Subspace Method for Blind Equalization of Multiple Time-Varying FIR Channels

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Abstract

Wireless communications is the fastest growing segment of communication technologies. In a wireless communication system, the inter-symbol interference (ISI) is a linear distortion which causes decision errors at the receiver. The equalizer is required to remove the ISI. In the past decade, the blind channel equalization has been a popular research topic in the area of wireless communication. A particular class of blind equalization approaches is based on the second order statistics (SOS) of the received signals. Within this framework, subspace methods exploit the orthogonality between the signal and noise subspaces in order to identify the channel characteristics so that the equalizer can be constructed.

This thesis investigates a new equalization algorithm for the time-varying (TV) channel under the single-input multiple-output (SIMO) framework. The TV channel is decomposed using arbitrary basis functions associated with time variable properties of the channels, and with expansion coefficients associated with multi-path delays. An equivalent time-invariant (TI) multiple-input multiple-output (MIMO) system is built for the TV SIMO system. The equivalent TI MIMO system is assumed to match the necessary and sufficient conditions of the SOS identification framework. The blind subspace method is exploited to identify the expansion coefficients when considered as channel characteristics of the MIMO system. The associated ambiguity matrix is identified by using the least square (LS) method. The zero forcing equalizer is realized based on the result of the subspace channel equalization and the ambiguity matrix. The simulation results indicate that the proposed equalizer can effectively recover the source signal in TV SIMO channel applications.

Résumé

La communication sans fil est le segment de croissance le plus dynamique parmi les techniques de la communication. Dans un système de communication sans fil, l'interférence inter-symboles (ISI) est une distorsion linéaire qui provoque des erreurs de décisions au niveau du récepteur. L'égaliseur est nécessaire pour éliminer l'ISI. Récemment, l'égalisation aveugle du canal est devenue un sujet de recherche populaire dans les domaines de la communication sans fil. Un des jalons de la technologie aveugle est fondé sur le cadre des statistiques du second ordre (SOS) du signal reçu. Tout particulièrement, la méthode du sous-espace exploite l'orthogonalité entre le sous-espace signal et le sous-espace bruit afin d'identifier les caractéristiques du canal de telle sorte que l'égaliseur puisse être construit.

Dans cette thèse, j'ai proposé un algorithme de péréquation pour le canal à variation temporelle (TV) des systèmes à entrée unique et sorties multiples (SIMO). Le canal TV est décomposé en fonctions arbitraires associées aux propriétés de TV du cacal, et avec les coefficients d'expansion associés à chacun des retards multi-trajet. Un système équivalent invariant dans le temps (TI), à entrée multiples et sorties multiples (MIMO) est conçu pour le TV SIMO. Le système équivalent TI MIMO est supposé correspondre aux conditions nécessaires et suffisantes dans le cadre de la théorie SOS. La méthode sous-espace aveugle est exploitée pour identifier les coefficients d'expansion quand ils sont considérés comme caractéristiques du canal du système MIMO. La matrice d'ambiguïté est déterminée par la méthode des moindres carrés (LS). La remise à zéro forcée de l'égaliseur est réalisée sur la base des résultats de l'égalisation des canaux de sous-espace et de la matrice d'ambiguïté. Des expériences de simulations numériques sont utilisées afin de démontrer le potential d'application de la nouvelle méthode.

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List of Acronyms

CMA	Constant modulus algorithm
CSI	Channel state information
DTFT	Discrete-time Fourier transform
EVD	Eigenvalue decomposition
FIR	Finite impulse response
HOS	Higher order statistics
i.i.d.	Independent and identically distributed
ISI	Inter-symbol interference
LS	Least squares
LTI	Linear time invariant
MAP	Maximum a posteriori
MIMO	Multiple-input multiple-output
ML	Maximum likelihood
MMSE	Minimum mean-square error
MSE	Mean square-error
QAM	Quadrature amplitude modulation
SER	Symbol error rate
SIMO	Single-input multiple-output
SISO	Single-input single-output
SNR	Signal-to-noise ratio

SOS	Second order statistics
TI	Time-invariant
TV	Time-varying
ZF	Zero forcing

Chapter 1

Introduction

As more and more multimedia services are being offered to the end users, service provider must adapt to the upcoming fourth generation (4G) of wireless communication systems in order to keep up with the demand for high data rates. However, there is a conflict between these increasing data rates and the limited radio spectrum currently available. The multiple-input multiple-output (MIMO) communication systems offer a significant performance gain in terms of capacity and coverage range. Through research studies on MIMO systems, it has been proven that improvements can be made on the spectral efficiency and against the distortion and interference incurred during transmission. In particular, the sensitivity to channel fading is reduced by the spatial diversity provided by multiple spatial channels. The power requirements of a high spectral efficiency transmission can also be significantly reduced by avoiding the compressive region of the information-theoretic capacity bound, if certain conditions are satisfied. Hence, the research associated with MIMO systems has become an exciting topic.

1.1 Time-Varying Channel Estimation and Equalization in Wireless Communications

In an ideal wireless communication system, the received signal $x(n)$ is identical with the transmitted signal $s(n)$, except for a transmission delay. In reality, signal distortion and noise are involved in the transmission of data over the wireless channel. Therefore, the channel state information (CSI) has to be obtained to adapt the transmission parameters to current channel conditions and hence optimize the transmission performance. The CSI indicates how a signal propagates from the transmitter to the receiver and represents the combined effect of, e.g. scattering, fading, and power decay with distance. It comprises both instantaneous and statistical information about the wireless channel. Instantaneous information characterizes short-term channel conditions such as the impulse response of a digital filter representation of the channel at a given time. Statistical information usually characterizes long-term channel conditions such as the average delay spread on the power delay profile.

The line-of-sight transmission paths between transmitters and receivers are usually obstructed, so that the propagation is via reflection, diffraction and scattering by hills, building and vehicles. These multiple transmission paths suffer different delays, which inevitably cause amplitude fading and unpredictable carrier phase shifts. Even worse, the transmitters and receivers could move relative to each other. If the scatterers separation increase, the spread between the channel multi-path delays increases. In addition to its instantaneous delay, phase and amplitude parameters, each multi-path component is characterized by a Doppler shift, which depends on the mobility (speed) of devices and scatterers. The Doppler effect manifests itself in the time variations for the above parameters. For low transmission rates, the normalized delay spread is relatively small so that the wireless channel may usually experience significant time variability but small frequency selectivity. Under this situation, the channel can be idealized as a time-varying (TV) channel, i.e. with TV impulse response representation.

Channel estimation is a procedure used to approximate the CSI of a communication link, since the channel conditions vary. The most popular approach is based on the use of a training sequence. In this method, a training sequence or pilot, which is already known by the receiver, is transmitted before the information sequence in order to estimate the channel impulse response in real-time. The received training sequence is analyzed and the unknown channel is estimated by minimizing the decoding error after the transmission. However, the use of training sequences at regular time intervals decreases the effective data rate. Accordingly, the so-called blind channel estimation algorithms have been a popular topic during the last two decades. The blind channel estimation is a process, which enables the determination of channel characteristics from the received signal, without using the training sequence.

Generally, an equalizer is a device used to compensate the distortion caused by propagation through the radio channel. The distortion occurs when a signal bandwidth exceeds the allowed passband, since physical transmission is band limited. In addition, the channel incurs additional effects due a non-flat frequency response and additive noise. An adaptive equalizer is an equalizer that automatically adapts to the TV properties of the communication channel based on constant monitoring of the training sequence. Hence, the adaptive equalizer is difficult to apply in real-time to high data rate wireless communication systems, because of the disadvantages of continually using training data over the communication link. For this reason, blind equalization algorithms have been developed where the training sequence is not needed in order to track variations of the radio channel. In this type of approach, the transmitted sequence is equalized from the received signal, while it only exploits the transmitted sequence statistics. Moreover, the equalization algorithm can take advantage from the results of a blind channel estimation algorithm, for example, the so-called subspace algorithm, to create an appropriate equalizer. This is the main topic under study in this thesis.

1.2 Literature Review

In wireless communications, the CSI needs to be estimated at the receiver and finely quantized so that the received signal can be recovered with reasonably low probability of error.

Estimation theory is a part of statistical signal processing that attempts to determine the value of one or more unknown parameters, from a set of measurements that usually take the form of a random signal. Different methods for designing estimators have been proposed, according to assumptions employed to model the measured signal. One of the classical estimation methods is the maximum likelihood (ML) for the estimation of one or more deterministic unknown parameters. It is used to find the estimator that maximizes the likelihood function describing the relation between the observed data and unknown parameters. With ML estimation, we search for the parameter values that maximize the conditional probability of observing the deterministic unknown parameters.

The maximum a posteriori probability (MAP) estimator is based on posterior distribution of Bayesian statistics. It can be used to obtain an estimator of an unobserved quantity, modelled as a random variable, on the basis of empirical data related to that quantity. ML and MAP achieve the same result if the unknown parameter follows a uniform distribution. Within the Bayesian framework, we also mention the minimum mean square error (MMSE) approach, which attempts to minimize the mean square error (MSE) of the estimator, a common measure of estimator quality.

The least squares (LS) estimation can be developed, whether the unknown parameter is deterministic or random. Unlike other estimation methods based on the use of probability models for the observed data or the parameters, the LS estimation simply attempts to minimizing the sum of squared errors between the measurements and a linear model containing the unknown parameters. In the present context, the LS estimation can be applied to construct blind linear estimators.

Practical algorithms for the estimation of wireless channels are based on the above theoretical framework. Generally, there exist three types of channel estimation techniques:

training based channel estimation, blind channel estimation, and semi-blind channel estimation [1]. Training based estimation, in which a known data sequence is transmitted for the purpose of channel estimation, is the most mature technology and has been used for several decades. However, since blind techniques can supply higher transmission efficiency, they have become a very attractive and promising research topic. Semi-blind techniques combine blind estimation and training sequences to achieve higher performance. Depending on the underlying statistical estimation approach, the blind channel estimation methods can be classified as higher order statistics (HOS)[1] [2] and second order statistics [1] based. The blind channel estimation techniques are very important for the blind channel equalization algorithms, since channel information can be obtained without the use of training sequences.

