}$ and $P_e^{(k)}$ denote the codeword detected by the maximum likelihood (ML) estimator and the average error probability of the *k*th user, respectively.

We can write $P_e^{(k)}$ as

$$P_e^{(k)} = \mathbb{E}_{\mathbf{h}_k}\left[\frac{1}{J}\sum_{i=1}^{J} \mathbb{P}\{\hat{\mathbf{C}}_{\mathrm{ML}}^{(k)} \neq \mathbf{C}_i \mid \mathbf{C} = \mathbf{C}_i, \mathbf{h}_k\}\right], \tag{3}$$

where $\mathbf{C}_i$ is the *i*th STBC codeword and *J* is the codebook size. Let $r_{i,j}^{(k)}$ and $\lambda_n^{(k)}(i,j)$ denote the rank and *n*th non-zero largest eigenvalue of $\mathbf{\Delta}_{i,j}^{(k)}$, respectively, where

$$\mathbf{\Delta}_{i,j}^{(k)} \triangleq (\mathbf{C}_i - \mathbf{C}_j)^H \mathbf{P}^H \mathbf{R}_k \mathbf{P}(\mathbf{C}_i - \mathbf{C}_j) \in \mathbb{C}^{T\times T}. \tag{4}$$

Applying the well-known union bound to (3) and using the results in [3], [7], we have

$$\begin{aligned} P_e^{(k)} &\leq \frac{1}{J}\sum_{i=1}^{J}\sum_{\substack{j=1\\ j\neq i}}^{J}\left(\prod_{n=1}^{r_{i,j}^{(k)}} \lambda_n^{(k)}(i,j)\right)^{-1} \rho^{-r_{i,j}^{(k)}} \\ &= \frac{2}{J}\sum_{i=2}^{J}\sum_{j=1}^{i-1}\left(\prod_{n=1}^{r_{i,j}^{(k)}} \lambda_n^{(k)}(i,j)\right)^{-1} \rho^{-r_{i,j}^{(k)}}. \end{aligned} \tag{5}$$

where the equality follows from the fact that $\mathbf{\Delta}_{i,j}^{(k)} = \mathbf{\Delta}_{j,i}^{(k)}$.

In order to adapt STBCs to massive-MIMO setups, precoding can be used to produce equivalent, spatially independent parallel channels for the STBC [3]. The sufficient condition for the corresponding precoding matrix $\mathbf{P} \in \mathbb{C}^{M\times N}$ is

$$\lim_{M\to\infty} \mathbf{P}^H \mathbf{R}_k \mathbf{P} = \frac{1}{N}\mathbf{I}_N. \tag{6}$$

In [3], a precoding matrix design satisfying this condition in the case of ULA (with additional assumptions of omnidirectional transmission and equal antenna power) is presented, which we use in our simulation work.

Combining (4), and (6) gives

$$\mathbf{\Delta}_\ell \triangleq \lim_{M\to\infty} \mathbf{\Delta}_{i,j}^{(k)} = \frac{1}{N}(\mathbf{C}_i - \mathbf{C}_j)^H(\mathbf{C}_i - \mathbf{C}_j), \tag{7}$$

where $\ell \triangleq (i-2)(i-1)/2 + j$, for $1 \leq j \leq i-1$ and $2 \leq i \leq J$. Thus, the eigenvalues of $\mathbf{\Delta}_{i,j}^{(k)}$ converge to the eigenvalues of $\mathbf{\Delta}_\ell$. Furthermore, since $\mathbf{P}^H\mathbf{R}_k\mathbf{P}$ converges to a nonsingular matrix, the rank of $\mathbf{\Delta}_{i,j}^{(k)}$ converges to the rank of matrix $\mathbf{\Delta}_\ell$. In the sequel, we refer to $\mathbf{\Delta}_\ell$ as a difference matrix. Letting r_ℓ and $\lambda_n^{(\ell)}$ denote the rank and *n*th non-zero eigenvalue of $\mathbf{\Delta}_\ell$, and passing to the limit in (5), we obtain

$$\lim_{M\to\infty} P_e^{(k)} \leq \frac{2}{J}\sum_{\ell=1}^{J'} \Lambda_\ell^{-1} \rho^{-r_\ell}, \tag{8}$$

where $J' \triangleq J(J-1)/2$ and $\Lambda_\ell \triangleq \prod_{n=1}^{r_\ell} \lambda_n^{(\ell)}$. The latter quantity, referred to as the *non-zero eigenvalue product*, is equal to the determinant of $\boldsymbol{\Delta}_\ell$ when the latter has full rank.

