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Spectrum sensing based on fractional lower order moments for cognitive radios in α-stable distributed noise

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ABSTRACT

The traditional spectrum sensing methods based on second order statistics are in general not applicable to detecting a primary user with unknown parameters in non-Gaussian noises. This paper presents a novel spectrum sensing scheme based on fractional lower order moment (FLOM) for the detection of a primary user in non-Gaussian noise that are modeled by the α-stable distribution. The new detector does not require any *a priori knowledge* about the primary user (PU) signal and channels. The statistics of the proposed FLOM detector are defined in a multi-user cooperative framework and its detection and false alarm probabilities as well as deflection coefficient are analyzed for both non-fading and Rayleigh fading communication channels between the primary and secondary users. The detection performance of the proposed method versus the generalized signal-to-noise ratio, the characteristic exponent α and the number of cooperative users is also studied along with comparison to the Cauchy detector through computer simulations. Analytical and simulation results show that the proposed FLOM detector has a much better performance than the Cauchy detector in the α-stable distributed noise environment. It is also shown that multi-user cooperative sensing leads to a significantly higher probability of detection than the single user version.

1. Introduction

Cognitive radio (CR) has been proposed as a key technology to improve the spectrum efficiency in next generation wireless networks through dynamic management and opportunistic use of radio resources. CR allows unlicensed (secondary) users (SU) to opportunistically access a frequency band allocated to a licensed (primary) user (PU), providing that the PU is not temporally using the spectrum or it can be adequately protected from the interference created by the SUs. Thus, the radio spectrum can be reused in an opportunistic manner or shared at all time, leading to an increase in the network capacity. One of the most important challenges in CR systems is to detect as quickly and reliably as possible the absence ($\mathcal{H}_0 =$ null hypothesis) or presence ($\mathcal{H}_1 =$ alternative hypothesis) of the PU in complex radio environments such as those characterized by fading effects and non-Gaussian noises.

Several spectrum sensing techniques such as [1–3] have been proposed for single-user and cooperative detection under the white Gaussian noise (WGN) assumption. In practice, however, the problem is more challenging as we need to detect various types of PU signals impaired by non-Gaussian noise and interference, as pointed out in [4]. Non-Gaussian noise impairments may include man-made

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impulsive noise, co-channel interference from other SUs, emission from microwave ovens, out of band spectral leakage, etc. [5,6]. It is shown in [7] that the performance of a spectrum detector optimized against Gaussian noise may degrade drastically when non-Gaussian noise or interference is present because of the heavy tail characteristics of its probability density function (PDF). In view of these problems, it is desirable to seek useful solutions to spectrum detection in practical non-Gaussian noises and to evaluate the detection performance.

Currently, several models are available in the literature to fit non-Gaussian noise or interference distributions, such as the α-stable distribution, the generalized Gaussian distribution (GGD) and the Gaussian mixture distribution (GMD). The α-stable distribution has proved to be very successful in modeling practical noises [8], including both Gaussian noise and non-Gaussian noises with different degrees of non-Gaussianity through the selection of its characteristic exponent, α. In [9], an α-stable distribution is proposed to fit noise and interference in wireless communication systems. In [10], the α-stable distribution is used to model the co-channel interference in wireless networks. Recently, a mathematical framework has been introduced, which uses α-stable distributions to model the interference in cognitive radio networks, wireless packet networks and networks with ultra wideband systems [11].

Unlike GGD and GMD, the α-stable distribution does not possess a compact analytical form of PDF, so most of the techniques for the detection of signals in non-Gaussian noise, which require the PDF of the noise, cannot be employed in the case of α-stable distributed noise. For this reason, the optimal detector in α-stable noises based on a likelihood ratio has a very complex structure, which requires numerical integration and/or DFT operations [12]. To overcome these difficulties, several suboptimal detectors have been proposed in the literature. For example, the conventional linear (Gaussian) detector, which is optimal when $\alpha=2$ only, is the least complex one, but it performs poorly when α is smaller. The Cauchy detector, which is optimum in the case of $\alpha=1$, was used as a suboptimal detector in α-stable noises with arbitrary values of α [13]. In [14], the α-stable distribution is approximated by a finite GMD; this modeling improves the performance but is still complicated. Other simple suboptimal detectors apply a nonlinearity to the received signal before using the linear detector, such as the soft limiter and matched myriad filter [15]. In [16], a novel soft limiter detector with adaptive threshold is proposed, which can significantly improve the performance of the conventional soft limiter. These detectors have better performance than the linear one or the conventional Gaussian detector, however, their performance is not optimal.

Spectrum sensing for CR networks in the presence of non-Gaussian noise has been addressed by several researchers recently [17–19]. However, the implementation of these detectors remains challenging as they require a priori knowledge of various side information, such as the variance of the noise [20] and the PU signal cyclic frequency [21], which may not be readily available in practice. In [20], the authors proposed a spectrum sensing method based on a Cauchy detector for CR with simulation results showing a better performance than the linear (Gaussian) detector; however, the performance of this Cauchy-based detector degrades rapidly when α is decreased and the test statistic requires a priori knowledge of the noise dispersion parameter, γ.

The L_p-norm based detector in [17] does not require any a priori knowledge about the noise distribution nor the PU signal. It can track changes in noise distributions with finite moments, but it is not applicable to α-stable distributed noise which has no finite moments when order $p \geq \alpha$ [17]. Although the α-stable distribution does not have finite moments $p \geq \alpha$, its moments of order $p < \alpha$ are finite. Hence, the fractional lower order statistics (FLOS) become a valid signal processing approach to deal with α-stable distributed noise in this case. Although FLOS has been applied to weak signal detection in non-Gaussian noises in [13], its application to spectrum sensing has not yet received much attention.

Multi-user cooperation is a commonly used technique in spectrum sensing due to its capability of overcoming the harmful effects of fading and shadowing by taking advantage of the spatial diversity. Many recent works have therefore employed multi-user cooperation for improving the performance of spectrum sensing in the presence of Gaussian noise [22,23]. Still, very little work has been found concerning multi-user cooperation for spectrum sensing in the presence of non-Gaussian noise.

In this paper, we model the background noise in CR systems by means of the α-stable distribution (SαS) and propose a cooperative spectrum sensing method based on the fractional lower order moment (FLOM), referred to as the FLOM detector. In this approach, the PU signal, the noise parameters and the channel gain are all assumed unknown. We analyze the performance of the FLOM detector in both non-fading and Rayleigh fading sensing channels. We also investigate the detection performance with different characteristic exponents of the SαS distribution as well as the effect of the characteristic exponent on the deflection coefficient.

Our main contributions in this paper include: (i) We derive the detection performance of the FLOM detector in the regime of low generalized signal-ro-noise ratio (GSNR) for non-fading sensing channels in terms of the probabilities of detection and false alarm. We also analyze the detection performance with respect to the degree of non-Gaussianity and the number of samples under different generalized signal-to-noise ratios (GSNRs). (ii) We investigate the performance of the proposed detector in terms of the deflection coefficient for the SαS distributed noise with various degrees of non-Gaussianity. (iii) We analyze the global detection and false alarm probabilities of the proposed FLOM-based cooperative sensing for Rayleigh fading channels. (iv)Through theoretical analysis and numerical simulations, we show that the FLOM detector can significantly enhance the detection performance over the Cauchy detection in the *SαS* noise and that the proposed cooperative spectrum sensing scheme has a significantly higher global probability of detection than the non-cooperative one.

The remainder of the paper is organized as follows. The CR system and the α-stable distributed noise model under consideration are presented in Section 2. The FLOM-based detection statistics are derived and analyzed in Section 3, while the theoretical performance of the FLOM detector for the SαS noise in non-fading sensing channels is derived in Section 4. The cooperative spectrum sensing scheme over Rayleigh fading channels is discussed in Section 5. Numerical and simulation results of the proposed scheme with comparison to the Cauchy detection are provided in Section 6. Finally, conclusions are drawn in Section 7.