In digital communication systems, inter-symbol interference (ISI) is caused by a sequence of symbols interfering with itself during transmission, as a result of channel spreading. Digital symbols are mapped into a given constellation scheme. Hence, one symbol interfering with subsequent symbols can cause an erroneous decision at the receiver. Mathematically, both channel and equalizer can be considered as a discrete-time filter, and the signal is a numerical sequence. Therefore, the problem is to create an equalizer filter to fix and compensate ISI, so that the signal sequence is recovered as accurately, and efficiently as possible at the receiver. High performance equalization approaches are studied to provide techniques to minimize the error rate during the process of symbol recovery. If the signal sequence and noise are considered as random sequences, the mathematical procedure thus concerns itself with reducing the probability of error.

The exact implementation of the ML criterion and the MAP criterion for symbol recovery is so complicated in practice that these algorithms can not be afforded in realistic communication systems, such as e.g. real-time communication at high bit rates. The Viterbi algorithm is the first effective solution to recover symbols in the presence of ISI [3]. It was originally proposed for decoding convolutionally encoded symbols based on the

[1]HOS indicates that statistics of order 3 or more are involved.

ML criterion [4]. As a result, the probability of error in detecting a complete transmitted sequence is (approximately) minimized. Subsequently, the BCJR algorithm, named after its inventors, was proposed based on the MAP criterion [5]. Unlike the Viterbi algorithm which aims to recover the best sequence, the signal is recovered on a symbol-by-symbol basis in the BCJR algorithm. In turn, the probability of error for each individual symbol is minimized.

In the presence of ISI, it is further necessary to apply an equalizer in order remove the signal distortion incurred during transmission, prior to the application of a data decoding technique. Several criteria are available for the design of the equalizing filter, such as the MSE and LS criteria. In practice, two options are possible: accounting for all of the transmitted symbols to obtain an optimized equalizer for the complete sequence, or adjusting the equalizer in real-time so that it can adapt to the most recent data as quickly as it is received. Finally, the use of *a priori* information is related to the possibility of realizing practical mechanisms for training the equalizer with known data sequence. If such a training mechanism is periodically implemented, we refer to the process as supervised equalization, otherwise, only inherent properties of the received information sequence are exploited and the process is called blind equalization.

Although the term "blind equalization" was first introduced in 1984 [6], many important research achievements had already been published by that time. In 1975, the classical blind equalization approach was proposed for pulse amplitude modulated signal under the single-input single-output (SISO) framework [7]. Later, the fundamental conditions of blind equalization were studied [8]. Also in 1984, researchers used the properties of complex signals in the design of blind equalization algorithm [9]. Indeed, structural properties of the source signal (i.e. constellation) can be exploited to design so-called constant modulus algorithms (CMA) [10]. These blind equalization algorithms are based on the idea of restoring the constant modulus properties of the source modulation. The theoretical conditions for blind equalization are further investigated in [11]. It is proved that if the 4^{th} order cumulant is maximal and the 2^{nd} order cumulant are constant, then the signal can be recovered, up to a scaled and rotated version of the source signal.

A blind equalization algorithm for multiple antenna receivers was first proposed in 1991 [12]. This blind equalization algorithm is applicable to time-invariant (TI) channels using only the second order statistics (SOS) of the received signal. Later, the subspace method [13], the prediction error structure [14], and the fractionally spaced method [15] were proposed which are also based on SOS. For TI MIMO systems, several different blind equalization algorithms have been proposed based on HOS and SOS frameworks. It has been proved in [14] that the MIMO finite impulse response (FIR) system can be identified by the SOS of the received signal under some assumptions. Also, the subspace method [16] and the matrix pencil decomposition method [17] were proposed. The subspace algorithm takes advantage of reduced computational complexity. It is based on the decomposition of the autocorrelation matrix of the received signal into orthogonal signal and noise subspaces [12, 13].

Based on the subspace framework, different approaches have also been proposed to estimate TV single-input multiple-output (SIMO) channels [18, 19]. First, the TV channels of the SIMO system are projected onto selected basis functions, which enable their equivalent representation as TI MIMO channel. Following this step, this latter channel can be estimated using standard subspace methods developed for TI channels. Once the parameters of the TV SIMO have been estimated, we can then proceed with the equalization. In [18], a zero forcing (ZF) equalizer was derived according to the channel estimation from the subspace approach.

The ZF equalizer is based on a worst case peak distortion criterion, that captures the effect of ISI on the error rate. The ZF algorithm was the first proposed method to be applied in FIR equalization in the 1960s [20]. It is proposed as an adaptive procedure to adjust the coefficients of an FIR equalizer so that the ISI is forced to be zero. Since all terms associated with the ISI are set to zero, the ZF equalizer tends to be the inverse of the channel impulse response in a noise-free situation. In practice, due to the effect of noise, only an approximate inverse can be found to design the ZF equalizer. As a result, ZF equalization tends to be sensitive to the measurement noise. A better approach is to use an MSE criterion with regularization [21]. Nevertheless, ZF equalization is commonly used in applications due to its simple structure. In [11], it has been proven that the ZF equalizer can be implemented if only two statistics of the involved signal are equalized.

1.3 Problem Addressed and Contributions

Under the SOS framework, the subspace method is exploited to identify multichannel impulse response [13]. It has been proven that the TV SIMO FIR channel can be identified up to an ambiguity matrix, if the TV channel is decomposed as expansion coefficients of complex exponential basis functions [18]. However, the blind equalization algorithm of the TV SIMO system associated with arbitrary basis functions has never been studied.

In this thesis, a multi-channel ZF equalizer is proposed based on the blind subspace-based identification of arbitrary TV SIMO FIR channel. The blind subspace channel identification associated with arbitrary basis functions was proposed and studied [19]. In this approach, the TV SIMO channel is expressed as a combination of expansion coefficients along arbitrary basis TV functions. The expansion coefficients are considered as channel impulse responses of an equivalent TI MIMO system, so that it can be estimated by existing bind subspace methods developed for the TI MIMO channel.

A ZF equalizer is suggested based on subspace channel estimation under the SOS framework. First, an approximated ZF equalizer, in the form of a FIR filter, is designed for the equivalent TI MIMO system based on the result of the blind subspace estimation. The necessary and sufficient conditions are discussed and proven so that the FIR MIMO system can be determined up to an ambiguity matrix under the SOS framework. Then, the ambiguity matrix is estimated by the LS method. Finally, the ZF equalizer is obtained by combining the approximated filter and the ambiguity matrix estimate.

Finally, the results of simulation experiments based on practical conditions show that the proposed equalizer for the TI SIMO system can recover the source signal, whether the expansion basis functions of the TV SIMO channel are exponential, or more general in nature.

1.4 Thesis Organization

The rest of this thesis is organized as follows. In Chapter 2, a classical subspace approach is introduced for the estimation of the TI FIR channels. Then, the basic ideas of linear equalization under the SISO and MIMO frameworks are introduced. In Chapter 3, an equivalent TI MIMO system is constructed for the TV SIMO system. The subspace method is exploited to identify the equivalent TI MIMO channels. After that, the ZF equalizer is designed. In Chapter 4, the necessary and sufficient conditions for the determination of the ambiguity matrix under the SOS framework are discussed. Next, an LS method is proposed to obtain it. In Chapter 5, the simulation results are presented for blind subspace-based TV SIMO channel estimation and associated channel equalization with the proposed method. Finally, Chapter 6 presents summary and conclusion.

Chapter 2

Background Material

The background material for the rest of thesis will be presented in this chapter. First, the classical subspace blind channel estimation approach is introduced based on the basic multichannel FIR model. In turn, the blind channel equalization is discussed for both the SISO and MIMO systems. Finally, previous research works on which the thesis is based are discussed.

2.1 Classical Subspace Approach for Blind TI SIMO Channel Estimation

The orthogonality between the signal and noise subspaces of the received signal can be used to identify the impulse response of a multiple-channel system [13]. Beginning with a single channel, the received continuous time baseband signal is defined as

$$x(t) = \sum_{m=-\infty}^{\infty} s[m]h(t - mT_s) + w(t), \tag{2.1}$$

where $s[m]$ is the emitted sequence of symbols, T_s is the symbol duration, and m is the symbol index. The signal is transmitted through a channel, characterized by a system impulse response $h(t)$. The received signal $x(t)$ is corrupted by the additive noise $w(t)$, which is modelled as a band-limited complex stationary process, independent from $s[m]$.

2.1.1 System Model for Channel Estimation

The classical subspace algorithm in [13] is based on following two fundamental assumptions:
(i) the channel impulse response $h(t)$ has finite support;
(ii) several measurements can be performed during the symbol period T_s.
In this algorithm, orthogonal bases of the received signal and noise subspaces are formed so that the channel impulse response can be identified. While the following presentation is for a single receiver system with oversampled received signal, it can also be applied to a multi-receiver system.

The oversampled signal in one symbol duration can be represented as

$$
\begin{aligned}
x_i[n] &= x(t_0 + i\Delta) && (2.2)\\
&= \sum_{m=0}^{M} s[n-m]h(t_0 + i\Delta + mT_s) + w_i[n], && (2.3)
\end{aligned}
$$

where: $0 \leq i \leq P-1$ is the sampling index over one symbol duration; $M+1$ is the number of the tap delays, i.e. duration of the channel in units of T_s; P is the oversampling factor and satisfies the condition $P = T_s/\Delta$; $t_0 + i\Delta$ represents the sampling epoch of the i^{th} sample; and $w_i[n]$ is the additive noise, defined as

$$w_i[n] = w(t_0 + i\Delta + nT_s). \tag{2.4}$$

Hence, we can define the channel impulse response corresponding with $x_i[n]$ as

$$\mathbf{h}_i = [h(t_0 + i\Delta), h(t_0 + i\Delta + T_s), \cdots, h(t_0 + i\Delta + MT_s)]^T, \tag{2.5}$$

where $[\cdot]^T$ denotes the matrix transpose. In other words, we may consider the oversampling system as a multichannel system with L *virtual channels*, within this case, $L = P$.

