A ROBUST SIDELOBE CANCELLER FOR REFLECTOR ANTENNA USING SIGNAL SUBSPACE EIGENVECTORS

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ABSTRACT

A new sidelobe canceller (SLC) using the signal subspace eigenvectors of the incident signal covariance matrix is proposed. Assuming independent point signal sources, it is analytically shown that the proposed SLC method produces exact nulls at the direction of interferences and thereby completely rejects the interference. Robustness of the proposed method is evaluated using computer simulations.

1 INTRODUCTION

In various applications such as in sonar, radar, satellite communications and other radio systems it is frequently required to reject the interfering signals. In the literature of array processing, several algorithms have been proposed that maximize the array output signal to interference and noise ratio (SINR) subject to knowning the direction of arrival (DOA) of the desired signal. Multiple sidelobe canceller (MSC) and the direct matrix inversion (DMI) are examples of these methods [1].

Sidelobe cancellation may be viewed as a special case of an adaptive array [2]. The structure of a sidelobe canceller (SLC) is shown in Fig. 1 which consists of a main directional antenna pointing toward the desired signal and L-auxiliary antennas, generally omni-directional.

Here, the object is to design the antenna weights (w_i's) in such a way that the overall pattern has nulls in the direction of interfering signals and a main-lobe in the direction of the desired signal. The traditional sidelobe cancellers maximize the output SINR. It is assumed that the spatial characteristics of elements are exactly known and that the elements are placed with a known geometry.

In some scenarios, the performance of demodulation and detection depends on the signal to interference ratio (SIR). For example, in spread spectrum communication, penetration of smart jammers into the system may cause destructive effects on the performance of the system. In such applications, maximization of the output SIR is much more important than the overall SINR improvement [3].

Some sidelobe cancellers filter the desired signal from a specified direction and consider the other signals as noise and interference. Generalized sidelobe canceller (GSC) and direct matrix inversion methods are examples of such SLCs [4]. These methods are sensitive to calibration; small errors in the estimation of the desired signal DOA and/or error in calibration will cause signal leakage which in turn causes signal cancellation [1, 5, 6]. Griffiths [7] and Jablon [8] have

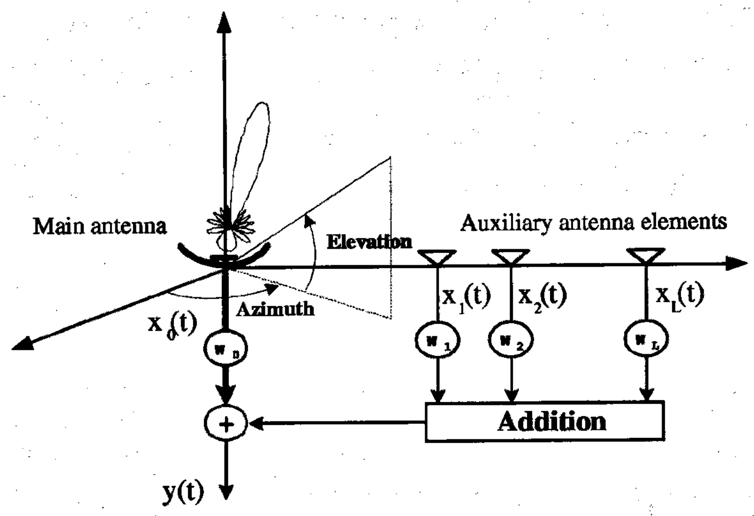

Figure 1: The structure of a sidelobe canceller.

proposed a filtering method to moderate this difficulty by inserting additional constraint on the desired signal filters. In these methods,the dimension of search for the optimum weight vector is reduced. Hence usually a degraded performance is achieved.

The problem of desired signal cancellation due to array imperfection was investigated in [9]. The method of diagonal loading [4, 10] was presented to reduce the array sensitivity to the SNR at the price of reducing the ability of interference suppression. Inadequate estimation of the covariance matrix results in adaptive antenna patterns with high sidelobes and a distorted mainlobe [11].

Beamforming methods have been proposed for signal extraction based on signal subspace eigenvectors for antenna arrays [12, 13, 14]. Here, we introduce a sidelobe cancellation method based on the eigen-decomposition of the incident signal covariance matrix of the main and auxiliary antenna elements. To apply this sidelobe canceling method, which is robust against the DOA and calibration errors, one should know the received noise power and the number of point jammers, which could be estimated through eigenvalues of the received signal correlation matrix [4, 15]. We prove that the introduced method is able to produce exact nulls at the direction of interferences. Simulation results show the robustness of the proposed algorithm as compared to the SLC based on DMI method.