Lemma 1: Asymptotic upper bounds $P_e^{\mathrm{ub}_1}$ and $P_e^{\mathrm{ub}_2}$ exist for P_e, where $\lim_{M\to\infty} P_e \le P_e^{\mathrm{ub}_1} \le P_e^{\mathrm{ub}_2}$ and

$$P_e^{\mathrm{ub}_1} \triangleq \frac{2K}{J}\sum_{\ell=1}^{J'} \Lambda_\ell^{-1}\rho^{-r_\ell} \tag{9a}$$

$$P_e^{\mathrm{ub}_2} \triangleq K(J-1)\left(\left(\max_\ell \Lambda_\ell^{-1}\right)^{-\frac{1}{D_g^{\mathrm{ub}_2}}}\rho\right)^{-\min_\ell r_\ell}. \tag{9b}$$

Proof: In the absence of cooperation, it is reasonable to assume that the UTs operate independently of each other. Moreover, in a common information broadcasting scenario, it is intended that the transmitted codeword can be decoded successfully by all the UTs [3], i.e., an error is rendered if at least one of the users cannot correctly detect its received signal. Consequently, the error probability P_e can be expressed as

$$P_e = 1 - \prod_{k=1}^{K}\left(1 - P_e^{(k)}\right). \tag{10}$$

We can also assume that $(J-1)(\max_\ell \Lambda_\ell^{-1})\rho^{-\min_\ell r_\ell} < 1$ and $\rho > 1$, which can be readily satisfied at moderate or larger SNRs. Then, the upper bound in (8) is guaranteed to be smaller than 1. Therefore, from (10) and (8) we have

$$\lim_{M\to\infty} P_e \le 1 - \left(1 - \frac{2}{J}\sum_{\ell=1}^{J'}\Lambda_\ell^{-1}\rho^{-r_\ell}\right)^K \tag{11}$$

$$\le \frac{2K}{J}\sum_{\ell=1}^{J'}\Lambda_\ell^{-1}\rho^{-r_\ell} = P_e^{\mathrm{ub}_1} \tag{12}$$

where (12) is due to the fact that $1 - Kx \le (1-x)^K$ for $0 \le x \le 1$x. This equation provides our new upper bound $P_e^{\mathrm{ub}_1}$ and by continuing this development, we have

$$P_e^{\mathrm{ub}_1} \le K(J-1)\Bigg(\overbrace{\left(\max_i \Lambda_\ell^{-1}\right)^{-\frac{1}{D_g^{\mathrm{ub}_2}}}}^{=C_g^{\mathrm{ub}_2}}\rho\Bigg)^{-\overbrace{\min_\ell r_\ell}^{=D_g^{\mathrm{ub}_2}}} = P_e^{\mathrm{ub}_2}, \tag{13}$$

where $P_e^{\mathrm{ub}_2}$ is the well-known upper bound commonly used for deriving the conventional design criteria [3], [4]. ■

According to (13), in order to design a good STBC for massive MIMO systems: *i*) $D_g^{\mathrm{ub}_2}$ should be maximized, *ii*) $C_g^{\mathrm{ub}_2}$ should be maximized. The former objective corresponds to the rank criterion while the latter corresponds to the MDC.

These criteria for massive MIMO settings are similar to those commonly employed in the design and performance analysis of conventional STBC [4]. However, as we will show in Section IV, there exist inconsistencies between the BER performance of some STBCs and their corresponding coding gains, which suggests that using $P_e^{\mathrm{ub}_2}$ to derive design criteria cannot always lead to accurate results [5]. In the next subsections, however, we explore the tighter bound $P_e^{\mathrm{ub}_1}$ to design more accurate design criteria.

B. Rank Criterion

As an alternative to $P_e^{\mathrm{ub}_2}$, we now explore the tighter upper bound $P_e^{\mathrm{ub}_1}$ in (12), focusing firstly on the asymptotic diversity gain.