Alternatively, in a multi-receiver system, K receivers (antennas) detect signals that have passed though the propagation channel. The signal received at the j^{th} receiver can expressed as

$$x_j[n] = \sum_{m=0}^{M} s[n-m]h_j(t_0 + mT_s) + w_j[n], \tag{2.6}$$

where $h_j(t)$ denotes the impulse response between the emitter and the j^{th} receiver antenna, and $w_j[n]$ is an additive noise at the j^{th} antenna.

Each channel is characterized by its impulse response vector

$$\mathbf{h}_j = [h_j(t_0), h_j(t_0 + T_s), \cdots, h_j(t_0 + MT_s)]^T, \tag{2.7}$$

where $0 \leq j \leq K-1$. In this case, $L = K$ is defined as the number of *virtual channels*.

Generally, we can consider both of the above systems as a multichannel system with L *virtual channels*, each one characterized by its impulse response vector $\mathbf{h}_l$, which can be expressed as a $(M+1) \times 1$ vector

$$\mathbf{h}_l = [h_l[0], h_l[1], \cdots, h_l[M]]^T, \tag{2.8}$$

where $1 \leq l \leq L$, and $h_l[m]$ represents the sampled values of the l^{th} continues-time impulse response. In turn, the channel impulse responses of these L virtual channels can be represented as a $L \times (M+1)$ matrix

$$\mathbf{h}[n] = [\mathbf{h}_1 \cdots \mathbf{h}_L]^T. \tag{2.9}$$

The output of the SIMO multichannel system can be expressed in vector form as

$$\mathbf{x}[n] = \sum_{m=0}^{M} s[m]\mathbf{h}[n - mT_s] + \mathbf{w}[n], \tag{2.10}$$

where the received signal vector $\mathbf{x}[n] = [x_1[n], \cdots, x_L[n]]^T$ and the noise signal vector $\mathbf{w}[n] = [w_1[n], \cdots, w_L[n]]^T$ have size $L \times 1$.

2.1.2 Subspace Approach for Channel Estimation

In the classical subspace-based approach for blind SIMO channel estimation, the aim is to estimate the set of channel coefficients, which are represented as a $L(M+1) \times 1$ vector

$$H = [\mathbf{h}_1^T \cdots \mathbf{h}_L^T]^T, \tag{2.11}$$

where $\mathbf{h}_l$, for $1 \leq l \leq L$, is a $(M+1) \times 1$ vector representing the impulse response of the l^{th} channel.

For the estimation, N successive samples of the received signals for each of the channels are represented by a $N \times 1$ vector

$$X_i[n] = [x_i[n], \cdots, x_i[n-(N-1)]]^T. \tag{2.12}$$

From (2.6), we can express $X_i[n]$ as

$$X_i[n] = \mathcal{H}_i S[n] + W_i[n], \tag{2.13}$$

where the corresponding source symbol $S[n]$ is defined as the $(N+M) \times 1$ vector

$$S[n] = [s[n], \cdots, s[n-(N+M-1)]]^T, \tag{2.14}$$

and the noise term $W_i[n]$ is defined as the $N \times 1$ vector

$$W_i[n] = [w_i[n], \cdots, w_i[n-(N-1)]]^T. \tag{2.15}$$

The $N \times (N+M)$ corresponding filter matrix $\mathcal{H}_i$ in (2.13) is defined as

$$\mathcal{H}_i = \begin{bmatrix} h_i[0] & \cdots & h_i[M] & 0 & \cdots & \cdots & 0 \\ 0 & h_i[0] & \cdots & h_i[M] & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & \cdots & 0 & h_i[0] & \cdots & h_i[M] \end{bmatrix}. \tag{2.16}$$

Hence, the corresponding N successively received signal samples of the multichannel system can be represented as an $LN \times 1$ vector

$$\begin{bmatrix} X_1[n] \\ \vdots \\ X_L[n] \end{bmatrix} = \begin{bmatrix} \mathcal{H}_1 \\ \vdots \\ \mathcal{H}_L \end{bmatrix} S[n] + \begin{bmatrix} W_1[n] \\ \vdots \\ W_L[n] \end{bmatrix}, \tag{2.17}$$

which can be expressed more compactly as

$$\mathbf{X}[n] = \mathcal{H}S[n] + \mathbf{W}[n], \tag{2.18}$$

where the unknown filter matrix $\mathcal{H}$ is restricted to be of a full column rank [13].

The subspace decomposition is based on the $LN \times LN$ correlation matrix $\mathbf{R}_X$ of the measurement vector $\mathbf{X}[n]$, defined as

$$\mathbf{R}_X = E[\mathbf{X}[n]\mathbf{X}[n]^H], \tag{2.19}$$

where $E[\cdot]$ denotes expectation, and the superscript ^{H} denotes Hermitian transpose. We assume that the additive noise is independent of the symbol sequence, so that, on the basis of (2.18), $\mathbf{R}_X$ can be expressed as

$$\mathbf{R}_X = \mathcal{H}\mathbf{R}_s\mathcal{H}^H + \mathbf{R}_w, \tag{2.20}$$

where $\mathbf{R}_s = E[S[n]S[n]^H]$ is the $(N+M) \times (N+M)$ correlation matrix of the symbol sequence $S[n]$ in (2.14), and $\mathbf{R}_w = E[\mathbf{W}[n]\mathbf{W}[n]^H]$ is the $LN \times LN$ correlation matrix of the additive noise measurement $\mathbf{W}[n]$. In this work, we assume that the noise is spatially and temporally white, so that

$$\mathbf{R}_w = \sigma^2 \mathbf{I}_{LN}, \tag{2.21}$$

where $\mathbf{I}_{LN}$ denotes an identity matrix of size $LN \times LN$ and σ^2 is the noise variance.

We assume that both $\mathbf{R}_s$ and $\mathcal{H}$ are full rank matrixes, and condition $LN > (M+N)$ is true, so that $\mathcal{H}$ is a tall matrix, i.e. with more rows than columns. Let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{LN}$ denote the eigenvalues of $\mathbf{R}_X$. Since the size of $\mathbf{R}_s$ and $\mathbf{R}_w$ are different, the eigenvalues can be grouped into two classes:

$$\lambda_1 \geq \cdots \geq \lambda_{M+N} > \sigma^2, \tag{2.22}$$

associated with orthonormalized eigenvectors $\mathbf{a}_1, \cdots, \mathbf{a}_{M+N}$; and eigenvalues

$$\lambda_{M+N+1} = \cdots = \lambda_{LN} = \sigma^2, \tag{2.23}$$

associated with eigenvectors $\mathbf{b}_1, \cdots, \mathbf{b}_{LN-(M+N)}$. For convenience, we use the $LN \times (M+N)$ matrix $\mathbf{A}$ and the $LN \times (LN - (M+N))$ matrix $\mathbf{B}$ to express the collections of above eigenvectors, i.e.:

$$\mathbf{A} = [\mathbf{a}_1 \cdots \mathbf{a}_{M+N}], \tag{2.24}$$

$$\mathbf{B} = [\mathbf{b}_1 \cdots \mathbf{b}_{LN-(M+N)}]. \tag{2.25}$$

The columns of matrix $\mathbf{A}$ span the *signal subspace*, which is identical to the columns span of channel matrix $\mathcal{H}$, and the columns of matrix $\mathbf{B}$ span the *noise subspace* [13]. Also, we express the $LN \times 1$ column vector $\mathbf{b}_i$, for $1 \leq i \leq LN - (M+N)$, as

$$\mathbf{b}_i = [\mathbf{b}_i^T(1) \cdots \mathbf{b}_i^T(N)]^T, \tag{2.26}$$

where each element $\mathbf{b}_i(j)$, for $1 \leq j \leq N$, is a $L \times 1$ vector

$$\mathbf{b}_i(j) = [b_{i,1}(j), \cdots, b_{i,L}(j)]^T. \tag{2.27}$$

Therefore, the covariance matrix $\mathbf{R}_X$ can be rewriten as

$$\mathbf{R}_X = \mathbf{A}\mathrm{diag}(\lambda_1 \cdots \lambda_{M+N})\mathbf{A}^H + \sigma^2\mathbf{B}\mathbf{B}^H. \tag{2.28}$$

Because of the orthogonality between the signal and noise subspaces, the columns of $\mathcal{H}$ are orthogonal to any vector in the noise subspace. That is:

$$\mathbf{b}_i^H\mathcal{H} = 0, \qquad 1 \leq i \leq LN - (M+N), \tag{2.29}$$

where $\mathbf{b}_i$ denotes a column of $\mathbf{B}$.