2. System model

Similar to our previous work [24], here we consider a CR sub-network comprised one PU and M SUs with one fusion center (FC). Each SU monitors the presence of the PU signal over a certain time-interval, through a wireless channel that is assumed to be frequency non-selective and time invariant. The M N-dimensional observation vectors from the SUs are forwarded to the FC where a final or global decision is made. With this cooperative strategy, the spectrum sensing in each SU can be formulated as a binary hypothesis testing problem, described by $\mathcal{H}_0$: *PU absent* and $\mathcal{H}_1$: *PU present*. Under these two hypotheses, the baseband signal $z_m(n)$, assumed to be real-valued in this paper and observed by the m-th SU, $m \in \{1,2,\ldots,M\}$, at discrete-time $n \in \{1,2,\ldots,N\}$, is formulated as

$$\begin{cases} \mathcal{H}_0: z_m(n) = w_m(n) \\ \mathcal{H}_1: z_m(n) = h_m s(n) + w_m(n) \end{cases} \quad (1)$$

where $w_m(n)$ is an additive background noise component present under both hypotheses, $s(n)$ is the signal sample emitted by the PU at time n only under $\mathcal{H}_1$, and h_m is the channel gain between the PU's transmitter and the m-th SU's receiver, which is assumed to be constant during spectrum sensing. In this paper, we consider two types of channels: (1) non-fading channel where $h_m = 1$; and (2) Rayleigh fading channel where h_m obeys the Rayleigh distribution.

Under both hypotheses, we model the noise sequence $w_m(n)$ as an independent and identically distributed (IID) symmetric α-stable (SαS) distribution with characteristic exponent α and dispersion γ. We assume that the noise sequences observed by different SUs are mutually independent. The PU signal $s(n)$ is modeled as an IID process with zero-mean and variance $\sigma_s^2 = E[|s(n)|^2]$, but with an arbitrary distribution. The PU signal $s(n)$ is independent of the noise processes $\{w_m(n)\}$. The channel gains h_m either assumed to be deterministic or random variables IID over the spatial index m with variances $\sigma_h^2 = E[|h_m|^2]$; in the latter case, they are independent of the PU signal and the SU noises.

In general, the SUs have no *a priori* knowledge about the emitted PU signal $s(n)$ nor the channel gains h_m, although they can extract relevant information about the noise $w_m(n)$ through measurements under $\mathcal{H}_0$. Therefore, a practical detector should not rely on a statistic that requires the knowledge about $s(n)$ or h_m.

In this paper, we model the non-Gaussian noise in the context of CR by means of the SαS distribution, with the characteristic exponent α controlling the degree of non-Gaussianity in the distribution. Due to the lack of a compact analytical form for its PDF, the SαS distribution is described by its characteristic function:

$$\varphi(t) = \exp\{j\mu t - \gamma |t|^{\alpha}\} \quad (2)$$

where $-\infty < \mu < \infty$ is the location parameter, i.e. the symmetry axis for the SαS distribution, $\gamma > 0$ is the noise dispersion, which measures the distribution's spread around its center, and $0 < \alpha \le 2$ is the characteristic exponent. The values $\alpha = 2$ and $\alpha = 1$ correspond to the Gaussian distribution and the Cauchy distribution, respectively, both having closed-form PDF expressions. For other values of α, the SαS noise does not have a closed-form PDF, but its moments of order $p < \alpha$ are finite. Furthermore, since there is no finite second moment for the SαS distribution when $\alpha < 2$, the noise variance is also infinite in this case. As such, we define the generalized signal-to-noise ratio (GSNR) for the m-th SU as

$$\mathrm{GSNR}_m = 10 \log_{10} \frac{\sigma_s^2 \sigma_h^2}{\gamma_m} \quad (3)$$

where γ_m denotes the noise dispersion of the m-th SU.

The PDFs of the SαS for $\gamma = 1$ and different values of the characteristic exponent α are plotted in Fig. 1. It is seen that α relates directly to the heaviness of the tails of the SαS PDF; a small value of α gives considerable probability mass in the tails which means a large degree of non-Gaussianity. For this reason, the SαS with $0 < \alpha < 2$ is well suited for the "heavier" than normal tail behavior found in practical CR systems.

Spectrum sensing for CR applications in non-Gaussian noise such as the one modeled by the SαS distribution ($\alpha \neq 2$) must take into account much sharper peaks and much heavier tails than the Gaussian PDF. Otherwise, the probability of false alarm will be high, which will in turn greatly reduce the spectrum resource sharing opportunities. For this reason, a good detector for non-Gaussian noise typically utilizes nonlinearities or clippers to reduce the noise spikes. In the next section, we propose a FLOM based detector, to reduce the degree of non-Gaussianity by a non-integer exponent.

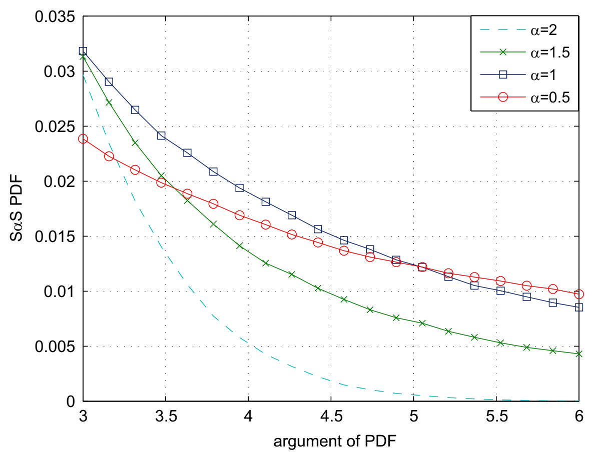

Fig. 1. PDF tails of SαS noise for $\gamma = 1$ and different values of α.

3. FLOM based spectrum sensing

Let us consider first an optimal detector for the spectrum sensing problem. Based on the hypothesis of the signal and noise in (1), the optimal detection statistic can be expressed as

$$y_{OP} = \log \frac{p(\mathbf{z}|\mathcal{H}_1)}{p(\mathbf{z}|\mathcal{H}_0)} = \log\left\{ \prod_{m=1}^{M} \prod_{n=1}^{N} \frac{p_{w_m}(z_m(n) - h_m s(n))}{p_{w_m}(z_m(n))} \right\}, \quad (4)$$

where $\mathbf{z}$ represents a vector of properly ordered observations $z_m(n)$ and $p(\mathbf{z}|\mathcal{H}_i)$ is the corresponding PDF conditioned on hypothesis $\mathcal{H}_i$ for i=0,1. The main difficulty in using (4) is that it requires an analytical expression for the PDF of the background noise. Moreover, it needs the knowledge of the PU signal $s(n)$ and channel gain h_m, which is usually not available in practical CR systems. This means that the likelihood ratio in (4) has uncertain parameters. To overcome this limitation, a generalized likelihood ratio test (GLRT) is proposed in [28,20], which replaces unknown parameters in the traditional likelihood ratio test with estimates. The detection statistic of the GLRT is given by

$$y_{GLRT} = \log\left\{ \prod_{m=1}^{M} \prod_{n=1}^{N} \frac{p_{w_m}(z_m(n) - \overline{h}_m \overline{s}(n))}{p_{w_m}(z_m(n))} \right\} \quad (5)$$

where $\overline{h}_m$ and $\overline{s}(n)$ are the maximum likelihood estimates of h_m and $s(n)$, respectively. The GLRT detector is considered optimal, but it needs to perform the maximum likelihood estimation (MLE) of the received signal under $\mathcal{H}_1$ and of the noise variance under $\mathcal{H}_0$ and as such, it suffers from a large computational burden. In addition, the noise PDF is required in the computation of GLRT. The non-Gaussian noise modeled by the $S\alpha S$ distribution in this paper does not possess an analytical expression for the PDF except for α=1, 2. When α=1, the $S\alpha S$ distribution reduces to Cauchy and the corresponding detector can be written as [20]

$$y_{Cauchy} = \sum_{m=1}^{M} \sum_{n=1}^{N} \log\left\{ 1 + \frac{|z_m(n)|^2}{\gamma^2} \right\} \quad (6)$$

While the Cauchy detector is not strictly applicable to the $S\alpha S$ noise with $\alpha \neq 1$, it is often used as a sub-optimal detector. Even in the case of α=1, one needs to known the dispersion parameter γ of the $S\alpha S$ noise, whose precise value may be difficult to obtain. Therefore, the Cauchy detector does not represent an attractive solution for practical application with arbitrary α. In this paper, we use it as a reference detector for the purpose of comparison. A suboptimal L_p-norm detector for primary signal detection in the presence of non-Gaussian noise was proposed in [17], in which a tunable parameter has to be optimized for the underlying type of noise, and does not require the knowledge of noise distribution. But this detector is not suitable for the $S\alpha S$ noise with no finite moments and moreover, its decision statistic requires knowledge of the powers of the fading channel gains and of the primary signal.