Theorem 1: For any STBC in a massive MIMO system, under condition (6), the asymptotic diversity gain $D_g^{\mathrm{ub}_1}$ corresponding to $P_e^{\mathrm{ub}_1}$ in (12) is given by

$$D_g^{\mathrm{ub}_1} = \min_{1\le \ell \le J'} r_\ell. \tag{14}$$

Proof: By definition of the diversity gain, it follows

$$D_g^{\mathrm{ub}_1} = \lim_{\rho\to\infty} -\frac{\log P_e^{\mathrm{ub}_1}}{\log\rho}.$$

Then, using (12) and applying L'Hopital's rule, we have

$$\begin{aligned}
D_g^{\mathrm{ub}_1} &= \lim_{\rho\to\infty} -\frac{\log\left(\frac{2K}{J}\sum_{\ell=1}^{J'}\Lambda_\ell^{-1}\rho^{-r_\ell}\right)}{\log\rho}\\
&= \lim_{\rho\to\infty}\frac{\min\limits_\ell r_\ell \sum_{\ell\in\mathscr{K}}\Lambda_\ell^{-1} + \sum_{\ell\in\overline{\mathscr{K}}}\Lambda_\ell^{-1}\rho^{-r_\ell+\min\limits_\ell r_\ell}}{\sum_{\ell\in\mathscr{K}}\Lambda_\ell^{-1} + \sum_{i\in\overline{\mathscr{K}}}\Lambda_\ell^{-1}\rho^{-r_\ell+\min_\ell r_\ell}}\\
&= \min_\ell r_\ell.
\end{aligned}$$

where $\mathscr{K} \triangleq \{k|1\le k\le J', r_k = \min_\ell r_\ell\}$ and $\overline{\mathscr{K}} \triangleq \{k|1 \le k \le J', r_k > \min_\ell r_\ell\}$. ■

Rank Criterion: In order to design an STBC with higher diversity order, the minimum rank of the difference matrices $\boldsymbol{\Delta}_\ell$ must be maximized for $1 \le \ell \le J'$.

Although this criterion is identical to the one obtained from $P_e^{\mathrm{ub}_2}$, it is more general since it is derived based upon a tighter upper bound which includes the effects of all the difference matrices $\boldsymbol{\Delta}_\ell$, rather than the ones with minimum rank.

C. Sum Determinant Criterion

Next, we extend our analysis to the asymptotic coding gain.

Lemma 2: Let x_i for $1 \le i \le N$ be positive real numbers. The following double inequality holds:

$$\sum_{i=1}^{N}\frac{1}{x_i} \le \xi_1 N + \xi_2\left(\sum_{i=1}^{N} x_i\right) \le \frac{N}{x_{\min}}, \tag{15}$$

where $\xi_1 \triangleq \frac{x_{\min}+x_{\max}}{x_{\min}x_{\max}}$, $\xi_2 \triangleq -\frac{1}{x_{\min}x_{\max}}$, with $x_{\min} \triangleq \min_i x_i$, and $x_{\max} \triangleq \max_i x_i$.

Proof: By defining $f(x) = 1/x$ and $g(x) = \xi_1 + \xi_2 x$ for $x > 0$, we can write $x_i = \alpha_i x_{\min} + (1-\alpha_i)x_{\max}$ for some α_i where $0 \le \alpha_i \le 1$ and $1 \le i \le N$. Since $f(x)$ is convex,

$$f(x_i) \le \alpha_i f(x_{\min}) + (1-\alpha_i) f(x_{\max}) = g(x_i),$$

where the equality can easily be verified. Taking the sum over all i, we obtain the first inequality in (15) results. The second inequality readily follows from the fact that $g(x_i) \le \frac{1}{x_{\min}}$. ■

Theorem 2: For any STBC in a massive MIMO system, under condition (6), the asymptotic coding gain $C_g^{\mathrm{ub}_1}$ corresponding to $P_e^{\mathrm{ub}_1}$ in (12) is given by

$$C_g^{\mathrm{ub}_1} \triangleq \left(\Gamma_1 J' + \Gamma_2\sum_{\ell=1}^{J'}\Lambda_\ell\right)^{-1/D_g^{\mathrm{ub}_1}} \tag{16}$$

where $\Gamma_1 \triangleq \frac{\Lambda_{\max}+\Lambda_{\min}}{\Lambda_{\max}\Lambda_{\min}}$, and $\Gamma_2 \triangleq -\frac{1}{\Lambda_{\max}\Lambda_{\min}}$, with $\Lambda_{\min} \triangleq \min_i \Lambda_i$, and $\Lambda_{\max} \triangleq \max_i \ \Lambda_i$.