2 RECEIVED SIGNAL MODEL AND PROPERTIES

We assume a main antenna, and L-omnidirectional elements as the auxiliary antennas arranged with arbitrary geometry. Each antenna is equipped with a receiver and an analog-to-digital (A/D) converter. We assume planar narrowband wavefronts. Let $\mathbf{x}(k)$ denote the complex envelop representation of the data received by the main and auxiliary elements at the k'th snapshot. Data vector can be expressed as

$$\mathbf{x}(k) = \mathbf{A}(k)\mathbf{s}(k) + \mathbf{n}(k), \tag{1}$$

where

$$\mathbf{x}(k) \;=\; [x_0(k)\;\; x_1(k)\;\; \cdots\;\; x_L(k)]^T, \tag{2}$$

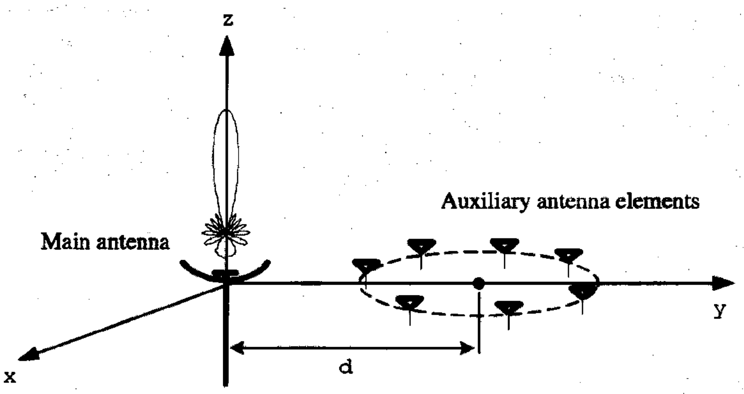

Figure 2: Structure of the sidelobe canceller used in simulations.

$$\mathbf{A}(k) = [\mathbf{a}(\theta_1,\phi_1)\ \mathbf{a}(\theta_2,\phi_2)\ \cdots\ \mathbf{a}(\theta_p,\phi_p)], \quad (3)$$

$$\mathbf{s}(k) = [s_1(k)\ s_2(k)\ \cdots\ s_p(k)]^T, \quad (4)$$

$$\mathbf{n}(k) = [n_0(k)\ n_1(k)\ \cdots\ n_L(k)]^T. \quad (5)$$

Here, the $p \times 1$ complex vector $\mathbf{s}(k)$ is the vector of the incident signals, and $\mathbf{n}(k)$ is the additive receiver and background noise which is assumed to be spatially and temporally white with variance σ^2. Non-white case can also be handled with pre-whitening. θ_n and ϕ_n, respectively, represent the azimuth and elevation of the n'th source (as shown in Fig. 1). Assuming that the main antenna is located at the origin of the rectangular coordinates, $\mathbf{a}(\theta,\phi)$ can be expressed as

$$\mathbf{a}(\theta,\phi) = [g(\theta,\phi)\ a_1(\theta,\phi)\ \cdots\ a_L(\theta,\phi)], \quad (6)$$

where $g(\theta,\phi)$ is the complex gain of the main antenna in the direction (θ,ϕ) , and

$$a_n(\theta,\phi) = \exp\{j\frac{2\pi}{\lambda}(x_n\cos(\phi)\cos(\theta) + y_n\cos(\phi)\sin(\theta) + z_n\sin(\phi))\}. \quad (7)$$

Here, (x_n, y_n, z_n) for $1 \leq n \leq L$ represents the location of the n'th auxiliary element in the rectangular coordinates.

Using (1), the covariance matrix of the received signals is

$$\mathbf{R}(k) = E\{\mathbf{x}(k)\mathbf{x}^H(k)\} = \mathbf{A}(k)\mathbf{P}\mathbf{A}(k)^H + \sigma^2\mathbf{I}, \quad (8)$$

where

$$\mathbf{P} = \mathrm{diag}(p_1,\cdots,p_p). \quad (9)$$

$\mathbf{P}$ is the source correlation matrix with p_i being the incident power of the sources. Diagonal form of $\mathbf{P}$ is a consequence of uncorrelated source assumption. In (8), $E\{.\}$ represents the expected value, and superscript H denotes Hermitian transposition. Given N snapshots of the incident signal, the covariance matrix can be estimated as

$$\hat{\mathbf{R}}(k) = \frac{1}{N}\sum_{i=0}^{N-1}\mathbf{x}(k-i)\mathbf{x}^H(k-i). \quad (10)$$

For the positive-definite Hermitian covariance matrix $\mathbf{R}$, there exist a set of $(L+1)$ orthonormal eigenvectors $\{\mathbf{q}_i\}$ and real eigenvalues $\{\lambda_i\}$ such that

$$\mathbf{R}\mathbf{q}_i = \lambda_i\mathbf{q}_i, \qquad \text{for} \qquad 1 \leq i \leq (L+1). \quad (11)$$