Although the second-order moment of an $S\alpha S$ random variable with $0<\alpha<2$ does not exist, its FLOMs of any order less than α do exist. Here, we propose a suboptimal detector called FLOM detector for spectrum sensing in the $S\alpha S$ noise. Inspired by the energy detection type of detector for $S\alpha S$ noise, the FLOM detector statistic is defined as

$$y_{FLOM} = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} |z_m(n)|^{p_m} \quad (7)$$

where p_m is the order of the fractional moment, which is the only parameter to be determined, and is tunable between 0 and $\alpha_m/2$. When p_m is given, one can compare y_{FLOM} to a pre-scribed threshold η: if $y_{FLOM} > \eta$, the detector decides that the PU is present, otherwise the PU is considered absent. In the FLOM detector, we limit the order p within the range $(0, \alpha_m/2)$ in order to ensure that the variance of the FLOM statistic also exists. The new detector does not use the power of the fading channel gains nor that of the PU signal as compared to the L_p-norm detector and therefore, it is more practical.

We note that the noise of each SU in general may have a different characteristic exponent α_m and dispersion γ_m. The choice of p_m for the m-th SU should therefore depend on the value of α_m in order to give the decision statistic some flexibility. For notational convenience, the statistic order parameter is denoted as vector $\boldsymbol{p} = [p_1, p_2, \ldots, p_M]$ which corresponds to the vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_M]$. Clearly, the FLOM detector does not need the knowledge of the noise PDF, nor that of the PU signal and the channel gain. It uses only the observations $z_m(n)$ and orders $p_m \in (0, \alpha_m/2), m = 1, 2, \ldots, M$. In order to determine appropriate values of p_m that satisfy $p_m < \alpha_m/2$, α_m for the m-th SU noise could be estimated by some estimators under $\mathcal{H}_0$ such as [12]. However, this process could be avoided in practice by always choosing sufficiently small values of p_m. This is because smaller values of p_m in general result in a better detection performance as will be seen in Section 6. In the rest of this paper we focus on the performance analysis of the FLOM detector by deriving and verifying through computer simulations the probabilities of detection and false alarm with respect to the GSNR, the $S\alpha S$ noise parameters and the detector order.

4. Performance of the FLOM detector in non-fading channels

In this section, analytical expressions for the false alarm probability P_{fa} and the detection probability P_d of the proposed FLOM detector are derived for non-fading channels under the assumption of low GSNR. For this, we assume that the channel gain of each SU is $h_m = 1,\ m = 1, 2, \ldots, M$.

4.1. Probabilities of false alarm and detection

The FLOM statistic y_{FLOM} can be regarded as the sum of N independent random variables no matter whether the PU is present or not. According to the Central Limit Theorem, when N is large enough, y_{FLOM} has an asymptotic Gaussian distribution. In the following, we first derive the mean and the variance of y_{FLOM} under the two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$.

Under hypothesis $\mathcal{H}_0$, the mean of y_{FLOM} can be calculated as

$$\mu_0 = E[y_{FLOM}|\mathcal{H}_0] = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} E\left[|w_m(n)|^{p_m}\right] \tag{8}$$

The FLOM of an SαS random variable X can be computed from its dispersion and characteristic exponent as follows [12]:

$$E[|X|^p] = C(p,\alpha)\gamma^{p/\alpha}, \quad p<\alpha \tag{9}$$

where

$$C(p,\alpha) = \frac{2^{p+1}\Gamma\left(\frac{p+1}{2}\right)\Gamma(-p/\alpha)}{\alpha\sqrt{\pi}\Gamma(-p/2)}, \tag{10}$$

with $\Gamma(a) = \int_0^\infty x^{a-1}e^{-x}\,dx$. According to (9) and (10), (8) can be rewritten as

$$\mu_0 = \frac{1}{M}\sum_{m=1}^{M} C(p_m,\alpha_m)\gamma_m^{p_m/\alpha_m} \tag{11}$$

where

$$C(p_m,\alpha_m) = \frac{2^{p_m+1}\Gamma\left(\frac{p_m+1}{2}\right)\Gamma(-p_m/\alpha_m)}{\alpha_m\sqrt{\pi}\Gamma(-p_m/2)}. \tag{12}$$

The variance of y_{FLOM} under $\mathcal{H}_0$ can be written as

$$\sigma_0^2 = E\{y_{FLOM}^2|\mathcal{H}_0\} - E^2\{y_{FLOM}|\mathcal{H}_0\} \tag{13}$$

Substituting (7) into (13), noting that $z_m(n) = w_m(n)$ and using (9), we obtain

$$\begin{aligned}\sigma_0^2 &= E\left[\left(\frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}\left[|w_m(n)|^{p_m}\right]\right)^2\right] - \left\{\frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}E\left[|w_m(n)|^{p_m}\right]\right\}^2 \\ &= \frac{1}{(MN)^2}\left\{NE\left[\sum_{m=1}^{M}|w_m(n)|^{2p_m}\right] + \sum_{\substack{m,i=1\\ m\neq i \text{ or } n\neq j}}^{M}\sum_{n,j=1}^{N}E\left[|w_m(n)|^{p_m}|w_i(j)|^{p_i}\right]\right. \\ &\quad \left. -N\sum_{m=1}^{M}E^2\left[|w_m(n)|^{p_m}\right] - \sum_{\substack{m,i=1\\ m\neq i \text{ or } n\neq j}}^{M}\sum_{n,j=1}^{N}E\left[|w_m(n)|^{p_m}|w_i(j)|^{p_i}\right]\right\} \\ &= \frac{1}{M^2N}\left\{\sum_{m=1}^{M}E[|w_m(n)|^{2p_m}] - \sum_{m=1}^{M}E^2[|w_m(n)|^{p_m}]\right\}\end{aligned} \tag{14}$$

As $p_m < \alpha_m/2$, using (9) into (14) leads to

$$\sigma_0^2 = \frac{1}{M^2N}\sum_{m=1}^{M}\left[C(2p_m,\alpha_m)\gamma_m^{2p_m/\alpha_m} - (C(p_m,\alpha_m)\gamma_m^{p_m/\alpha_m})^2\right] \tag{15}$$

Similarly, it can be shown that the mean and variance of y_{FLOM} under $\mathcal{H}_1$ are given by (see Appendix A for detail)

$$\mu_1 = E[y_{FLOM}|\mathcal{H}_1] = \mu_0 + \sum_{m=1}^{M}\beta_{0,m} \tag{16}$$

where

$$\beta_{0,m} = \frac{\sigma_s^2 p_m(p_m-1)C(p_m-2,\alpha_m)}{2M}\gamma_m^{(p_m-2)/\alpha_m} \tag{17}$$

$$\sigma_1^2 = E\{y_{FLOM}^2\big|\mathcal{H}_1\} - E^2\{y_{FLOM}|\mathcal{H}_1\}$$

$$= \sigma_0^2 + \frac{\sigma_s^2}{M^2N}\sum_{m=1}^{M}\beta_{1,m} \tag{18}$$

where

$$\begin{aligned}\beta_{1,m} &= p_m(2p_m-1)C(2p_m-2,\alpha_m)\gamma_m^{2(p_m-1)/\alpha_m} \\ &\quad - p_m(p_m-1)C(p_m,\alpha_m)\gamma_m^{p_m/\alpha_m}C(p_m-2,\alpha_m)\gamma_m^{(p_m-2)/\alpha_m}\end{aligned} \tag{19}$$