Proof: Since $\rho > 1$, we have $\rho^{-r_i} \leq \rho^{-D_g^{\mathrm{ub}_1}}$ for $1 \leq i \leq J'$. Applying this result along with Lemma 2 to (12), we obtain

$$P_e \leq \frac{2K}{J}\sum_{\ell=1}^{J'}\Lambda_\ell^{-1}\rho^{-r_i} \leq \frac{2K\rho^{-D_g^{\mathrm{ub}_1}}}{J}\sum_{\ell=1}^{J'}\Lambda_\ell^{-1} \tag{17a}$$

$$\leq \frac{2K}{J}\left(\left(\Gamma_1 J' + \Gamma_2\sum_{\ell=1}^{J'}\Lambda_\ell\right)^{-1/D_g^{\mathrm{ub}_1}}\rho\right)^{-D_g^{\mathrm{ub}_1}}. \tag{17b}$$

Thus, $C_g^{\mathrm{ub}_1}$ is the asymptotic coding gain. ■

Remark 1: From Lemma 2 and (17b), it follows that:

$$P_e \leq \frac{2K}{J}(C_g^{\mathrm{ub}_1}\rho)^{-D_g^{\mathrm{ub}_1}} \leq K(J-1)(C_g^{\mathrm{ub}_2}\rho)^{-D_g^{\mathrm{ub}_2}} \tag{18}$$

which verifies the advantage of the presented asymptotic coding gain over the conventional coding gain. Consequently, if the conventional design criteria are valid for some STBCs, the proposed criteria are also valid. Hence, the proposed criteria also cover the STBC designs in [3]. Moreover, it can be observed form (14) and (16) that both proposed asymptotic coding and diversity gains are independent of the number of UTs *K*.

According to Theorem 2 and (18), the higher the coding gain $C_g^{\mathrm{ub}_1}$, the better the performance of the code. This, in turn, can be achieved by maximizing the sum determinant of the code which is defined as follows:

$$S \triangleq (\sum_{\ell=1}^{J'}\Lambda_\ell)^{1/D_g^{\mathrm{ub}_1}} = (\sum_{i=1}^{J'}\prod_{n=1}^{r_i}\lambda_n^{(i)})^{1/D_g^{\mathrm{ub}_1}}. \tag{19}$$

Sum Determinant Criterion: In order to reduce $P_e^{\mathrm{ub}_1}$ and thereby improve the performance of the code, the sum determinant *S* must be maximized.

Note that the exponent $1/D_g^{\mathrm{ub}_1}$ in the definition (19) is used mainly for consistency with the conventional definition of the coding gain and normalization purposes. In essence, the main intuition behind the SDC is that the sum of the non-zero eigenvalue products of all the difference matrices $\mathbf{\Delta}_i$ leads to a more accurate design criterion than the conventional determinant criterion, which only considers the minimum of the non-zero eigenvalue products.

D. STBC Design Criteria for Small-Scale MIMO

STBCs were initially designed for small-scale MIMO to combat the fading effect in wireless channels by exploiting spatial diversity. For conventional STBC-based MIMO, the received vector can be expressed as (1) by setting $\mathbf{P} = \mathbf{I}$. Under the common assumption of independent Rayleigh fading paths, the channel covariance matrix can be written as $\mathbf{R}_k = \frac{1}{N}\mathbf{I}_N$ [4], [8]. Therefore, we can write $P_e^{(k)}$ as (5) with $r_{i,j}^{(k)}$ and $\lambda_n^{(k)}(i,j)$ being the rank and *n*th non-zero largest eigenvalue of $\mathbf{\Delta}_{i,j}^{(k)}$, respectively, where

$$\mathbf{\Delta}_{i,j}^{(k)} \triangleq (\mathbf{C}_i - \mathbf{C}_j)^H(\mathbf{C}_i - \mathbf{C}_j). \tag{20}$$

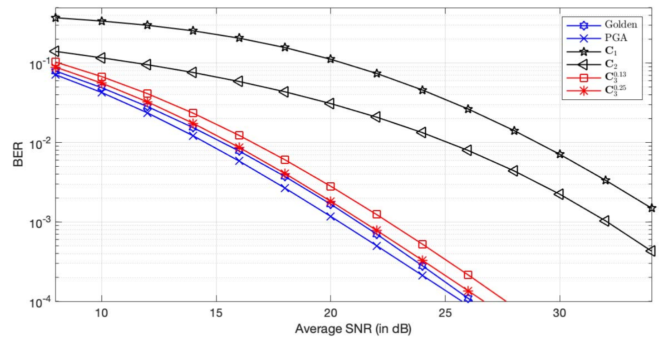

Fig. 1. BER performances of different STBCs.