In practice, only an estimated value of $\mathbf{b}_i$, denoted as $\hat{\mathbf{b}}_i$, is available. The least squares method is therefore suggested to solve the linear equation (2.29) in the presence of estimation error. Hence, the associate problem becomes

$$\min_{\mathcal{H}} \sum_{i=1}^{LN-M-N} \|\hat{\mathbf{b}}_i^H\mathcal{H}\|^2, \tag{2.30}$$

where $\mathcal{H}$ is defined in terms of the coefficient matrix H (2.11) via the construction in (2.16)-(2.18). Therefore, based on the Lemma 1 in [13], the expression in (2.30) can be equivalently represented as

$$\min_{H} H^H Q H, \tag{2.31}$$

where we define

$$Q = \sum_{i=1}^{LN-M-N} \hat{\mathcal{B}}_i\hat{\mathcal{B}}_i^H. \tag{2.32}$$

In (2.32), the matrices $\hat{\mathcal{B}}_i$, for $1 \leq i \leq LN - (M+N)$, have size $L(M+1) \times (M+N)$ and are constructed as follow

$$\hat{\mathcal{B}} = [\hat{\mathcal{B}}_{i,1}^T \cdots \hat{\mathcal{B}}_{i,L}^T]^T, \tag{2.33}$$

where

$$\hat{\mathcal{B}}_{i,1} = \begin{bmatrix} \hat{b}_{i,1}(0) & \cdots & \hat{b}_{i,1}(N) & 0 & \cdots & \cdots & 0 \\ 0 & \hat{b}_{i,1}(0) & \cdots & \hat{b}_{i,1}(N) & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & \cdots & 0 & \hat{b}_{i,1}(0) & \cdots & \hat{b}_{i,1}(N) \end{bmatrix} \tag{2.34}$$

is a $(M+1) \times (M+N)$ matrix, whose entries are the estimated entires of vector $\mathbf{b}_i(j)$ in (2.27). Finally, the estimated value of the channel impulse response matrix $\mathcal{H}$ can be obtained by minimizing (2.31) with respect to H. In practice, to avoid the trivial solution $H = 0$, the search needs to be restricted. This can be achieved by imposing a constraint on the norm of H, i.e. $\|H\| = 1$ in [13].

2.2 Blind Channel Equalization

Blind equalization is a process of recovering an unknown input data sequence from an observed noisy signal at the output of an unknown channel. The main advantage of blind channel equalization is that it does not require a training sequence, which would usually cause a reduction in the data rate. Generally speaking, blind equalization approaches can be classified into direct and indirect approaches. Direct blind equalization can be performed without channel identification. Indirect blind equalization requires the identification of the channel impulse response, so that channel estimation algorithms have to be involved. This thesis will focus on an indirect blind channel equalization algorithm for TV SIMO channels.

2.2.1 Blind SISO Equalization

Blind equalization is first applied to a single-input single-output (SISO) discrete linear system, which is a simple single variable control system with one input and one output, by Sato in 1975 [7]. Fig. 2.1 shows the block diagram of the discrete-time SISO system with a linear equalizer, which consists of an unknown complex linear time invariant (LTI) channel, with impulse response $h[n]$, and a linear equalizer $g[n]$ to remove the ISI.

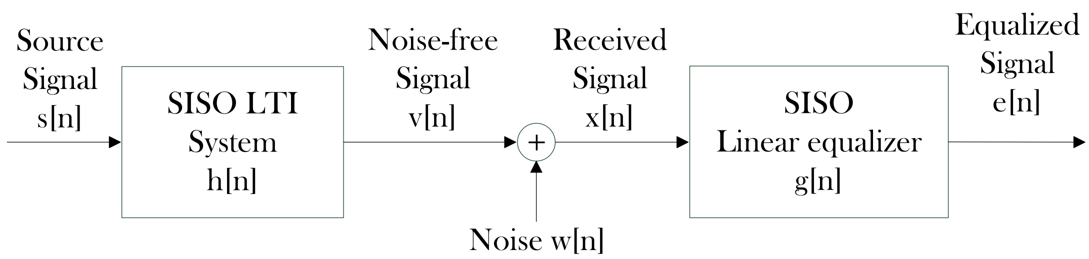

Fig. 2.1 Block diagram of a SISO system with linear equalization

The system output results from a quadrature amplitude modulation (QAM) symbol sequence $s[n]$, passing though a complex LTI channel $h[n]$. It can be expressed as

$$v[n] = s[n] * h[n] = \sum_{m=-\infty}^{\infty} s[m]h[n-m]. \tag{2.35}$$

The observed signal $x[n]$ at the receiver is given by

$$x[n] = v[n] + w[n], \tag{2.36}$$

where the additive noise $w[n]$ is assumed to be a zero-mean random sequence, independent from $s[n]$. The desired equalized signal $e[n]$ can be expressed as

$$e[n] = x[n] * g[n] = \sum_{m=-\infty}^{\infty} x[m]g[n-m], \tag{2.37}$$

where $g[n]$ represents the impulse response of the designed blind equalizer. Since the received signal $x[n]$ involves an additive noise, the equalized signal $e[n]$ consists of a signal component $e_S[n]$ and a noise component $e_N[n]$, i.e.:

$$e[n] = e_S[n] + s_N[n] \tag{2.38}$$

where

$$e_S[n] = s[n] * h[n] * g[n], \tag{2.39}$$

$$e_N[n] = w[n] * g[n]. \tag{2.40}$$

From the above mathematical description of the overall system, the aim of direct blind equalization approach is to design an equalizer $g[n]$ directly from the received signal $x[n]$, so that the equalized signal component $e_S[n]$ approximates the original source signal $s[n]$ as accurately as possible within the limits imposed by the additive noise. In contrast, indirect blind equalization takes advantage of available channel estimation: firstly, a blind channel estimation algorithm is applied to estimate channel impulse response $h[n]$; then the equalizer is designed based on the estimated response, say $\hat{h}[n]$, or other estimated parameters related to $h[n]$.

2.2.2 Blind MIMO Equalization

A MIMO communication system consists of P transmitters and L receivers, where we assume for simplicity that $L \geq P$. Fig. 2.2 shows the block diagram of a MIMO system, where the source signal vector $\mathbf{s}[n]$ and received signal vector $\mathbf{x}[n]$, at time n, are defined as

$$\mathbf{s}[n] = [s_1[n], \cdots, s_P[n]]^T, \tag{2.41}$$

$$\mathbf{x}[n] = [x_1[n], \cdots, x_L[n]]^T. \tag{2.42}$$

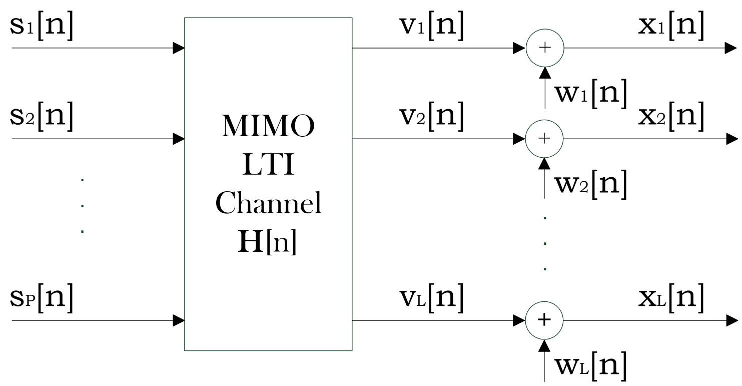

Fig. 2.2 Block diagram of a MIMO communication system.

The noise-free output of the MIMO system is represented by vector $\mathbf{v}[n]$

$$\mathbf{v}[n] = [v_1[n], \cdots, v_L[n]]^T = \mathbf{H}[n] * \mathbf{s}[n]. \tag{2.43}$$

At the receiver, additive noise is also included in the form of vector

$$\mathbf{w}[n] = [w_1[n], \cdots, w_L[n]]^T. \tag{2.44}$$

The noisy output of the MIMO LTI channel in Fig. 2.1 can be expressed as

$$\mathbf{x}[n] = \mathbf{H}[n] * \mathbf{s}[n] + \mathbf{w}[n] = \sum_{m=-\infty}^{\infty} \mathbf{H}[m]\mathbf{s}[n-m] + \mathbf{w}[n], \tag{2.45}$$

where $\mathbf{H}[n]$ is the impulse response of the P inputs and L outputs LTI MIMO channel, and can be represented as an $L \times P$ matrix

$$\mathbf{H}[n] = \begin{bmatrix} h_{11}[n] & h_{12}[n] & \cdots & h_{1P}[n] \\ h_{21}[n] & h_{22}[n] & \cdots & h_{2P}[n] \\ \vdots & & \ddots & \\ h_{L1}[n] & h_{L2}[n] & \cdots & h_{LP}[n] \end{bmatrix}. \tag{2.46}$$

That is, the number of columns of $\mathbf{H}[n]$ equals the number of inputs of the MIMO system, and the number of rows of $\mathbf{H}[n]$ equals the number of outputs. Thus, the $(l,p)^{th}$ entry $h_{lp}[n]$ in (2.46) represents the LTI channel impulse response between the l^{th} output, for $1 \le l \le L$, and the p^{th} input, for $1 \le p \le P$. For convenience, $\mathbf{H}[n]$ can be expressed as

$$\mathbf{H}[n] = [\mathbf{h}_1[n] \cdots \mathbf{h}_L[n]]^T, \tag{2.47}$$

where

$$\mathbf{h}_l[n]^T = [h_{l1}[n], \cdots, h_{lP}[n]] \tag{2.48}$$

is the l^{th} row of matrix $\mathbf{H}[n]$. Therefore, the noise-free signal at the l^{th} receiver can be represented as

$$v_l[n] = \mathbf{h}_l[n] * \mathbf{s}[n] = \sum_{p=1}^{P} \sum_{m=-\infty}^{\infty} h_{lp}[m] s_p[n-m], \qquad 1 \le l \le L \tag{2.49}$$

and the corresponding noisy measurement can be represented as

$$x_l[n] = v_l[n] + w_l[n]. \tag{2.50}$$

In this work, the source signal vector $\mathbf{s}[n]$ and noise signal vector $\mathbf{w}[n]$ are modelled as independent, stationary vector random processes. In turn, the noise-free signal vector $\mathbf{v}[n]$ and received signal vector $\mathbf{x}[n]$ are also considered as vector random processes. Therefore, the auto-correlation matrix function and power spectral matrix of the received signal vector $\mathbf{x}[n]$ can be represented as

$$\mathrm{R}_{\mathbf{x}}[m] = E\{\mathbf{x}[n]\mathbf{x}^H[n-m]\} = \mathrm{R}_{\mathbf{v}}[m] + \mathrm{R}_{\mathbf{w}}[m] \tag{2.51}$$

$$\mathcal{S}_{\mathbf{x}}(\omega) = \mathcal{F}\{\mathrm{R}_{\mathbf{x}}[m]\} = \mathcal{S}_{\mathbf{v}}(\omega) + \mathcal{S}_{\mathbf{w}}(\omega). \tag{2.52}$$

where we define

$$\mathrm{R}_{\mathbf{v}}[m] = E\{\mathbf{v}[n]\mathbf{v}^H[n-m]\}, \tag{2.53}$$

$$\mathcal{S}_{\mathbf{v}}(\omega) = \mathcal{F}\{\mathrm{R}_{\mathbf{v}}[m]\}, \tag{2.54}$$

$$\mathrm{R}_{\mathbf{w}}[m] = E\{\mathbf{w}[n]\mathbf{w}^H[n-m]\}, \tag{2.55}$$

$$\mathcal{S}_{\mathbf{w}}(\omega) = \mathcal{F}\{\mathrm{R}_{\mathbf{w}}[m]\}, \tag{2.56}$$

and $\mathcal{F}\{\cdot\}$ is the discrete-time Fourier transform (DTFT) operator. The signal-noise ratio (SNR) at the output of the MIMO system in Fig. 2.2 can be represented as

$$\mathrm{SNR} = \frac{E\{||\mathbf{v}[n]||^2\}}{E\{||\mathbf{w}[n]||^2\}}. \tag{2.57}$$