Recall that y_{FLOM} is a Gaussian random variable when N is large. With the above derived mean μ_i and variance σ_i^2 $(i=0,1)$, we can easily obtain the probability of false alarm,

$$P_{fa} = \{y_{FLOM} > \eta\big|\mathcal{H}_0\} = Q\left(\frac{\eta-\mu_0}{\sqrt{\sigma_0^2}}\right) \tag{20}$$

and the probability of detection

$$P_d = \{y_{FLOM} > \eta\big|\mathcal{H}_1\} = Q\left(\frac{\eta-\mu_1}{\sqrt{\sigma_1^2}}\right) \tag{21}$$

where η is the sensing threshold and $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{(-t^2/2)}\,dt$. By choosing a proper value of η, we can achieve a desired trade-off between the two probabilities. According to Neyman–Pearson rule, in order to maximize the opportunity of SU transmission through the spectrum hole, the probability of false alarm should be limited by a maximum value $\overline{P}_{fa}$, while the probability of detection should be maximized to protect the PU transmission. For a given value of $P_{fa} \in (0,\overline{P}_{fa}]$, the sensing threshold η can be expressed as

$$\eta = \sqrt{\sigma_0^2}Q^{-1}(P_{fa}) + \mu_0 \tag{22}$$

By substituting (22) into (21), the probability of detection is rewritten as

$$P_d = Q\left(\frac{\eta-\mu_1}{\sqrt{\sigma_1^2}}\right) = Q\left(\frac{\sqrt{\sigma_0^2}Q^{-1}(P_{fa})+\mu_0-\mu_1}{\sqrt{\sigma_1^2}}\right) \tag{23}$$

By substituting (16) and (18) into (23), we can finally obtain a closed-form expression for the probability of the FLOM detector over non-fading channel, as given by

$$P_d = Q\left(\frac{\sqrt{\sigma_0^2}Q^{-1}(P_{fa}) - \sum_{m=1}^{M}\beta_{0,m}}{\sqrt{\sigma_0^2 + \frac{\sigma_s^2}{M^2N}\sum_{m=1}^{M}\beta_{1,m}}}\right) \tag{24}$$

where σ_0^2, $\beta_{0,m}$, and $\beta_{1,m}$ are functions of the order vector $\boldsymbol{p}$. When P_{fa} is given, we can obtain optimal P_d by searching $\mathbf{p}_m$. However, it is very difficult to find the maximum of P_d in theory due to the high complexity of the function. In Section 6, we will conduct a numerical computation of the detection probability based on (24) as well as Monte Carlo simulations to validate our theoretical analysis.

4.2. Noise uncertainty analysis

From (22), (11) and (15), we can see that η depends on the knowledge of the parameters γ_m and α_m of the noise, which are usually not available and difficult to estimate precisely in practical CR systems. Like the basic energy detection, the well-known noise uncertainty analysis is needed for the FLOM detector.

Here, we just discuss the sensitivity of the spectrum sensing performance on the uncertainty of γ_m; a similar development can be carried out for the parameter α_m. The noise dispersion γ_m, which measures the α-stable distribution noise's spread around its center, can be any positive number and behaves like the variance (for instance, in the Gaussian case when $\alpha=2$, it is equal to half the variance). We define

$$e_m = \frac{\gamma'_m}{\gamma_m} \tag{25}$$

where γ'_m is the practically estimated value of the noise dispersion γ_m by the m-th SU. The degree of noise uncertainty b_m can be defined as $b_m = 10\log_{10} e_m$, which is modeled by a uniform distribution within $[-B, B]$ [25]. When $\gamma'_m = \gamma$ with certainty, $e_m = 1$, and $b_m = 0$ dB. When the noise dispersion γ is uncertain, i.e., $\gamma'_m = e_m\gamma_m$, (11) (15) and (22) can be rewritten as

$$\mu_{0_{un}} = \frac{1}{M}\sum_{m=1}^{M} C(p_m, \alpha_m)(e_m\gamma_m)^{p_m/\alpha_m} \tag{26}$$

$$\sigma^2_{0_{un}} = \frac{1}{M^2 N}\sum_{m=1}^{M}\left[C(2p_m, \alpha_m)(e_m\gamma_m)^{2p_m/\alpha_m} - (C(p_m, \alpha_m)(e_m\gamma_m)^{p_m/\alpha_m})^2\right] \tag{27}$$

and

$$\eta_{un} = \sqrt{\sigma^2_{0un}}\,Q^{-1}(P_{fa}) + \mu_{0un} \tag{28}$$

Comparing (28) to (22), we can see that η_{un} is not a fixed value because of the uncertain e_m, so when the FLOM detector uses the fixed η as the sensing threshold, this would affect the detection performance. In that case, we can use multi-user cooperation or the double sensing threshold method as in the basic energy detection [26] to decrease the effect of noise uncertainty. In Section 6, we will conduct Monte Carlo simulations to analyze the effect of noise uncertainty on spectrum sensing performance.

4.3. Deflection coefficient

The deflection coefficient d^2 is another way to evaluate the detection performance, which gives an overall consideration of P_{fa} and P_d. The FLOM detector includes the orders p_m as free parameters, and its deflection coefficient can be denoted as [27]

$$d^2(\mathbf{p}) = \frac{(\mu_1 - \mu_0)^2}{\sigma_0^2} \tag{29}$$

By substituting (11), (15) and (16) into (29), we obtain

$$d^2(\mathbf{p}) = \frac{N\sigma_s^4\left(\sum_{m=1}^{M} p_m(p_m-1)C(p_m-2, \alpha_m)\gamma_m^{(p_m-2)/\alpha_m}\right)^2}{4\sum_{m=1}^{M}[C(2p_m, \alpha_m)\gamma_m^{2p_m/\alpha_m} - (C(p_m, \alpha_m)\gamma_m^{p_m/\alpha_m})^2]} \tag{30}$$

In the above expression, we have M unknowns p_m's. To make the problem tractable, we assume that $p_1 = p_2 = \cdots = p_M = p$ even though the noise of each SU has a different value for α. Hence, (30) can be simplified as

$$d(p) = \frac{\sqrt{N}\sigma_s^2\sum_{m=1}^{M} p(p-1)C(p-2, \alpha_m)\gamma^{(p-2)/\alpha_m}}{2\sqrt{\sum_{m=1}^{M}[C(2p, \alpha_m)\gamma^{2p/\alpha_m} - (C(p, \alpha_m)\gamma^{p/\alpha_m})^2]}} \tag{31}$$

Given α_m, $d(p)$ depends only on p, and an optimal value of p that maximizes $d(p)$ may be found. We will discuss this issue through computer simulations in Section 6. We now give a simplified expression of (23) in terms of $d(p)$ by using the assumption of low GSNR. Note that under a low GSNR we have $\sigma_1^2 \approx \sigma_0^2$, and thus the detection probability (23) can be simplified using (29) with $p_1 = p_2 = \cdots = p_M = p$ as

$$P_d = Q(Q^{-1}(P_{fa}) - d(p)) \tag{32}$$

According to properties of Q function, the probability of detection will change monotonically with $d(p)$, and thus P_d reaches the maximum when $d(p)$ is maximized.

5. Performance of the FLOM detector in Rayleigh fading channels

In this section, we consider the performance of the FLOM detector when the communication channels between PU and SUs undergo Rayleigh fading.

Under $\mathcal{H}_0$, the mean and the variance of y_{FLOM} do not depend on the channel gain, so they have the same expressions as in the non-fading case. Under $\mathcal{H}_1$, with $y_{FLOM} = (1/MN)\sum_{m=1}^{M}\sum_{n=1}^{N}|z_m(n)|^{p_m}$ as defined in (7), the channel gain h_m involved in z_m will undergo Rayleigh fading. In this case, the mean and the variance depend on the channel gain h_m and their computation is similar to the non-fading case under $\mathcal{H}_1$, which leads to the following expressions (see Appendix A for details):

$$\mu_1 = E[y_{FLOM}|\mathcal{H}_1] = \mu_0 + \sum_{m=1}^{M}\beta_{0,m}|h_m|^2 \tag{33}$$

$$\sigma_1^2 = E[(y_{FLOM} - E[y_{FLOM}])^2|\mathcal{H}_1] = \sigma_0^2 + \frac{\sigma_s^2}{M^2 N}\sum_{m=1}^{M}\beta_{1,m}|h_m|^2 \tag{34}$$

where

$$\beta_{0,m} = \frac{\sigma_s^2 p_m(p_m-1)C(p_m-2, \alpha_m)}{2M}\gamma_m^{(p_m-2)/\alpha_m} \tag{35}$$

$$\begin{aligned}\beta_{1,m} = {} & p_m(2p_m-1)C(2p_m-2, \alpha_m)\gamma_m^{2(p_m-1)/\alpha_m} \\ & - p_m(p_m-1)C(p_m, \alpha_m)\gamma_m^{p_m/\alpha_m}C(p_m-2, \alpha_m)\gamma_m^{(p_m-2)/\alpha_m}.\end{aligned} \tag{36}$$