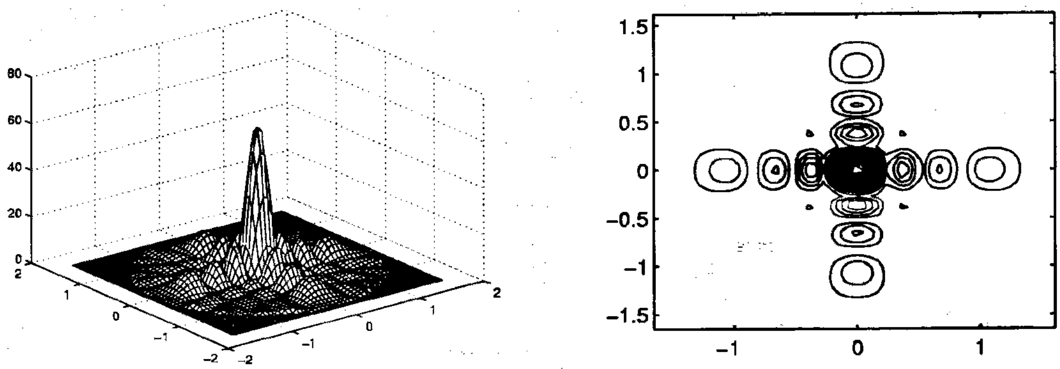

Figure 3: Assumed beam-pattern of the main antenna, (Left) 3-dimensional pattern, (Right) contour plot.

We assume that λ's are in decreasing order as $\lambda_1 \geq \cdots \geq \lambda_p > \lambda_{p+1} = \cdots = \lambda_{L+1} = \sigma^2$. Thus

$$\mathbf{R}\mathbf{q}_i = \sigma_i^2 \mathbf{q}_i, \qquad \text{for} \qquad (p+1) \leq i \leq (L+1). \tag{12}$$

Substituting (8) in (12), we get

$$\mathbf{A}(k)\mathbf{P}\mathbf{A}^H(k)\mathbf{q}_i = \mathbf{0}, \qquad \text{for} \qquad (p+1) \leq i \leq (L+1). \tag{13}$$

This will only be satisfied if the following condition is met [16]

$$\mathbf{A}^H(k)\mathbf{q}_i = \mathbf{0}, \qquad \text{for} \qquad (p+1) \leq i \leq (L+1). \tag{14}$$

Based on our assumptions the following relation holds (see annex):

$$\mathbf{A}^H \mathbf{Q}_s (\mathbf{\Lambda}_s - \sigma^2 \mathbf{I})^{-1} \mathbf{Q}_s^H \mathbf{A} = \mathbf{P}^{-1}, \tag{15}$$

where $\mathbf{I}$ is identity matrix.

3 PROPOSED SLC METHOD

In DMI method, the desired weight vector for SLC (to extract the target signal located at (θ_1, ϕ_1)) is computed based on the following minimization

$$\min_{\mathbf{w}} P_{out} = \min_{\mathbf{w}} E\{\mathbf{x}^H \mathbf{x}\} = \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \qquad \text{subject to} \qquad \mathbf{w}^H \mathbf{a}(\theta_1, \phi_1) = 1. \tag{16}$$

The solution of (16), using the Lagrange multipliers techniques, is [18]

$$\mathbf{w} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_1, \phi_1)}{\mathbf{a}^H(\theta_1, \phi_1)\mathbf{R}^{-1}\mathbf{a}(\theta_1, \phi_1)}. \tag{17}$$

In [19] it is shown that for high input SNR the above weight vector produces relative nulls in the direction of interferers.

To extract the desired signal we propose the following weight vector for SLC [12],

$$\mathbf{w}_1 = \frac{\mathbf{D}\mathbf{a}(\theta_1, \phi_1)}{\mathbf{a}^H(\theta_1, \phi_1)\mathbf{D}\mathbf{a}(\theta_1, \phi_1)}, \tag{18}$$

where

$$\mathbf{D} \overset{\text{def}}{=} \sum_{i=1}^{p} \frac{\mathbf{q}_i \mathbf{q}_i^H}{\lambda_i - \sigma^2} = \mathbf{Q}_s (\mathbf{\Lambda}_s - \sigma^2 \mathbf{I})^{-1} \mathbf{Q}_s^H, \tag{19}$$

and (θ_1, ϕ_1) represent the DOA of the target signal.