In MIMO systems, the ISI can be removed by a linear equalizer. Fig. 2.3 shows the block diagram of a MIMO linear equalizer, whose output can be defined as

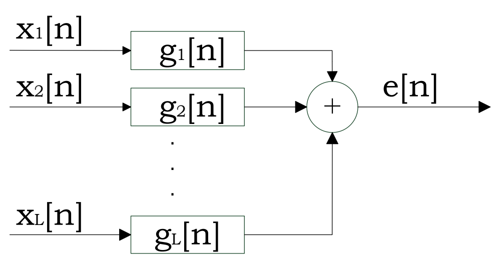

Fig. 2.3 Block diagram of MIMO linear equalization.

$$\mathbf{e}[n] = \mathbf{G}[n]^T * \mathbf{x}[n] = \sum_{l=1}^{L}\sum_{m=-\infty}^{\infty}\mathbf{g}_l[m]x_l[n-m], \tag{2.58}$$

where $\mathbf{G}[n]^T$ denotes the $P \times L$ impulse response matrix of the linear equalizer. It can be represented as

$$\mathbf{G}[n]^T = [\mathbf{g}_1[n] \cdots \mathbf{g}_L[n]], \tag{2.59}$$

where $\mathbf{g}_l[n]$, for $1 \leq l \leq L$, can be expressed as a $P \times 1$ vector

$$\mathbf{g}_l[n] = [g_{l1}[n], \cdots, g_{lP}[n]]^T. \tag{2.60}$$

The equalized $P \times 1$ signal vector $\mathbf{e}[n]$ consists of a signal component $\mathbf{e}_S[n]$ and a noise component $\mathbf{e}_N[n]$, and can be expressed as

$$\mathbf{e}[n] = \mathbf{e}_S[n] + \mathbf{e}_N[n], \tag{2.61}$$

where $\mathbf{e}_S[n]$ and $\mathbf{e}_N[n]$ can be defined as

$$\mathbf{e}_S[n] = \mathbf{G}^T[n] * \mathbf{v}[n], \tag{2.62}$$

$$\mathbf{e}_N[n] = \mathbf{G}^T[n] * \mathbf{w}[n]. \tag{2.63}$$

Finally, by combining (2.49) and (2.62) the equalized received signal can be expressed as

$$\mathbf{e}_S[n] = \mathbf{G}^T[n] * \mathbf{H}[n] * \mathbf{s}[n] = \sum_{l=1}^{L} \mathbf{g}_l^T[n] * \mathbf{h}_l[n] * \mathbf{s}[n]. \tag{2.64}$$

2.3 Related Works on TV Channel Estimation and Equalization

In indirect blind channel equalization, estimation of the channel characteristic, i.e. the channel impulse response $\mathbf{H}[n]$, is the first problem that has to be considered. Several algorithms have been proposed to estimate the LTI MIMO channel using a classical subspace algorithms [13, 14, 16]. These algorithems can be used to obtain an estimate of $\mathbf{H}[n]$, i.e. $\hat{\mathbf{H}}[n]$, and use it to construct an equalizer matrix $\mathbf{G}[n]$ that inverts the effect of $\mathbf{H}[n]$ in (2.64), i.e.

$$\mathbf{G}^T[n] * \hat{\mathbf{H}}[n] = I_P, \tag{2.65}$$

where I_P denotes a $P \times P$ identity matrix. For the case of TV channels, which are often encountered in mobile radio applications, the basic principle of direct equalization is similar, that is:

- First estimate the parameters of the unknown $L \times P$ TV impulse response matrix;
- Use the estimated response to construct an equalizer that compensate the channel effects.

For a TV MIMO channel, the basic convolution in the signal model (2.45) is replaced by

$$\mathbf{x}[n] = \sum_{m} \mathbf{H}[n,m]\mathbf{s}[n-m] + \mathbf{w}[n], \tag{2.66}$$

where $\mathbf{H}[n,m]$ now represents the $L \times P$ impulse response matrix of the MIMO channel at discrete-time n, with the second index m used to denote the lag. Clearly, because of the dependence on current time n, the estimation (and equalization) of MIMO TV channels is considerably more challenging.

An interesting approach to the problem of TV channel estimation is presented in [22]. First consider the case of a TV SISO FIR channel with impulse response $h[n,m]$. This channels is then expended as a linear combination of a finite number of basis functions, i.e.:

$$h[n,m] = \sum_{p=1}^{P} \kappa_p[m] f_p[n], \tag{2.67}$$

where P is the number of the basis functions, $f_p[n]$ is the p^{th} basis function, and $\kappa_p[m]$ is the corresponding expansion coefficient. In [22], it is shown that the TV SISO FIR channel with impulse response $h[n,m]$ can be realized as a TI SIMO channel, with each sub-channel characterized by the impulse response $\kappa_p[m]$. Therefore, classical blind subspace methods developed for TI SIMO channels can be applied to estimate the parameter $\kappa_p[m]$, which in turn enables the estimation of the TV SISO $h[n,m]$ via (2.67).

However, in this method, $f_p[n]$ has to obey several assumptions to ensure that the TI SIMO channel can be identified by second order statistics [22]. An alternative algorithm that links the TV SIMO system and the TI MIMO system by expanding the TV channel characteristic as a linear combination of complex exponential basis functions, i.e. $f_p[n] =$

$e^{j\omega_p n}$, where ω_p denotes an angular frequency, is investigated in [18]. Again, blind subspace methods developed for the estimation of TI MIMO channels can be used to estimate the expansion coefficients. In [18], the channel estimates obtained in this way are used to indirectly build a channel equalizer.

In [19], it has been shown that to a first order of approximation (i.e. for slow channel variations), the basis functions in [18] need not be restricted to the complex exponential type. The TV SIMO FIR channel, with $L\times 1$ impulse response vector $\mathbf{h}[n,m]$, is represented as the linear combination in terms of scalar basis functions, i.e.

$$\mathbf{h}[n,m] = \sum_{p=1}^{P} \boldsymbol{\kappa}_p[m] f_p[n], \tag{2.68}$$

where P is the numbers of basis function, $f_p[n]$ is the p^{th} basis function and $\boldsymbol{\kappa}_p[m]$ denotes the unknown $L \times 1$ expansion coefficients vector. In [19], it is shown that the TV SIMO FIR system is equivalent to a TI MIMO FIR system, represented by the set of coefficient vectors $\{\boldsymbol{\kappa}_p[m]\}$. Then, these expansion coefficients are estimated by applying existing subspace methods for the estimation of TI MIMO FIR channels.

In this thesis, by proceeding as in [18], but using the more general estimation approach in [19], our goal is to develop and study an indirect approach for the equalization of TV SIMO channel represented in terms of arbitrary basis functions.

Chapter 3

Indirect Equalization via Channel Estimation

Blind equalization can be classified into indirect and direct approaches. The direct blind equalization is achieved by using input statistics and the received signals through the unknown channels to create the equalizer filters. The indirect blind equalization is achieved by identifying the channel impulse response before building the appropriate equalizer filters, that is, the equalizer is designed based on the knowledge of estimated channel impulse responses. A blind method is proposed in [13] that exploits the orthogonality between the noise and signal subspaces of the received signals in order to identify multiple FIR channel. This classical subspace channel estimation algorithm has been applied to several communication areas, including indirect blind channel equalization.

In particular, the classical algorithm in [13] is applied to the estimation of a TV channel in [22]. In [18], this approach is further extended to the blind estimation of TV SIMO channels. Indeed, by making use of an expansion in terms of Fourier basis functions, the TV SIMO system is converted into an equivalent TI MIMO system which can be identified with the help of the classical subspace technique in [14]. In [19], it is further proven that the classical subspace channel estimation algorithm can be used in the same way to identify TV SIMO channels expanded along arbitrary basis functions.

In this chapter, we first review the blind subspace method for TV SIMO channel estimation based on SOS proposed in [19]. Then, a novel indirect ZF equalizer is developed by combining this method with a peak distortion criterion.

3.1 Blind Estimation of Arbitrary Time-Varying SIMO Channels

In this section, we discuss a recently proposed blind estimation algorithm for TV SIMO channels. We first describe the system model under consideration. We then present an approach to convert the TV SIMO system into an equivalent TI MIMO system using arbitrary basis expansion functions [19]. Finally, we consider the estimation of the resulting TI MIMO channels using a classical subspace algorithm.