To make the problem tractable, we consider again the low GSNR case where $\sigma_s^2 \approx 0$ and $\sigma_1^2 \approx \sigma_0^2$. Using this approximation along with (23) and (33), we can express P_d under Rayleigh fading as

$$\begin{aligned}P_d &= E_{\mathbf{h}}\left[Q\left(\frac{\eta - \mu_1}{\sqrt{\sigma_1^2}}\right)\right] \\ &\approx E_{\mathbf{h}}\left[Q\left(Q^{-1}(P_{fa}) + \frac{\mu_0 - \mu_1}{\sigma_0}\right)\right] \\ &= E_\Delta[Q(Q^{-1}(P_{fa}) - \Delta)]\end{aligned}$$

$$= \int_0^\infty Q(Q^{-1}(P_{fa}) - \Delta) p_\Delta(\Delta)\, d\Delta \tag{37}$$

where

$$\Delta = \sum_{m=1}^{M} \frac{\sigma_s^2 p_m(p_m-1)C(p_m-2,\alpha_m)\gamma_m^{(p_m-2)/\alpha_m}}{2M\sigma_0} |h_m|^2 \tag{38}$$

$p_\Delta(\Delta)$ is the PDF of Δ, and P_{fa} is given by (20).

For the calculation of (37), the exact PDF $p_\Delta(\Delta)$ is needed. We consider the channel gain h_m for $m=1,\ldots,M$ as IID Rayleigh random variables with the PDF $f_{h_m}(h) = (h/\sigma^2)e^{-h^2/2\sigma^2}$. It can be shown that the distribution of Δ is given by [29]

$$p_\Delta(\Delta) = \frac{\Delta^{M-1}}{(M-1)!\Delta_c^M} e^{(-\Delta/\Delta_c)} \tag{39}$$

where

$$\Delta_c = \sum_{m=1}^{M} \frac{\sigma_s^2 p_m(p_m-1)C(p_m-2,\alpha_m)\gamma_m^{(p_m-2)/\alpha_m}}{2M\sigma_0} E\left[|h_m|^2\right] \tag{40}$$

with $E[|h_m|^2] = \sigma_h^2$. By substituting (39) into (37), the probability of detection of the FLOM detector for Rayleigh fading channel is given by

$$P_d = \frac{1}{\sqrt{2\pi}} \int_0^\infty \int_{Q^{-1}(P_{fa})-\Delta}^\infty e^{-x^2/2}\, dx \frac{\Delta^{M-1}}{(M-1)!\Delta_c^M} e^{(-\Delta/\Delta_c)} d\Delta \tag{41}$$

Finding a closed-form expression for P_d is difficult in general. Thus, we consider the following special cases and approximations to evaluate the integral in (41).

(1) $M=1$: We have $\Delta = \sigma_s^2 p(p-1)C(p-2,\alpha)\gamma^{(p-2)/\alpha}|h|^2/2\sigma_0$, $\Delta_c = \sigma_s^2 p(p-1)C(p-2,\alpha)\gamma^{(p-2)/\alpha}\sigma_h^2/2\sigma_0$, and $p_\Delta(\Delta) = (1/\Delta_c)e^{(-\Delta/\Delta_c)}$. The corresponding probability of detection is given by

$$P_d = \frac{1}{\sqrt{2\pi}\Delta_c} \int_0^\infty \int_{Q^{-1}(P_{fa})-\Delta}^\infty e^{-x^2/2}\, dx\, e^{(-\Delta/\Delta_c)}\, d\Delta \tag{42}$$

Now the double integration in (42) can be evaluated easily by numerical means.

(2) $M>1$: Here we study a special case where all the SUs share the same parameters $p_m = p$ and $\alpha_m = \alpha$ in order to simply the calculation. From (38) and (40), we have

$$\Delta = \sum_{m=1}^{M} \frac{\sigma_s^2 p(p-1)C(p-2,\alpha)\gamma^{(p-2)/\alpha}}{2M\sigma_0} |h_m|^2 \tag{43}$$

and

$$\Delta_c = \frac{\sigma_h^2\sigma_s^2 p(p-1)C(p-2,\alpha)\gamma^{(p-2)/\alpha}}{2\sigma_0} \tag{44}$$

where $\sigma_0 = \sqrt{(C(2p,\alpha)\gamma^{2p/\alpha} - (C(p,\alpha)\gamma^{p/\alpha})^2)/MN}$. With (43) and (44) above, (41) can be easily evaluated numerically to obtain P_d.

6. Numerical and simulation results

In this section, we present numerical and simulation results to evaluate the performance of the FLOM detector as opposed to the Cauchy detector.

6.1. Performance of the FLOM detector in non-fading channels

Here, we investigate the performance of the FLOM detector in terms of the probabilities of false alarm and detection as well as of the deflection coefficient when the communication channels between the PU and SUs at the CR are fading-free, $h_1 = h_2 = \cdots = h_M = 1$. We assume that the PU signal is Gaussian with zero-mean, variance σ_s^2, and the noises are IID $S\alpha S$ with dispersion $\gamma = 1$. The simulation results are obtained based on 10,000 Monte Carlo runs where the sample size is set to $N=1000$, unless otherwise specified.

Fig. 2 shows plots of the probability of false alarm versus the sensing threshold for different values of α, p and M, where $M=1$ means a detection case involving one SU only and $M=2$ represents a cooperative detection with two SUs. In the case of cooperative sensing, we have set $\alpha_1 = 2, p_1 = 0.8$ for SU_1 and $\alpha_2 = 0.8, p_2 = 0.3$ for SU_2. The curves for the probability of false alarm are obtained both by simulation using (7) in a binary detection test, indicated as "simulation", and by evaluation of analytical expression (20) indicated as "analysis". Clearly, the theoretical results agree well with the simulations for both the single SU and the two-SU cooperative detection.

Fig. 3 shows the receiver operating characteristic (ROC) curves of the FLOM detector for different values of noise dispersion γ with $B=0$ dB, 1 dB and 2 dB, where we have set $p=0.4$, $\alpha=1$, $M=1$, GSNR$=0$ dB. From Fig. 3, we can see that the detection performance decreases with increasing the degree of noise uncertainty. For example, for $P_{fa}=0.1$, probability of detection is 88% when $B=0$ dB, 70% when $B=1$ dB, and 45% when $B=2$ dB.

Fig. 4 shows the probability of detection versus GSNR for different values of noise dispersion γ with B=0 dB,1 dB and 2 dB, where we have set $p=0.4$, $\alpha=1$, $M=1$, $P_{fa}=0.1$. From Fig. 4, we can see that the detection performance decreases with increasing the degree of noise uncertainty but increases with increasing GSNR. For example, the probability of detection is 100% when GSNR=5 dB, 88% when GSNR=0 dB, and 0.17% when GSNR$=-10$ dB with $B=0$ dB.

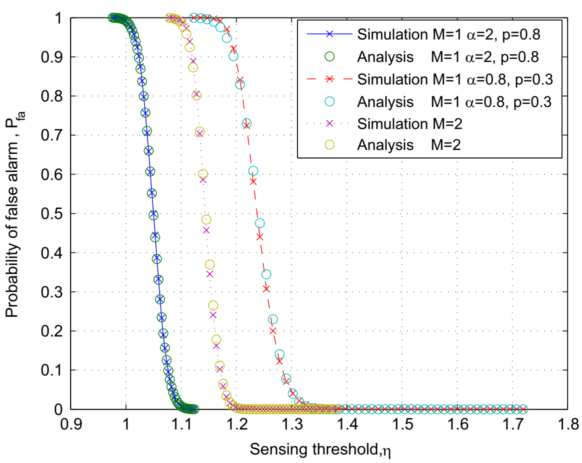

Fig. 2. Probability of false alarm versus sensing threshold of the FLOM detector with $N=1000$.