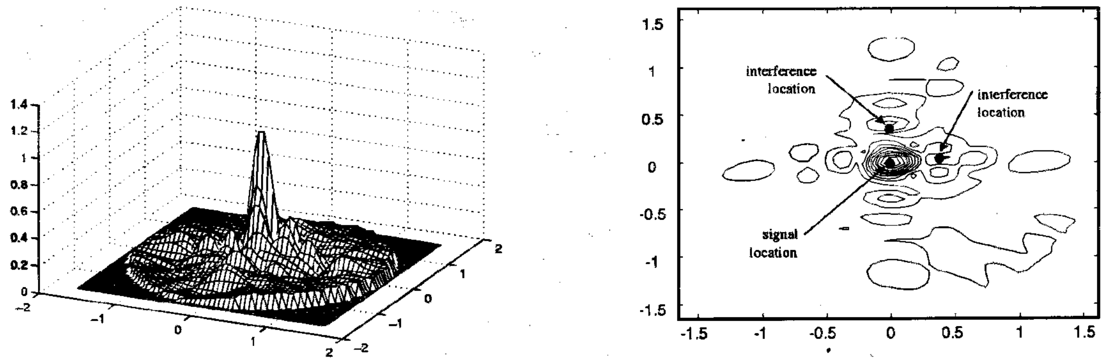

Figure 4: The produced beam-pattern using the proposed SLC method, (Left) 3-dimensional plot, (Right) contour plot.

Theorem 1 *The produced pattern for the SLC (18) has nulls (exact zeros) in the direction of interferences and unit gain in the direction of the desired signal, i.e.,*

$$\mathbf{w}_1^H \mathbf{a}(\theta_n, \phi_n) = \delta_{1,n} = \begin{cases} 1 & for \quad n = 1, \\ 0 & for \quad n = 2, \cdots, p. \end{cases} \tag{20}$$

Proof: Using (15), (18) and (19) and noting that $\mathbf{P}^{-1}$ is a diagonal matrix, we have

$$\begin{aligned} \mathbf{w}_1^H \mathbf{a}(\theta_n, \phi_n) &= \frac{\mathbf{a}^H(\theta_n, \phi_n)\mathbf{Q}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1}\mathbf{Q}_s^H\mathbf{a}(\theta_1, \phi_1)}{\mathbf{a}^H(\theta_1, \phi_1)\mathbf{Q}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1}\mathbf{Q}_s^H\mathbf{a}(\theta_1, \phi_1)} \\ &= \frac{[\mathbf{P}^{-1}]_{n,1}}{[\mathbf{P}^{-1}]_{1,1}} = \begin{cases} 1 & \text{for} \quad n = 1, \\ 0 & \text{for} \quad n = 2, \cdots, p. \end{cases} \end{aligned} \tag{21}$$

♦

where $[\mathbf{P}^{-1}]_{i,j}$ is the (i, j)'th element of the matrix $\mathbf{P}^{-1}$. Hence, we call the proposed SLC, the minimum interference (MI) method.

Theorem 2 *The output SINR using (18) as weight vector for SLC is restricted to* $(\lambda_1 - \sigma^2)/\sigma^2$ *and* $(\lambda_p - \sigma^2)/\sigma^2$ *, i.e.*

$$\frac{\lambda_p - \sigma^2}{\sigma^2} \leq SINR_{out} \leq \frac{\lambda_1 - \sigma^2}{\sigma^2}, \tag{22}$$

and for the case of no interference ($p = 1$), *the output SINR=SNR is equal to* $(\lambda_1 - \sigma^2)/\sigma^2$.

The proof for the above theorem can be verified using (20) and (26).

We represent the benefit of using the SLC by introducing SLC Improvement Factor (IF_{SLC}), defined as the ratio of the output SINR using auxiliary array (SINR_{out}) to the output SINR of the main antenna (SINR_{main}),

$$\text{IF}_{SLC} = \frac{\text{SINR}_{out}}{\text{SINR}_{main}}. \tag{23}$$

Using this definition and (22) the improvement factor for the proposed MI method is restricted to

$$\frac{\lambda_p - \sigma^2}{\sigma^2 \text{SINR}_{main}} \leq \text{IF}_{SLC} \leq \frac{\lambda_1 - \sigma^2}{\sigma^2 \text{SINR}_{main}}. \tag{24}$$

For the case of no interferer ($p = 1$) and regarding (26), this relation is simplified as

$$\text{IF}_{SLC} = \frac{\lambda_1 - \sigma^2}{\sigma^2 \text{SINR}_{main}} = \frac{\lambda_1 - \sigma^2}{g(\theta_1, \phi_1)p_1} = 1 + \frac{L}{g(\theta_1, \phi_1)}, \tag{25}$$

where p_1 is the incident power of the signal.

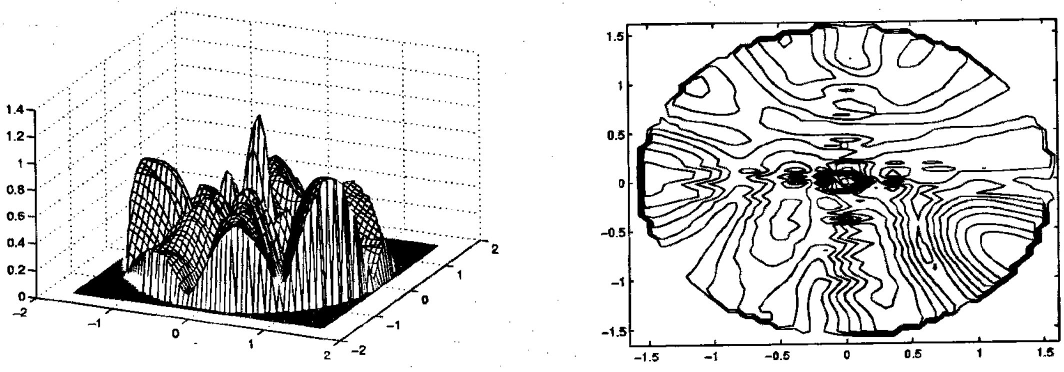

Figure 5: The produced beam-pattern using the DMI method for SLC, (left) 3-dimensional plot, (right) contour plot.