The error probability can be written as (10) and we have $P_e \leq P_e^{\mathrm{ub}_1} \leq P_e^{\mathrm{ub}_2}$ where $P_e^{\mathrm{ub}_1}$ and $P_e^{\mathrm{ub}_2}$ are defined as (9) (see Lemma 1). Then, to derive the design criteria, the steps in Section III-B and III-C can be similarly taken. Hence, the proposed criteria, i.e., rank and SDC, are valid for conventional small-scale MIMO, and hold regardless of the number of BS antennas.

IV. Simulation Results

In our simulations, we let $K = 1$ since according to Theorems 1 and 2, the number of UTs does not affect the diversity and coding gain of the STBCs. We consider a BS equipped with a ULA of 128 antenna elements with half-wavelength spacing. The truncated Gaussian distribution is considered for angular power spectrum where the mean angle of departure and angle spread are set to 0 and $\frac{5\pi}{180}$ as in [3]. We examine different 2×2 STBCs with BPSK modulation where, in each case, the 128×2 precoder matrix $\mathbf{P}$ is designed as in [3]. As a performance benchmark, we consider two well-known full-rate STBCs, namely, the Golden [9] and the Paredes–Gershman–Alkhansari (PGA) [10] codes. To highlight the advantages of SDC over MDC, we also consider the following rate-one codes: $\mathbf{C}_1 = \frac{1}{2}\,\mathrm{diag}([2-0.2s_1[2-s_2)], 2+0.2s_1[2-s_2]])$ and $\mathbf{C}_2 = \frac{1}{2}\,\mathrm{diag}([(s_1+1)(2-0.2s_2)/2) - j(s_1-1)(s_2), (s_1+1)(2+0.2s_2)/2) + j(s_1-1)(s_2)])$. We designed these codes solely to illustrate how SDC can correctly represent the BER performance of the codes in an extreme case where MDC cannot provide the consistent results. Finally, to demonstrate that SDC can be superior to MDC for STBCs design and optimization, a rate-one STBC with parameter α is designed as

$$\mathbf{C}_3^\alpha = \frac{3}{4}\mathrm{diag}\begin{bmatrix}\frac{s_1+1}{2}(2+s_2) - \frac{s_1-1}{2}(2-s_2)e^{j2\pi\alpha} \\ \frac{s_1+1}{2}(2-s_2) + \frac{s_1-1}{2}(2-s_2)e^{j2\pi\alpha}\end{bmatrix}. \tag{21}$$

The optimal values of α maximizing the MDC and the SDC are obtained by exhaustive search as: $\alpha_{\mathrm{MDC}} = 0.13$ and $\alpha_{\mathrm{SDC}} = 0.25$, respectively.

In our experiments, we evaluate the BER performance of these codes in the above massive MIMO setup. We also compute and compare their sum determinant (SD) as given by (19) and minimum determinant (MD) as given by $C_g^{\mathrm{ub}_2}$ in (13). Fig. 1 depicts the BER performance of the different STBCs versus SNR, while the corresponding MD and SD values are presented in Table I. Although both the Golden and PGA codes

TABLE I
SD AND MD OF DIFFERENT STBCS

	Golden	PGA	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3^{0.13}$	$\mathbf{C}_3^{0.25}$
MD	1.7888	1.5118	0.0392	0.0392	0.6400	0.6400
SD	24.7870	35.2622	0.5100	5.6293	6.1646	7.1490