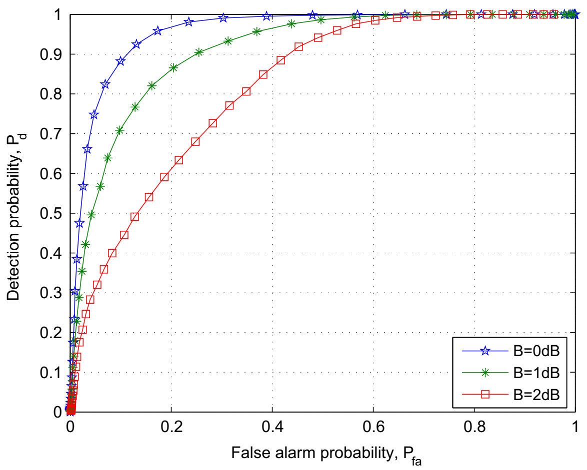

Fig. 3. ROC of the FLOM detector with different degrees of noise uncertainty with $N=1000$, $p=0.4$, $\alpha=1$, $M=1$, GSNR=0 dB.

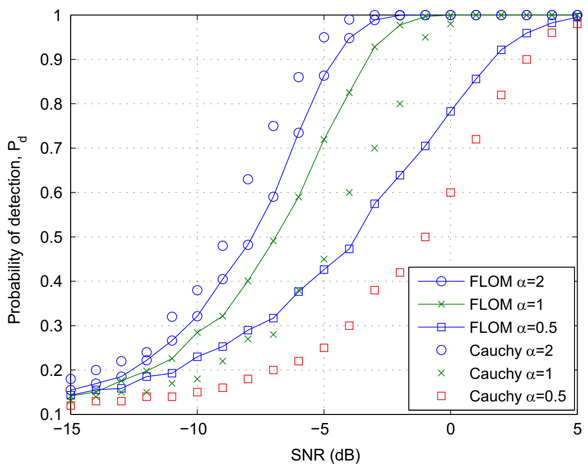

Fig. 5. Probability of detection versus GSNR for FLOM and Cauchy detectors with $p=0.3$, $P_{fa}=0.1$, $M=1$, $N=1000$.

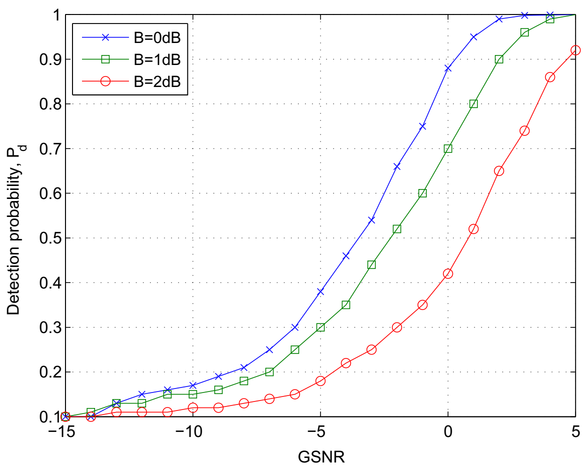

Fig. 4. Probability of detection versus GSNR with different degrees of noise uncertainty with $N=1000$, $p=0.4$, $\alpha=1$, $M=1$, $P_{fa}=0.1$.

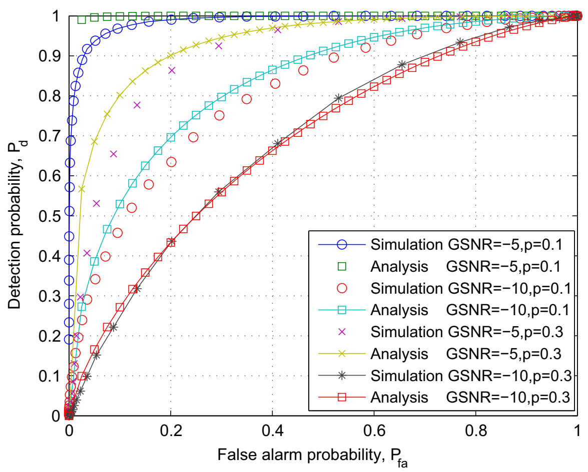

Fig. 6. ROC of FLOM detector with the same order p and GSNR $=-10$ dB, $N=5000$, $M=2$, $\alpha_1=2$, $\alpha_2=0.8$.

Fig. 5 shows the probability of detection of the proposed FLOM detector as a function of GSNR for different values of α, with comparison to that of the Cauchy detector obtained by simulation using (7) and (6), respectively, where $P_{fa}=0.1$, $p=0.3$, and a single SU is assumed. It can be seen that the probability of detection increases with increasing the GSNR or the value of α. Moreover, our proposed detector has a better detection performance than the Cauchy detector under almost all levels of GSNR for $\alpha=1$ and 0.5. For example, when GSNR $=-5$ dB, $P_{fa}=0.1$ and $\alpha=1$, the probability of detection of our detector is 72%, but that of the Cauchy detector is 45% only, which fails to meet the requirement of spectrum sensing. Note that when $\alpha=2$, the detection probability of the FLOM detector is lower than that of the Cauchy detector. For this special case, where the noise Gaussian distribution, but we could increase p towards $p=2$ to increase P_d, while for the Cauchy detector, P_d is fixed.

In the next experiment, we focus on the cooperative sensing case with $M=2$, and set the noise models of the two SUs as $\alpha_1=2$, $\alpha_2=0.8$, respectively, and the detector orders as $0<p_1=p_2=p<\alpha_{min}/2$, where α_{min} is the smallest of the α values for the two SUs.

Fig. 6 shows the ROC curves of the FLOM detector for different values of p and two values of GSNR (-10 dB and -5 dB), as obtained by using simulation and (24). It is seen that the FLOM detector performs better for smaller values of p. For example, the probability of detection is nearly to 70% with $p=0.1$, but just 43% with $p=0.3$, when GSNR $=-10$ dB and $N=5000$.

Note that based on (7), and assuming that all the observed samples are non-zero, the decision statistic y_{FLOM} approaches the fixed value of 1 when p is zero. Here, the minimum value of p is set to 0.05. Fig. 7 plots the deflection coefficient of the FLOM detector calculated using (31) when the fractional lower order p varies within [0.05, 0.4) for two different values of GSNR (0 dB and -5 dB). It is seen that the deflection coefficient $d(p)$ decreases as p increases, and it approaches the maximum when $p=0.05$.

Fig. 8 shows the relationship between the probability of detection and the deflection coefficient for two different values of GSNR (0 dB and -5 dB) based on (32) when

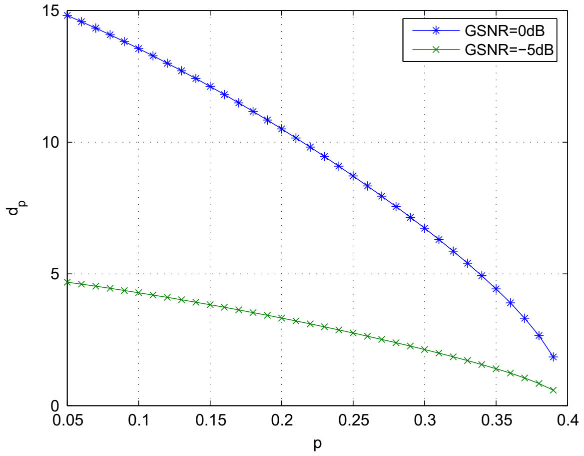

Fig. 7. Deflection coefficient versus order p of the FLOM detector with $N=5000$, $M=2$, $\alpha_1=2$, $\alpha_2=0.8$.

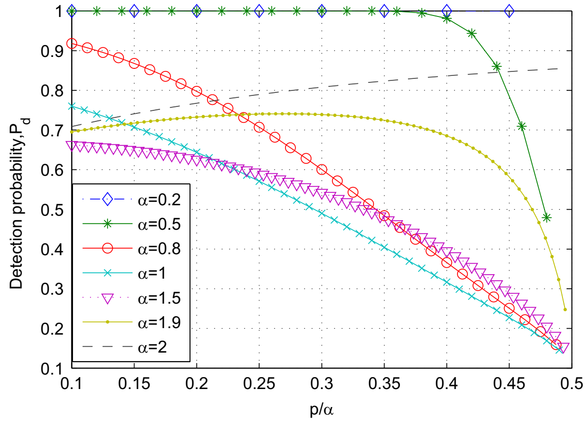

Fig. 9. Detection probability versus p/α of the FLOM detector for GSNR $=-8$ dB with $P_{fa}=0.1$, $N=5000$, $M=2$.