4 PERFORMANCE EVALUATION

In this section, the performance of proposed method for interference cancellation is evaluated. Analytical results indicate that interferences are completely suppressed using the proposed SLC method. In practice, however, only an estimate of **R** is available, with the consequence that the cancellation is not perfect. Besides, auxiliary array imperfection causes degradation in the SLC performance. For comparison we have taken into account the DMI method in parallel to ours, for our simulations.

In all of our simulations:

1-We intentionally have put interference signal on the peak of sidelobes of the main antenna radiation pattern.

2-The sources illuminate the main and auxiliary antenna elements with equal power.

3-The desired signal was taken at the main-lobe of the main antenna.

4-The noise was modeled as a zero–mean, temporally and spatially white Gaussian process.

Each simulation point is the average of 1000 independent runs with independent noise samples and independent signal samples for each run.

For simulation, we have assumed a main antenna at the origin of coordinate system, and L–auxiliary omnidirectional antennas located on the xy-plane arranged as a Uniform Circular Array (UCA). Other geometrical arrangements are also possible and could be considered. The assumed inter-element spacing of the auxiliary UCA is $\lambda/2$ and the distance d is considered to be 6λ where λ is the signal wavelength (see Fig. 2). Fig. 3 shows the 3-dimensional pattern and contour plot of the assumed main antenna.

For the case of $+0.02\lambda$ error in d the sample produced beam-pattern using the proposed SLC method is shown in Fig. 4. Here $\text{SNR}_{input} = 10dB$ and $L = 8$ auxiliary elements are assumed. Signal locations are indicated using arrows. Fig. 5, shows the sample produced beam-pattern using DMI method. Huge sidelobe levels are the result of error in d for DMI method (for the case of desired signal DOA estimation error, same effect is observed).

Fig. 6 represents the SLC output SNR, SINR, and SIR using DMI and the proposed method (MI) for $L = 4,\ 6,\ 8$ auxiliary antenna elements knowing the exact **R**. Here, it is assumed that there is $+0.02\lambda$ error in the location of the auxiliary UCA elements. As seen for the proposed method the output SNR, SINR, and SIR monotonically increases with input SNR and L, however for the DMI method they decrease with input SNR and L after reaching a maximum point.

The effect of covariance matrix estimation on the output SNR, SINR, and SIR is respectively shown in Fig. 7 versus the main antenna input SNR for DMI and the proposed method using