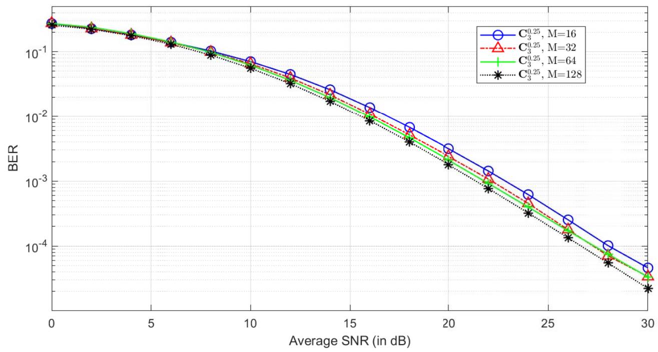

Fig. 2. BER performances for different number of BS antennas.

achieve a diversity gain of 2, we can observe from Fig. 1 that PGA has slightly better performance than Golden. However, we note from Table I that MD of the Golden code exceeds that of PGA, which is inconsistent with the former observation. In contrast, the behavior of the SD values is consistent with the BER performance of these two codes. This inconsistency of MD in predicting the BER performance is further emphasized with $\mathbf{C}_1$ and $\mathbf{C}_2$. While $\mathbf{C}_2$ exhibits a much better performance than $\mathbf{C}_1$, their MDs are identical; this problem is avoided by the SDs whose values are consistent with the BER performance. Interestingly, in the case of $\mathbf{C}_3$ a better performance (by about 1dB) is achieved when α is optimized using SDC instead of MDC. All in all, simulation results show that not only SDC exhibits more consistency when used as a performance metric, it can also lead to better STBC design than MDC in the context of massive MIMO.

In order to show the advantage of massive-MIMO over conventional MIMO, the BER performance of STBC $\mathbf{C}_3^0.25$ for different numbers of transmit antennas (under constant transmit power) is plotted in Fig. 2. The precoder $\mathbf{P}$ is designed using the approach in [3] for $M = \{16, 32, 46, 128\}$. As seen from the figure, increasing the number of antennas improves the performance of the omnidirectional STBC in massive MIMO systems. For example, the required SNR for a BER of 10^{-3} is approximately reduced by 2 dB when the number of BS antennas increases from 16 to 128.

V. CONCLUSION

In this letter, we derived two design criteria for omnidirectional STBC-based massive MIMO systems, i.e., the rank criterion and the SDC. While the rank criterion is similar to that for the conventional MIMO, its derivation is based on a tighter upper bound on the average error probability. The SDC, is shown to be more accurate than the MDC for both performance comparison and designing purposes. Finally, we conducted computer simulations which validate our theoretical findings.

REFERENCES

[1] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.
[2] A. Morsali, A. Haghighat, and B. Champagne, "Realizing fully digital precoders in hybrid A/D architecture with minimum number of RF chains," *IEEE Commun. Lett.*, vol. 21, no. 10, pp. 2310–2313, Oct. 2017.
[3] X. Meng, X.-G. Xia, and X. Gao, "Omnidirectional space-time block coding for common information broadcasting in massive MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 17, no. 3, pp. 1407–1417, Mar. 2018.
[4] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.
[5] S. S. Hosseini, S. Talebi, and J. Abouei, "Comprehensive study on a 2×2 full-rate and linear decoding complexity space time block code," *IET Commun.*, vol. 9, no. 1, pp. 122–132, Jan. 2015.
[6] L. You, X. Gao, X.-G. Xia, N. Ma, and Y. Peng, "Pilot reuse for massive MIMO transmission over spatially correlated Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3352–3366, Jun. 2015.
[7] S. Siwamogsatham, M. P. Fitz, and J. H. Grimm, "A new view of performance analysis of transmit diversity schemes in correlated Rayleigh fading," *IEEE Trans. Inf. Theory*, vol. 48, no. 4, pp. 950–956, Apr. 2002.
[8] A. Morsali and S. Talebi, "On permutation of space-time-frequency block codings," *IET Commun.*, vol. 8, no. 3, pp. 315–323, Feb. 2014.
[9] J. C. Belfiore, G. Rekaya, and E. Viterbo, "The golden code: A 2×2 full-rate space-time code with nonvanishing determinants," *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1432–1436, Apr. 2005.
[10] J. M. Paredes, A. B. Gershman, and M. Gharavi-Alkhansari, "A new full-rate full-diversity space-time block code with nonvanishing determinants and simplified maximum-likelihood decoding," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2461–2469, Jun. 2008.