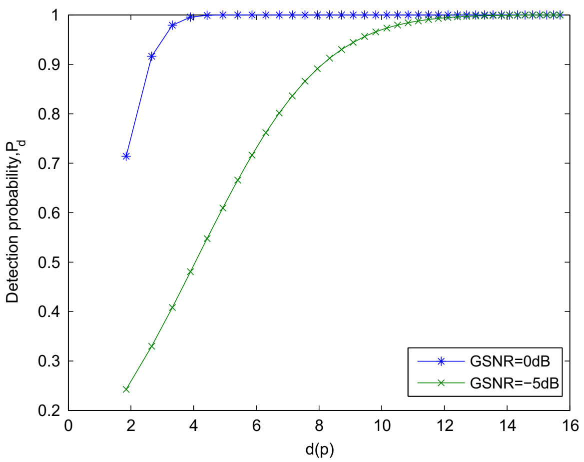

Fig. 8. Detection probability versus deflection coefficient of the FLOM detector with $P_{fa}=0.1$, $N=5000$, $M=2$, $\alpha_1=2$, $\alpha_2=0.8$.

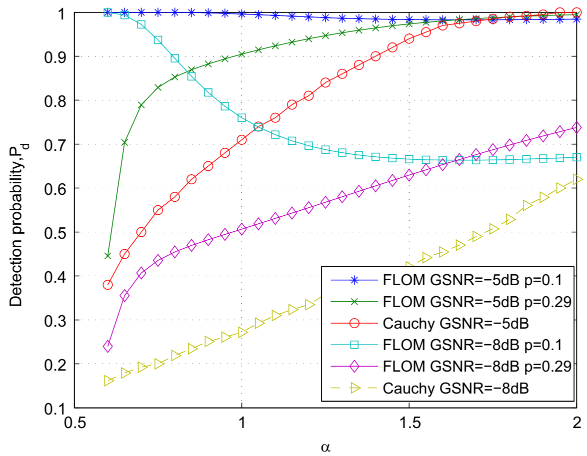

Fig. 10. Detection probability versus α of the FLOM and the Cauchy detectors with $P_{fa}=0.1$, $N=5000$, $M=2$.

$P_{fa}=0.1$. It can be seen that P_d increases as $d(p)$ increases in general. It is noted from Fig. 8 that the detection probability depends largely on the GSNR when P_{fa} is fixed.

Fig. 9 shows the relationship between the probability of detection and $p/\alpha \in [0.1, 0.5)$ for different SαS noises with given $\alpha_1=\alpha_2=\alpha$, when $P_{fa}=0.1, N=1000, M=2$, GSNR $=-8$ dB. It is seen that P_d increases as p/α decreases for almost all given SαS except for Gaussian noise with $\alpha=2$, which is consistent with the result in Fig. 7. In brief, as the order $p \in (0, \alpha_{min}/2)$ decreases, both $d(p)$ and P_d increase and thus the performance of the FLOM detector is greatly improved. From Fig. 9, we can see that the FLOM detector also exhibits good performance under Gaussian noise, where P_d increases as p/α increases.

Fig. 10 shows the performances of the FLOM and Cauchy detectors as the heaviness of the SαS noises tail varies (i.e., characteristic exponent $\alpha \in [0.6, 2]$) for two different values of GSNR (-5 dB and -8 dB). Here, the detection probability is calculated using (32) for the FLOM detector with $p=0.1$ and $p=0.29<0.6/2$ and using (6) for the Cauchy detector when $P_{fa}=0.1$ and $\alpha_1=\alpha_2=\alpha$. From Fig. 10, it can be seen that as the heaviness of the tail of the SαS noise decreases (α increases), both the FLOM detector with $p=0.29$ and the Cauchy detector give a better performance. It can be seen that the detection probability of $p=0.1$ is higher than that of $p=0.29$ when $\alpha \in [0.6, 1.7)$, but lower when $\alpha \in [1.7, 2]$. However, the detection probability of the proposed detector is much better than that of the Cauchy detector under almost all values of $\alpha \in [0.6, 2]$.

6.2. *Performance of the FLOM detector in Rayleigh fading channels*

In this subsection, the performance of the FLOM detector is investigated against Rayleigh fading communication channels.

We consider the channel gains h_m to be constant during spectrum sensing time with $E[|h_m|^2]=1$. In addition, we assume that the noises are IID SαS with dispersion $\gamma_m=1$. The simulation results are obtained based on 10,000 Monte Carlo runs and the sample size is set to $N=10{,}000$, unless otherwise specified.

In Fig. 11, the ROC curves of FLOM and Cauchy detectors for different values of α are shown when $M=1, N=10{,}000, p=0.2$ and GSNR $=-10$ dB. The simulation results show that at small α values, the FLOM detector using (42) gives a better performance than the Cauchy detector using (6). Also, as α decreases, the probability of detection of the FLOM detector decreases much more slowly than that of the Cauchy detector. In particular, when $P_{fa}=0.1$ and α changes from 2 to 0.5, P_d of the Cauchy detector decreases from 90% to 70%, while P_d of the FLOM detector decreases from 90% to 85% only, meaning that the FLOM detector provides a more robust spectrum sensing in CR systems under the SαS noise.

Next, we investigate the performance of the FLOM detector for different numbers of samples and SUs. In Fig. 12, the detection probability of the FLOM detector is shown as a function of the number of samples for $p=0.2$, $\alpha=1$ and $P_{fa}=0.1$. It can be seen that, the detection probability increases as N and M increase. Especially, when GSNR $=-15$ dB and $N=20{,}000$, P_d is 33% for $M=1$, 67% for $M=2$ and 97% for $M=4$, indicating that multi-user cooperative sensing can improve the detection performance significantly.

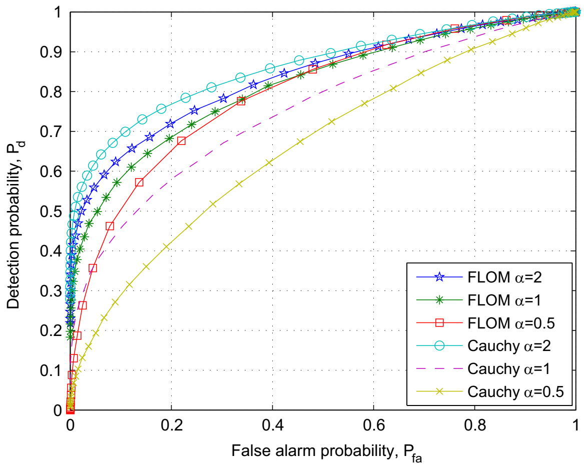

Fig. 11. ROC of the FLOM and the Cauchy detectors under Rayleigh fading with $M=1, N=10{,}000, p=0.2$, GSNR $=-10$ dB.

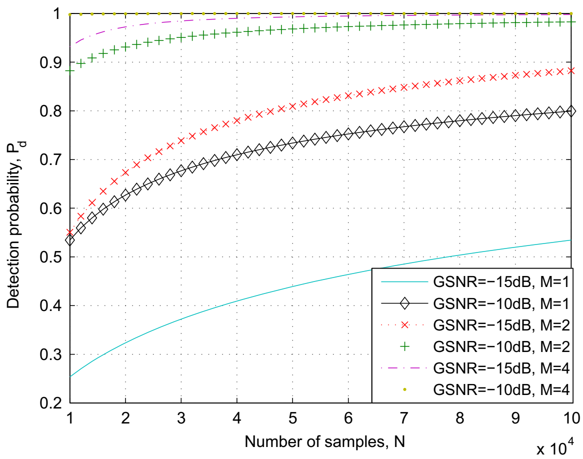

Fig. 12. Probability of detection versus number of samples under Rayleigh fading with $p=0.2$, $\alpha=1$ and $P_{fa}=0.1$.

7. Conclusion

We have studied cooperative spectrum sensing in non-Gaussian noise environments that can be modeled by symmetric α-stable distribution. We have focused on a scenario where the PU signal and the Rayleigh fading channel gains are unknown to the CR users and the noise does not have a compact analytical form of PDF. A new cooperative scheme for spectrum sensing in symmetric α-stable noise has been derived in the form of the proposed FLOM detector. Numerical and simulation results have shown that the proposed FLOM detector achieves a much better performance than the traditional Cauchy detector does, and the proposed cooperative scheme improves further the detection probability which makes the FLOM detector more useful when the GSNR is very small.