3.1.1 Time-Varying SIMO System Model

The TV SIMO system under consideration, with single input and L outputs, is described as

$$\mathbf{x}[n] = \sum_{m=0}^{M} \mathbf{h}[n,m]s[n-m] + \mathbf{w}[n], \tag{3.1}$$

where:

- The received signal $\mathbf{x}[n]$ at discrete-time n is an $L \times 1$ vector, defined as

$$\mathbf{x}[n] = [x_1[n], \cdots, x_L[n]]^T. \tag{3.2}$$

- $s[n]$ represents the digital symbol transmitted by the source [1] at time n .

- The $L \times 1$ vector $\mathbf{h}$[n,m] represents TV SIMO channel impulse response at time n and lag m, i.e.:

$$\mathbf{h}[n,m] = [h_1[n,m], \cdots, h_L[n,m]]^T, \tag{3.3}$$

[1]In this work, we shall focus mainly on a QAM constellation.

where $m \in \{0, \cdots, M\}$, $M+1$ is the number of channel taps (or delays), and element $h_l[n,m]$ represents the impulse response between the transmitter and the l^{th} receiver at time n and lag m.

- $\mathbf{w}[n]$ denotes the additive noise and is defined as an $L \times 1$ vector

$$\mathbf{w}[n] = [w_1[n], \cdots, w_L[n]]^T. \tag{3.4}$$

To apply the classical subspace method to estimate the impulse response of the TV SIMO channel, we stack N successively received signal vector $\mathbf{x}[n]$ to construct an $LN \times 1$ vector $X[n]$

$$X[n] = [\mathbf{x}[n]^T \cdots \mathbf{x}[n-N+1]^T]^T. \tag{3.5}$$

It follows from (3.1) that $X[n]$ can be described by

$$X[n] = \mathcal{H}[n]S[n] + W[n], \tag{3.6}$$

where the $(N+M) \times 1$ vector $S[n]$ is considered as the source and defined as

$$S[n] = [s[n], \cdots, s[n-(N+M)+1]]^T, \tag{3.7}$$

and $W[n]$ is the $LN \times 1$ additive noise vector

$$W[n] = [\mathbf{w}[n]^T \cdots \mathbf{w}[n-N+1]^T]^T. \tag{3.8}$$

The filtering matrix $\mathcal{H}[n]$, of dimension $LN \times (N+M)$, can be expressed as

$$\mathcal{H}[n] = \begin{bmatrix} \mathbf{h}[n,0] & \cdots & \mathbf{h}[n,M] & 0 & \cdots & \cdots & 0 \\ 0 & \mathbf{h}[n-1,0] & \cdots & \mathbf{h}[n-1,M] & 0 & \cdots & 0 \\ \vdots & & & \cdots & & & \\ 0 & \cdots & \cdots & 0 & \mathbf{h}[n-N+1,0] & \cdots & \mathbf{h}[n-N+1,M] \end{bmatrix}. \tag{3.9}$$

The sequences $S[n]$ and $W[n]$ are modelled as statistically independent vector random processes with zero mean, and correlation matrices denoted as

$$R_S = E[S[n]S^H[n]], \tag{3.10}$$

$$R_W = E[W[n]W^H[n]]. \tag{3.11}$$

We assume that both R_S and R_W are full rank matrices, and that the noise is spatially white, that is:

$$R_W = \sigma_w^2 I_{LN}, \tag{3.12}$$

where $\sigma_w^2 = E[|w_l[n]|^2]$ is the instantaneous noise power in each channel.

3.1.2 Equivalent Time Invariant MIMO System

In [19], it is proposed that the TV SIMO channel impulse response $\mathbf{h}[n,m]$ can be represented as a linear combination of arbitrary basis functions, as opposed to the complex exponential basis functions in [18], which require the *a priori* estimation of frequency parameters.

We assume that the TV channel impulse response $\mathbf{h}[n,m]$ in (3.1) admits a linear expansion in terms of a known set of P basis functions, that is

$$\mathbf{h}[n,m] = \sum_{p=1}^{P} \mathbf{k}_p[m]\phi_p[n], \tag{3.13}$$

where:

- $\mathbf{k}_p[m] \in \mathbb{C}^L$, for $1 \leq p \leq P$, are the arbitrary expansion coefficient vectors of dimension $L \times 1$

  $$\mathbf{k}_p[m] = [k_{1,p}[m], \ldots, k_{L,p}[m]]^T \tag{3.14}$$

  and $k_{l,p}[m]$ is the l^{th} entry of $\mathbf{k}_p[m]$. The latter can be interpreted as the channel impulse response between p^{th} virtual transmitters and the l^{th} receiver of equivalent TI MIMO system.

- $\phi_p[n]$ represents the p^{th} basis function. All P basis functions are assumed to be linearly independent [19]. The collection of these P basis functions can be conveniently represented as the $P \times 1$ vector

  $$\Phi[n] = [\phi_1[n], \cdots, \phi_P[n]]^T. \tag{3.15}$$

It follows from (3.13) that the TV filtering matrix (3.9) can be expressed as

$$\mathcal{H}[n] = \sum_{p=1}^{P} \mathcal{H}_p[n], \tag{3.16}$$

where $\mathcal{H}_p[n]$ is a $LN \times (N+M)$ matrix

$$\mathcal{H}_p[n] = \begin{bmatrix} \mathbf{k}_p[0]\phi_p[n] & \cdots & \mathbf{k}_p[M]\phi_p[n] & 0 & \cdots & 0 \\ 0 & \mathbf{k}_p[0]\phi_p[n-1] & \cdots & \mathbf{k}_p[M]\phi_p[n-1] & \cdots & 0 \\ \vdots & & & \cdots & & \vdots \\ 0 & \cdots & 0 & \mathbf{k}_p[0]\phi_p[n-N+1] & \cdots & \mathbf{k}_p[M]\phi_p[n-N+1] \end{bmatrix}. \tag{3.17}$$

In [19], based on the assumption that the channel impulse response will not vary significantly over the duration N of a stacked data vector $\mathbf{X}[n]$, it is proposed to approximated

(3.17) as

$$\mathcal{H}_p[n] = \phi_p[n]\mathcal{K}_p, \tag{3.18}$$

where

$$\mathcal{K}_p = \begin{bmatrix} \mathbf{k}_p[0] & \cdots & \mathbf{k}_p[M] & 0 & \cdots & 0 \\ 0 & \mathbf{k}_p[0] & \cdots & \mathbf{k}_p[M] & \cdots & 0 \\ \vdots & & & \cdots & & \vdots \\ 0 & \cdots & 0 & \mathbf{k}_p[0] & \cdots & \mathbf{k}_p[M] \end{bmatrix}. \tag{3.19}$$

Therefore, the system model (3.6) can be expressed as

$$X[n] = \sum_{p=1}^{P} \mathcal{K}_p \phi_p[n] S[n] + W[n]. \tag{3.20}$$

According to this expression, the original TV SIMO system with L outputs, is converted to a TI MIMO system with P inputs, L outputs, and corresponding source signals $\phi_p[n]S[n]$ for $p \in \{1, \cdots, P\}$. This equivalent TI MIMO system can also be approximately described as

$$\mathbf{x}[n] = \sum_{m=0}^{M} \mathbf{K}[m]\mathbf{t}[n-m] + \mathbf{w}[n], \tag{3.21}$$

where the equivalent source signal $\mathbf{t}[n]$ is defined as the $P \times 1$ vector

$$\mathbf{t}[n] = [\phi_1[n]s[n], \cdots, \phi_P[n]s[n]]^T, \tag{3.22}$$

and the $L \times P$ matrix $\mathbf{K}[m]$ represents the impulse responses of the equivalent TI MIMO channels, that is

$$\mathbf{K}[m] = [\mathbf{k}_1[m] \cdots \mathbf{k}_P[m]]. \tag{3.23}$$

Note that $\mathbf{K}[m] = 0$ and $\mathbf{k}_p[m] = 0$ for $m \notin \{0, \cdots, M\}$. Finally, the set of channel impulse responses of the TI MIMO system can be compactly represented by

$$K = [\mathbf{K}[0]^T \cdots \mathbf{K}[M]^T]^T, \tag{3.24}$$

which is a $L(M+1) \times P$ matrix.

We can also represent the N successively received signal vectors of the equivalent TI MIMO system as

$$X[n] = \mathcal{K}T[n] + W[n], \tag{3.25}$$

where $P(N+M) \times 1$ vector $T[n]$ is formed from $N+M$ successive source signal samples of the TI MIMO system, and is defined as

$$T[n] = S[n] \otimes \Phi[n], \tag{3.26}$$

where $\otimes$ denotes the Kronecker product. The equivalent filtering matrix $\mathcal{K}$ is the $LN \times P(N+M)$ matrix

$$\mathcal{K} = \begin{bmatrix} \mathbf{K}[0] & \cdots & \mathbf{K}[M] & 0 & \cdots & 0 \\ 0 & \mathbf{K}[0] & \cdots & \mathbf{K}[M] & \cdots & 0 \\ \vdots & & & \cdots & & \\ 0 & \cdots & 0 & \mathbf{K}[0] & \cdots & \mathbf{K}[M] \end{bmatrix}. \tag{3.27}$$

3.1.3 Estimation of the Equivalent TI MIMO System

Instead of estimating the channel impulse responses of the TV SIMO system defined in (3.1), we will estimate the channel impulse responses of the equivalent TI MIMO system described in (3.21); to this end we will use a blind subspace-based estimation algorithm. Let N_a denote the integration time, i.e. the required time interval over which the received signal vector $\mathbf{x}[n]$ in (3.1) is collected to approximate its correlation matrix. In the traditional application of subspace method for the estimation of TI channel, the integration time N_a is required to be large enough to limit the effect of noise and statistical fluctuations, but also small enough to ensure that the unknown channel can be considered as time invariant. For the TV channel, this assumption will be violated in general. However, it has been shown in [19] that the classical subspace algorithm is applicable for the blind channel estimation of the TI MIMO system in (3.20), even if $\mathbf{x}[n]$ is not stationary over the integration time, which allows us to use $N_a \gg N$.