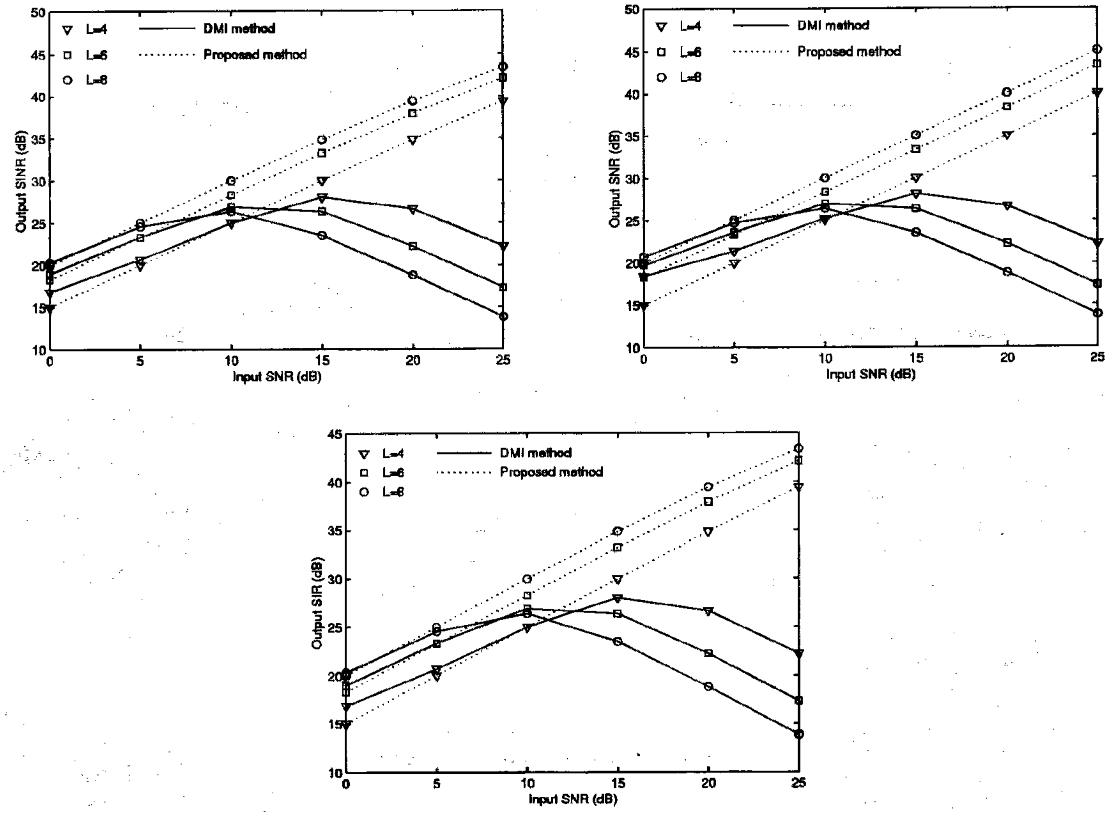

Figure 6: SINR (top–left), SNR (top–right), and SIR (bottom) at the output of SLC for the case of 0.02λ error in the position of the auxiliary array elements using $L = 4,\ 6,\ 8$ auxiliary elements.

N=10, 50, 100 snapshots. The results show the proposed method outperforms DMI specially for high input SNR and low N.

The phase error at the mixer and the local oscillator of the antenna elements, different lengths for the transmission lines, synchronization error in the sampling time for A/D converters are sources of phase error at the auxiliary elements of SLC which cause degradation in the performance of the SLC in practice. Assuming a uniform random phase shift up to 1 degree at the auxiliary antenna elements, Fig. 8 represents the output SNR, SINR, and SIR using $L = 4,\ 6,\ 8$. The results show the robustness of the proposed MI method with respect to random phase errors.

The output SINR of the SLC for different number of interferers ($p = 1, \cdots, 11$) using exact and sample covariance matrix is shown in Fig. 9. It is seen that the proposed SLC method outperforms DMI, specially for low number of interferers.

5 CONCLUSION

This paper presented a new robust method for interference cancellation in presence of the desired signal using the signal subspace eigenvectors of the received covariance matrix. Assuming that the antenna elements characteristics and the DOA of the desired signal are known, it is proved analytically and by computer simulations that the method completely filters out the interferences at the SLC output. Simulation results reveal the effectiveness and robustness of the proposed method in presence of (position) calibration error, phase shift errors and covariance matrix error