Acknowledgment

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Appendix A. Mean and variance of y_{FLOM} under $\mathcal{H}_1$

Here we derive the mean and variance of y_{FLOM} under $\mathcal{H}_1$ for fading channels, since the non-fading channels can be regarded as a special case of the fading channels with $h_m=1$, for $m=1,2,\dots,M$. Under $\mathcal{H}_1$, the mean of y_{FLOM} for fading channels can be expressed as

$$\mu_1 = E[y_{FLOM}|\mathcal{H}_1] = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} E\left[|h_m s(n)+w_m(n)|^{p_m}\right] \tag{45}$$

Recalling the assumption in (1) that the PU signal $s(n)$, the channel gains h_m and the SαS noise $w_m(n)$ are mutually independent and IID processes, we have $E[\sum_{n=1}^{N}|z_m(n)|_m^p] = NE[|z_m(n)|_m^p]$, based on which (45) can be rewritten as

$$\mu_1 = E\left[y_{FLOM}|\mathcal{H}_1\right] = \frac{1}{M}\sum_{m=1}^{M} E\left[|h_m s(n)+w_m(n)|^{p_m}\right]. \tag{46}$$

We now use the generalized binomial theorem [30] to approximate $|h_m s(n)+w_m(n)|^{p_m}$, leading to

$$\mu_1 = \frac{1}{M}\sum_{m=1}^{M} E\left[|w_m(n)|^{p_m} + p_m|h_m s(n)||w_m(n)|^{p_m-1} + \frac{p_m(p_m-1)}{2!}|h_m s(n)|^2|w_m(n)|^{p_m-2} + \cdots\right] \tag{47}$$

Under the assumption of low GSNR, we have $|h_m s(n)| \ll |w_m(n)|$. By ignoring the higher-order terms, noting that h_m remains constant during the sensing period, $s(n)$ has zero mean, and using (9), we can obtain

$$\mu_1 \approx \frac{1}{M}\sum_{m=1}^{M} E\left[|w_m(n)|^{p_m} + \frac{p_m(p_m-1)}{2!}|h_m s(n)|^2 |w_m(n)|^{p_m-2}\right]$$
$$= \mu_0 + \frac{1}{2M}\sum_{m=1}^{M} p_m(p_m-1)E\left[|h_m s(n)|^2|w_m(n)|^{p_m-2}\right]$$
$$= \mu_0 + \frac{\sigma_s^2}{2M}\sum_{m=1}^{M} |h_m|^2 p_m(p_m-1)C(p_m-2,\alpha_m)\gamma^{(p_m-2)/\alpha_m}$$
$$= \mu_0 + \sum_{m=1}^{M}\beta_{0,m}|h_m|^2 \tag{48}$$

where

$$\beta_{0,m} = \frac{\sigma_s^2 p_m(p_m-1)C(p_m-2,\alpha_m)}{2M}\gamma_m^{(p_m-2)/\alpha_m}.$$

The variance of y_{FLOM} under $\mathcal{H}_1$ can be calculated as

$$\sigma_1^2 = E[(y_{FLOM}^2 - E^2 y_{FLOM})|\mathcal{H}_1] \tag{49}$$

Using (1) and (7) in (49), we have

$$\sigma_1^2 = E\left\{\left[\frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}|h_m s(n)+w_m(n)|^{p_m}\right]^2\right\} - \left\{\frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}E\left[|h_m s(n)+w_m(n)|^{p_m}\right]\right\}^2 \tag{50}$$

By using the independence assumption for the PU signal $s(n)$, the channel gains h_m and the SαS noise $w_m(n)$, we have $E[\sum_{n=1}^{N}|z_m(n)|^{p_m}] = NE[|z_m(n)|^{p_m}]$. Applying this property to (50) yields

$$\sigma_1^2 = \frac{1}{(MN)^2}\left\{N\sum_{m=1}^{M}E[|h_m s(n)+w_m(n)|^{2p_m}] + E\left[\sum_{\substack{m,i=1\\ m\neq i\,\mathrm{or}\,n\neq j}}^{M}\sum_{n,j=1}^{N}|h_m s(n)+w_m(n)|^{p_m}|h_i s(n)+w_i(j)|^{p_i}\right] - N\sum_{m=1}^{M}E^2\left[|h_m s(n)+w_m(n)|^{p_m}\right] - \sum_{\substack{m,i=1\\ m\neq i\,\mathrm{or}\,n\neq j}}^{M}\sum_{n,j=1}^{N}E[|h_m s(n)+w_m(n)|^{p_m}|h_i s(n)+w_i(j)|^{p_i}]\right\}$$
$$= \frac{1}{M^2N}\left\{\sum_{m=1}^{M}E[|h_m s(n)+w_m(n)|^{2p_m}] - \sum_{m=1}^{M}E^2[|h_m s(n)+w_m(n)|^{p_m}]\right\} \tag{51}$$

Now we use the generalized binomial theorem to approximate $|h_m s(n)+w_m(n)|^{2p_m}$ and $|h_m s(n)+w_m(n)|^{p_m}$, leading to

$$\sigma_1^2 = \frac{1}{M^2N}\left\{\sum_{m=1}^{M}E\left[|w_m(n)|^{2p_m} + 2p_m|h_m s(n)||w_m(n)|^{2p_m-1} + \frac{2p_m(2p_m-1)}{2!}|h_m s(n)|^2|w_m(n)|^{2p_m-2}+\cdots\right] - E^2\left[|w_m(n)|^{p_m} + p_m|h_m s(n)||w_m(n)|^{p_m-1} + \frac{p_m(p_m-1)}{2!}|h_m s(n)|^2|w_m(n)|^{p_m-2}+\cdots\right]\right\} \tag{52}$$

Under the assumption of low GSNR, we have $|h_m s(n)| \ll |w_m(n)|$. By ignoring the higher-order terms and noting that h_m remains constant during the sensing period and $s(n)$ has zero mean, we can obtain

$$\sigma_1^2 \approx \frac{1}{M^2N}\left\{\sum_{m=1}^{M}E\left[|w_m(n)|^{2p_m} + \frac{2p_m(2p_m-1)}{2!}|h_m s(n)|^2|w_m(n)|^{2p_m-2}\right] - E^2\left[|w_m(n)|^{p_m} + \frac{p_m(p_m-1)}{2!}|h_m s(n)|^2|w_m(n)|^{p_m-2}\right]\right\}$$
$$= \frac{1}{M^2N}\sum_{m=1}^{M}\{E\left[|w_m(n)|^{2p_m}\right] - E^2\left[|w_m(n)|^{p_m}\right] + \sigma_s^2|h_m|^2 p_m(2p_m-1)E[|w_m(n)|^{2p_m-2}] - \sigma_s^2|h_m|^2 p_m(p_m-1)E[|w_m(n)|^{p_m}]E[|w_m(n)|^{p_m-2}]\} \tag{53}$$

Using (9) and (14) into (53), we have

$$\sigma_1^2 \approx \sigma_0^2 + \frac{\sigma_s^2}{M^2N}\sum_{m=1}^{M}|h_m|^2\{p_m(2p_m-1)C(2p_m-2,\alpha_m)\gamma_m^{2(p_m-1)/\alpha_m} - p_m(p_m-1)C(p_m,\alpha_m)\gamma_m^{(p_m)/\alpha_m}C(p_m-2,\alpha_m)\gamma_m^{(p_m-2)/\alpha_m}\}$$
$$= \sigma_0^2 + \frac{\sigma_s^2}{M^2N}\sum_{m=1}^{M}\beta_{1,m}|h_m|^2 \tag{54}$$

where

$$\beta_{1,m} = p_m(2p_m-1)C(2p_m-2,\alpha_m)\gamma_m^{2(p_m-1)/\alpha_m} - p_m(p_m-1)C(p_m,\alpha_m)\gamma_m^{p_m/\alpha_m}C(p_m-2,\alpha_m)\gamma_m^{(p_m-2)/\alpha_m}. \tag{55}$$

Obviously, the mean and variance given by (48) and (54), respectively, also apply to non-fading channel as long as h_m is replaced by 1.

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