In this work, to satisfy the relationship (3.18), it is assumed that, over the duration of the stacked vector $X[n]$, the variations of the basis functions can be neglected [19], that is:

$$\Phi[n_1] \approx \Phi[n_2], \tag{3.28}$$

where any n_1, n_2 must satisfy

$$|n_1 - n_2| \leq N. \tag{3.29}$$

To identify the channel impulse response of the TV SIMO system defined in (3.3), we firstly estimate the channel coefficient matrix $\mathbf{K}[m]$ of the equivalent TI MIMO system (3.21) based on the received signal $X[n]$, $0 \leq n < N_a$, over the integration time. Secondly, the results can be used to construct the estimated value of the TV SIMO channel impulse response defined by means of expansion (3.13).

We define the sample correlation of the successive received signal vector $X[n]$ of the equivalent TI MIMO system over the integration time, $0 \leq n < N_a$, as

$$\hat{R}_X(N_a) = \frac{1}{N_a} \sum_{n=0}^{N_a-1} X[n]X[n]^H, \tag{3.30}$$

which is an $LN \times LN$ matrix. Similarly, let

$$\hat{R}_T(N_a) = \frac{1}{N_a} \sum_{n=0}^{N_a-1} T[n]T[n]^H, \tag{3.31}$$

where $T[n]$ is defined in (3.26). It follows from (3.25) that

$$E[\hat{R}_X(N_a)] = \mathcal{K}E[\hat{R}_T(N_a)]\mathcal{K}^H + \sigma_w^2 I_{LN}. \tag{3.32}$$

Under fairly general conditions, it is shown in [19] that matrix $E[\hat{R}_T(N_a)]$ is full rank

$P(N+M)$, which motivates the application of a subspace method as explained below.

Let $E[\hat{R}_X(N_a)]$ be represented in terms of its eigenvalues decomposition (EVD):

$$E[\hat{R}_X(N_a)] = U\Lambda U^H, \tag{3.33}$$

where $\Lambda = \text{diag}(\lambda_1 \cdots \lambda_{LN})$ and $\lambda_1 \geq \cdots \geq \lambda_{LN}$ represent the eigenvalues, $U = [U_1 \cdots U_{LN}]$ represents the corresponding orthonormalized eigenvectors U_i for $1 \leq i \leq LN$, each of which is a $LN \times 1$ vector, such that $U^H U = UU^H = I$. Assuming that $LN > P(N+M)$, we may write $U = [U_S\ U_N]$ where U_S denotes the collection of the first $P(N+M)$ columns of the matrix U, and U_N denotes the submatrix $[U_{P(N+M)+1} \ldots U_{LN}]$, which contains the remaining columns. The corresponding eigenvalues can be characterized via

$$\lambda_1 \geq \cdots \geq \lambda_{P(N+M)} > \sigma_w^2, \tag{3.34}$$

$$\lambda_{P(N+M)} = \cdots = \lambda_{LN} = \sigma_w^2. \tag{3.35}$$

Note that the columns of U_S span the signal subspace, defined as the range space of matrix $\mathcal{K}$ (3.27), and the columns of U_N span its orthogonal complement, the noise subspace.

Since the columns of $\mathcal{K}$ are orthogonal to all the column vectors of the noise subspace matrix U_N, we obtain

$$U_i^H \mathcal{K} = 0, \quad \text{all} \quad i \in \mathcal{I}, \tag{3.36}$$

where we define $\mathcal{I} = \{P(N+M)+1, \cdots, LN\}$ and U_i denotes the i^{th} column of noise subspace matrix U_N. Hence, the channel transformation matrix $\mathcal{K}$ of the TI MIMO system in (3.25) can be obtained (up to some ambiguity) from the solution of (3.36). In turn, this allows the identification of the channel matrix K in (3.24).

In practice, the channel estimation algorithm is realized based on the available observations, i.e. the stacked received signal vector $X[n]$ for $0 \leq n < N_a$. Therefore, the EVD of (3.33) is approximated from the sample covariance matrix $\hat{R}_X(N_a)$ in (3.30):

$$\hat{R}_X(N_a) = \hat{U}\hat{\Lambda}\hat{U}^H, \tag{3.37}$$

where $\hat{\Lambda}$ is the estimated eigenvalue matrix, and $\hat{U}$ is the corresponding estimated eigenvector matrix. Due to estimation, (3.36) is no longer satisfied exactly. Therefore, we solve the equation (3.36) by the least squares method. That is, we search the estimate $\hat{K}$ which minimizes

$$\min_K \sum_{i \in \mathcal{I}} \|\hat{U}_i^H \mathcal{K}\|^2, \tag{3.38}$$

where according to (3.27), matrix $\mathcal{K}$ is a function of the search variable $K = [\mathbf{K}[0]^T \cdots \mathbf{K}[M]^T]^T$. To avoid the trivial zero solution, we enforce the additional condition

$$KK^H = I. \tag{3.39}$$

However, other types of constraints can be used [13]. Hence, after certain mathematical manipulations, the estimated value of K can be expressed as

$$\hat{K} = \arg \min_{KK^H = I} \mathrm{tr}[K^H \mathcal{P} K], \tag{3.40}$$

where

$$\mathcal{P} = \sum_{i \in \mathcal{I}} \hat{\mathcal{U}}_i \hat{\mathcal{U}}_i^H \tag{3.41}$$

and $\hat{\mathcal{U}}_i$ is a filtering matrix constructed from the estimated eigenvectors $\hat{U}_i$ in the following way. Let

$$\hat{U}_i = [\hat{U}_{i,1}^T \cdots \hat{U}_{i,N}^T]^T, \tag{3.42}$$

where $\hat{U}_{i,l} \in \mathbb{C}^{L \times 1}$, for $l \in \{1, \cdots, N\}$. Hence, $\hat{\mathcal{U}}_i$ is constructed as the $L(M+1) \times (N+M)$

matrix

$$\hat{\mathcal{U}}_i = \begin{bmatrix} \hat{U}_{i,1} & \cdots & \hat{U}_{i,N} & 0 & \cdots & \cdots & 0 \\ 0 & \hat{U}_{i,1} & \cdots & \hat{U}_{i,N} & 0 & \cdots & 0 \\ \vdots & & & \cdots & & & \\ 0 & \cdots & \cdots & 0 & \hat{U}_{i,1} & \cdots & \hat{U}_{i,N} \end{bmatrix}. \tag{3.43}$$

Let the eigenvectors of $\mathcal{P}$ in (3.41) be given as the column of the $L(M+1) \times L(M+1)$ matrix

$$\mathcal{V} = [\mathcal{V}_1 \cdots \mathcal{V}_{L(M+1)}], \tag{3.44}$$

where the $\mathcal{V}_i$ are arranged by the decreasing order of their eigenvalues. The solution to the constrained optimization problem of function (3.40), i.e. the estimated channel impulse response matrix $\hat{K}$, is given by

$$\hat{K} = [\mathcal{V}_{L(M+1)-P+1} \cdots \mathcal{V}_{L(M+1)}], \tag{3.45}$$

which is a $L(M+1) \times P$ matrix. For more clarity, (3.45) can be represented as

$$\hat{K} = \begin{bmatrix} \hat{\mathbf{k}}_1[0] & \cdots & \hat{\mathbf{k}}_P[0] \\ \hat{\mathbf{k}}_1[1] & \cdots & \hat{\mathbf{k}}_P[1] \\ \vdots & \cdots & \vdots \\ \hat{\mathbf{k}}_1[M] & \cdots & \hat{\mathbf{k}}_P[M] \end{bmatrix}, \tag{3.46}$$

where each vector $\hat{\mathbf{k}}_p[m]$ has dimension $L \times 1$. Finally, the estimated channel impulse response of the TV SIMO system defined in (3.13) can be constructed with help of (3.46), that is

$$\hat{\mathbf{h}}[n,m] = \sum_{p=1}^{P} \hat{\mathbf{k}}_p[m]\phi_p[n]. \tag{3.47}$$

In the next section, we present a novel indirect blind equalization algorithm which is based on the above estimation approach.

3.2 Indirect Blind Channel Equalization

Since we now have knowledge of the channel impulse response of the TI MIMO system by mean of the subspace estimation algorithm presented above, we may now proceed with the discussion of the indirect blind channel equalizer. A ZF equalization algorithm is considered in this section, similar to the work in [18], but no longer restricted to the set of complex exponential basis functions.

3.2.1 Peak Distortion Equalization Criterion

When combining the structures in Fig. 2.2 and Fig. 2.3, the result can be considered as a MIMO system with a linear equalizer. In the absence of the noise, i.e. $\mathbf{w}[n] = \mathbf{0}$, the $P \times P$ impulse response of the overall system can be written as

$$\mathbf{Y}[n] = \mathbf{G}[n]^T * \mathbf{H}[n], \tag{3.48}$$

where $\mathbf{H}[n]$ is the $L \times P$ impulse response matrix of the LTI MIMO channel in (2.46) and $\mathbf{G}[n]$ is the $L \times P$ impulse response matrix of the equalizer. Hence, the equalized signal in (2.64) can be represented as

$$\begin{aligned} \mathbf{e}_S[n] &= \mathbf{Y}[n] * \mathbf{s}[n] \\ &= \sum_{m=-\infty}^{\infty} \mathbf{Y}[m]\mathbf{s}[n-m] \\ &= \mathbf{Y}[n_o]\mathbf{s}[n-n_o] + \sum_{\substack{m=-\infty \\ m \neq n_o}}^{\infty} \mathbf{Y}[m]\mathbf{s}[n-m], \end{aligned} \tag{3.49}$$

where n_o is the desired delay index of the overall system, and the second term of the right hand side is the ISI term which causes a distortion.