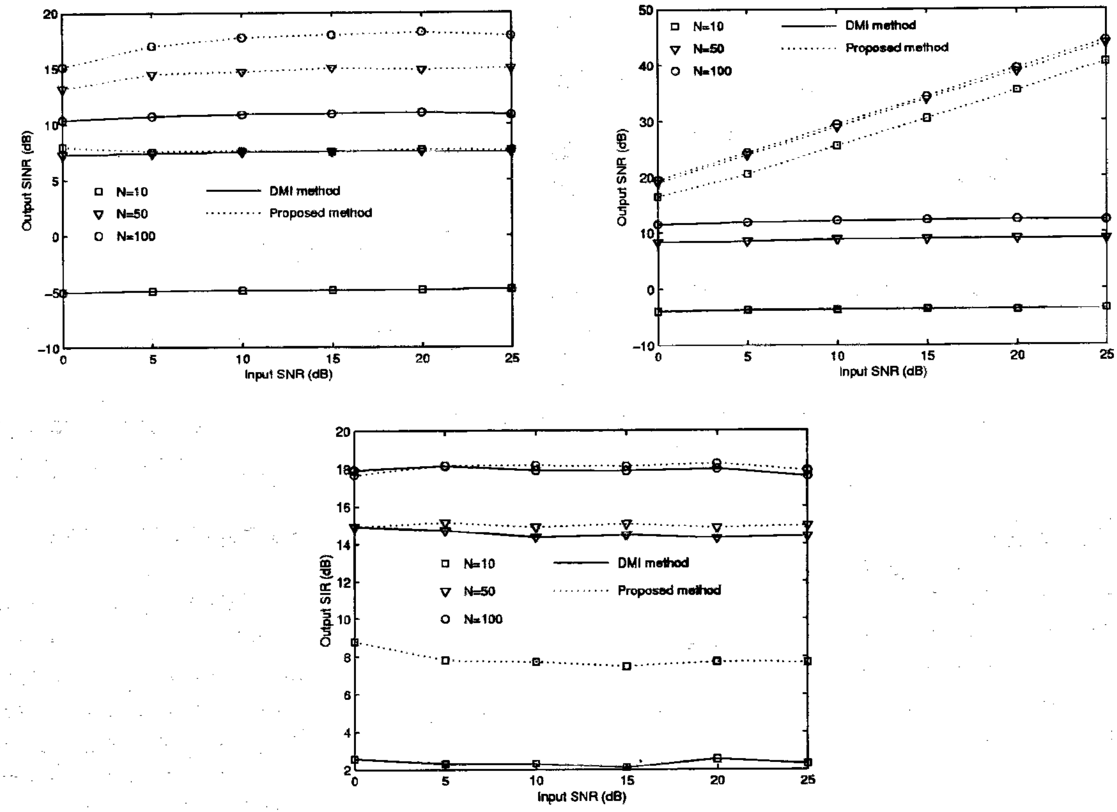

Figure 7: SINR (top–left), SNR (top–right), and SIR (bottom) at the output of SLC for sample covariance matrix using $N = 10,\ 50,\ 100$. There are two interfering signals ($p = 3$) and $L = 8$ auxiliary elements.

estimation and desired signal DOA error when compared to the DMI method. It could be shown that the method is also robust with respect to the DOA error but the related simulations have not been presented here.

ANNEX

Based on the assumption of section-2 the following properties hold

Property 1 *The total received signal power P_T may be computed as*

$$\begin{aligned} P_T &= \sum_{i=1}^{p} g(\theta_i, \phi_i) p_i + L \sum_{i=1}^{p} p_i + \sigma^2 (L+1) \\ &= \mathrm{tr}(\mathbf{R}) = \sum_{i=1}^{L+1} \lambda_i = \sum_{i=1}^{p} \lambda_i + \sigma^2 (L + 1 - p). \end{aligned} \tag{26}$$

Property 2 *(Karhunen-Loève expansion) The covariance matrix* $\mathbf{R}$ *and its inverse can be expressed as [17]*

$$\mathbf{R} = \sum_{i=1}^{L} \lambda_i \mathbf{q}_i \mathbf{q}_i^H = \mathbf{Q}_s \mathbf{\Lambda}_s \mathbf{Q}_s^H + \mathbf{Q}_n \mathbf{\Lambda}_n \mathbf{Q}_n^H, \tag{27}$$

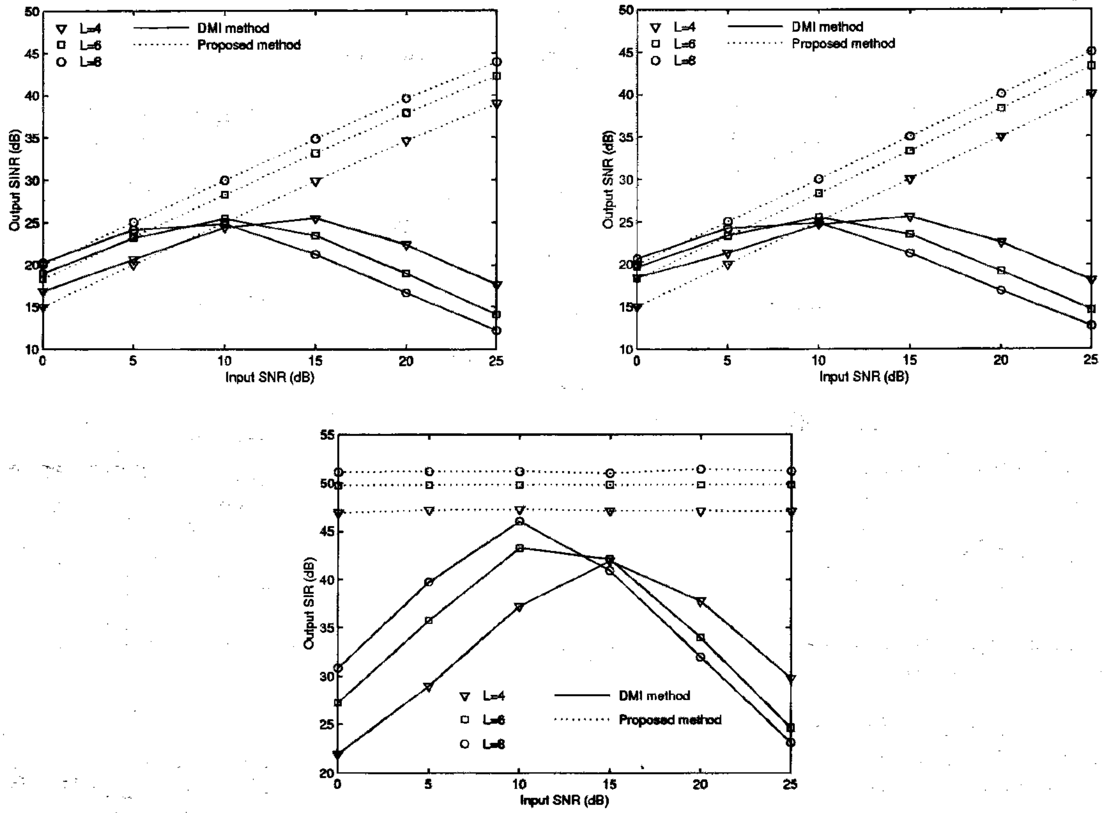

Figure 8: SINR (top–left), SNR (top–right), and SIR (bottom) for the case of 1 degree uniform phase error at each auxiliary antenna elements using the estimated covariance matrix with $N = 100$ and assuming two interfering signals ($p = 3$).

$$\mathbf{R}^{-1} = \sum_{i=1}^{L} \lambda_i^{-1}\mathbf{q}_i\mathbf{q}_i^H = \mathbf{Q}_s\mathbf{\Lambda}_s^{-1}\mathbf{Q}_s^H + \mathbf{Q}_n\mathbf{\Lambda}_n^{-1}\mathbf{Q}_n^H, \tag{28}$$

where

$$\mathbf{Q}_s = [\mathbf{q}_1, \cdots, \mathbf{q}_p], \tag{29}$$

$$\mathbf{Q}_n = [\mathbf{q}_{p+1}, \cdots, \mathbf{q}_{L+1}], \tag{30}$$

$$\mathbf{\Lambda}_s = \mathrm{diag}(\lambda_1, \cdots, \lambda_p), \tag{31}$$

$$\mathbf{\Lambda}_n = \mathrm{diag}(\lambda_{p+1}, \cdots, \lambda_{L+1}). \tag{32}$$

The columns of $\mathbf{Q}_s$ span the signal subspace and the columns of $\mathbf{Q}_n$ span the so–called noise subspace. Since $\mathbf{A}$ is orthogonal to the noise subspace (according to (14)), the column span of the matrix $\mathbf{A}$ is the signal subspace. In other words, the following relation holds

$$\mathbf{A} = \mathbf{Q}_s\mathbf{K}, \tag{33}$$

where $\mathbf{K} \in C^{p\times p}$ with $C^{p\times p}$ being the $p \times p$ complex vector space.

Lemma 1 *Columns of* $\mathbf{Q}_s^H\mathbf{A}(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1/2}$ *are orthogonal to each other.*

Proof: From (8) and (11), it follows that

$$\mathbf{R}\mathbf{Q}_s = \mathbf{Q}_s\mathbf{\Lambda}_s, \tag{34}$$

$$\mathbf{A}\mathbf{P}\mathbf{A}^H\mathbf{Q}_s = \mathbf{Q}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}), \tag{35}$$

$$\mathbf{A}\mathbf{P}\mathbf{A}^H\mathbf{Q}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1} = \mathbf{Q}_s. \tag{36}$$

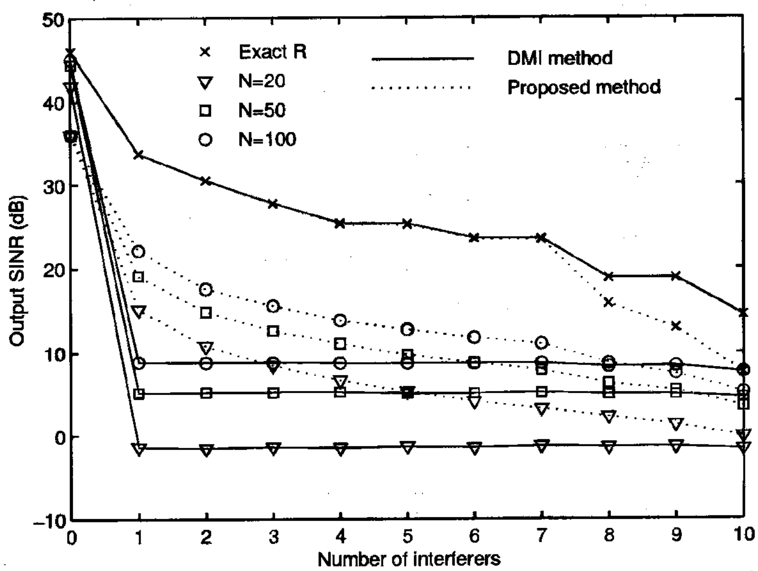

Figure 9: The SINR at the output of SLC for different number of interfering signals using $L = 12$ auxiliary elements (the exact and estimated covariance matrices).

Define

$$\tilde{\mathbf{K}} \stackrel{\text{def}}{=} \mathbf{P}\mathbf{A}^H\mathbf{Q}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1}. \tag{37}$$

Using (33), (36) and (37), it is straight forward to show that

$$\mathbf{A} = \mathbf{A}\tilde{\mathbf{K}}\mathbf{K}. \tag{38}$$

Since it is assumed that $\mathbf{A}$ is full column rank, we have

$$\tilde{\mathbf{K}}\mathbf{K} = \mathbf{I}. \tag{39}$$

Applying (33) and (37) to (39), we deduce (15). As such the proof is completed.

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