The goal of the MIMO equalization is to design a filter such that the equalized signal $\mathbf{e}_S[n]$ can approximate the source signal $\mathbf{s}[n-n_o]$. Consider the two terms on the right-hand side in (3.49). The 'optimal' processing would keep the first term to track $\mathbf{s}[n-n_o]$ as accurately as possible, while pushing the ISI term as small as possible at the same time. Because the norm of the ISI term satisfies the following inequality:

$$\begin{aligned} \| \sum_{\substack{m=-\infty \\ m\neq n_o}}^{\infty} \mathbf{Y}[m]\mathbf{s}[n-m] \| &\leq \sum_{\substack{m=-\infty \\ m\neq n_o}}^{\infty} \|\mathbf{Y}[m]\|_F \|\mathbf{s}[n-m]\| \\ &\leq \sqrt{P}\zeta \sum_{\substack{m=-\infty \\ m\neq n_o}}^{\infty} \|\mathbf{Y}[m]\|_F \end{aligned} \tag{3.50}$$

where $\|\cdot\|_F$ denotes the Forbenius norm [23] and $\zeta \geq 0$ is an upper bound on the source signal constellation values, that is $|s_p[n]| \leq \zeta$ far all p and n. Considering that $\mathbf{Y}[n]$ is related to the equalizer matrix $\mathbf{G}[n]$ via (3.48), we may design the specific $\mathbf{G}[n]$ to force

$$\begin{aligned} \mathbf{Y}[m] &= 0; && m \neq n_o, \\ \mathbf{Y}[n_o] &= \alpha; && \text{otherwise}, \end{aligned} \tag{3.51}$$

for some positive constant $\alpha > 0$. This ZF approach is closely related to the peak distortion equalization criterion for the MIMO system defined as [1]

$$\mathrm{ISI}\{\mathbf{G}[n]\} = \frac{\sum_{n=-\infty}^{\infty} \|\mathbf{Y}[n]\|_F^2 - \max_{j,i,n}\{|y_{j,i}[n]|^2\}}{\max_{j,i,n}\{|y_{j,i}[n]|^2\}} \geq 0, \tag{3.52}$$

where $y_{j,i}[n]$ denotes the ij^{th} entry of matrix $\mathbf{Y}[n]$ in (3.48). The solution to minimize the peak distortion equalization criterion is discussed in [1], where it is shown that $\mathbf{G}[n]$ must satisfy

$$\mathbf{G}[n]^T * \mathbf{H}[n] = \mathrm{diag}(\alpha_1\delta[n-n_1], \cdots, \alpha_P\delta[n-n_P]), \tag{3.53}$$

where α_p, for $1 \leq p \leq P$, are constants, and n_p are integer delays. Equation (3.53) generalizes (3.51) in that the desired delays for each source signal components, i.e. $s_j[n-n_j]$, are controlled separately. In this work, we use the solution of (3.53) to define the ZF equalizer.

Let $\mathbf{H}(z)$ denote the z-transform of the MIMO channel impulse response matrix $\mathbf{H}[n]$, that is

$$\mathbf{H}(z) = \sum_{n=0}^{\infty} \mathbf{H}[n] z^{-n}, \tag{3.54}$$

where $\mathbf{H}[n]$ is assumed to be causal. The following important result is proven in [24, pp. 295-296]:

> "There exists a stable ZF equalizer $\mathbf{G}[n]$ for the MIMO system $\mathbf{H}[n]$, if and only if matrix $\mathbf{H}(z)$ is full column rank for all $|z| = 1$."

In general, the impulse response of the equalizer $\mathbf{G}[n]$ would be doubly infinite, i.e. extending from $n = -\infty$ to $+\infty$. However, it can be shown that a finite length equalizer exists if $\mathbf{H}[n]$ is a finite impulse response, i.e. $\mathbf{H}[n] = 0$ for $n < 0$ and $n > L$, where $L > P$.

In conclusion, there exist a FIR ZF equalizer, if the following conditions are satisfied:
A1: $\mathbf{H}[n]$ is MIMO FIR with $L > P$;
A2: $\mathbf{H}(z)$ is full column rank for all $|z| = 1$.

3.2.2 Zero Forcing Equalizer of TI MIMO System

According to the equivalent TI MIMO system represented in (3.21), the coefficient matrix $\mathbf{K}[m]$, for $m \in \{0, \cdots, M\}$, now provide a characterization of this MIMO FIR system. Hence, we may design a ZF equalizer based on existing method for the TI MIMO system, and through appropriate manipulations, obtain a corresponding equalizer for the original TV SIMO system. This is the approach discussed below.

The TI MIMO system under consideration has P inputs, L outputs, with a channel impulse response of length $M + 1$ or less for each sub-channel. In the z-domain, the transfer function associated to the FIR matrix sequence $\mathbf{K}[m]$ can be expressed as

$$\mathbf{K}(z) = \sum_{m=0}^{M} \mathbf{K}[m] z^{-m}. \tag{3.55}$$

We shall assume that $\mathbf{K}[m]$ satisfies the general assumption in A2. The impulse response of the ZF equalizer can also be represented in terms of its transfer function in the z-domain given by

$$\mathbf{G}(z) = \sum_{d=0}^{D} \mathbf{G}[d] z^{-d}, \tag{3.56}$$

where $D+1$ is the length of the ZF equalizer, and $\mathbf{G}[d]$ denotes the d^{th} matrix tap of the impulse response in the time domain. It is convenient to express the $L \times P$ matrix $\mathbf{G}[d]$ in terms of its columns as follows,

$$\mathbf{G}[d] = [\mathbf{g}_1[d] \cdots \mathbf{g}_P[d]], \tag{3.57}$$

where $\mathbf{g}_p[d]$, for $p \in \{1, \cdots, P\}$, is the $L \times 1$ vector

$$\mathbf{g}_p[d] = [g_{1p}[d], \cdots, g_{Lp}[d]]^T. \tag{3.58}$$

In the z-domain, the system function of the TI MIMO system and corresponding ZF equalizer satisfies

$$\mathbf{G}^T(z)\mathbf{K}(z) = \mathrm{diag}(\alpha_1 z^{-n_1}, \cdots, \alpha_P z^{-n_P}). \tag{3.59}$$

In this thesis, we focus on the special case where $\alpha_1 = \cdots = \alpha_P = 1$ and $n_1 = \cdots = n_P = n_o$. The first condition accounts for a common scaling of the equalized signal, while the second condition (identical delays) is possible if A2 is satisfied [18]. Therefore, we can have

$$\mathbf{G}^T(z)\mathbf{K}(z) = z^{-n_o} I. \tag{3.60}$$

In the time domain, (3.59) can be expressed as

$$\sum_{d=0}^{D} \mathbf{G}[d]^T \mathbf{K}[n-d] = \mathrm{diag}(\delta(n-n_o), \cdots, \delta(n-n_o)), \tag{3.61}$$

where the domain of n can restricted to

$$n \in \{0, \cdots, M+D\}, \tag{3.62}$$

since $\mathbf{G}[d] = 0$ for $d \notin \{0, \cdots, D\}$ and $\mathbf{K}[m] = 0$ for $m \notin \{0, \cdots, M\}$.

The right-hand side of (3.61) will be denoted by θ_n. The left-hand side of (3.61) can be expanded as

$$\begin{aligned}
&\sum_{d=0}^{D} \begin{bmatrix} \mathbf{g}_1^T[d] \\ \vdots \\ \mathbf{g}_P^T[d] \end{bmatrix} [\mathbf{k}_1[n-d] \cdots \mathbf{k}_P[n-d]] \\
&= \begin{bmatrix} \mathbf{g}_1^T[D] & \cdots & \mathbf{g}_1^T[0] \\ \vdots & \ddots & \vdots \\ \mathbf{g}_P^T[D] & \cdots & \mathbf{g}_P^T[0] \end{bmatrix} \begin{bmatrix} \mathbf{k}_1[n-D] & \cdots & \mathbf{k}_P[n-D] \\ \vdots & \ddots & \vdots \\ \mathbf{k}_1[n-0] & \cdots & \mathbf{k}_P[n-0] \end{bmatrix}.
\end{aligned} \tag{3.63}$$

To simplify the presentation in the next chapter we shall use the following notations: the first term of the right-hand side will be denoted by G_D which is a $P \times L(D+1)$ matrix; the second term of the right-hand side will be denoted by K_n which is a $L(D+1) \times P$ matrix.

We know that the range of n in (3.63) is restricted by (3.62), so we may now formulate the ZF equalization problem in the compact form

$$G_D \Gamma = \Theta, \tag{3.64}$$

where we also define

$$\Gamma = [K_0 \cdots K_{M+D}], \tag{3.65}$$

$$\Theta = [\theta_0 \cdots \theta_{M+D}]. \tag{3.66}$$

It should be noted that the delay of the equalizer, n_o, is usually chosen as the median of the value $\{0, \cdots, M+D\}$, which we denote as ξ. In practice, therefore, (3.66) reduces to

$$\Theta = [0 \cdots I_\xi \cdots 0], \tag{3.67}$$

where the identity matrix is the median $P \times P$ block. Further, the desired ZF equalizer can be expressed as

$$G_D = \Theta \Gamma^\dagger, \tag{3.68}$$

where $[\cdot]^\dagger$ denotes the pseudo-inverse of a matrix argument [23] . It is important to point out that for the equalization problem in (3.63) or equivalently (3.64), there are $LP(D+1)$ unknowns, i.e. the entries $\{g_{lp}[d]\}$ of matrix $\mathbf{G}[d]$, and $P^2(M+D+1)$ equations. In general, to ensure that the non-homogeneous system (3.64) admits one (or more) solution, we must ensure that it is underdetermined, i.e. that the number of equation does not exceed the number of unknowns. That is, we require $P^2(M+D+1) \leq LP(D+1)$, which indicates that

$$D + 1 \geq \frac{PM}{L-P}. \tag{3.69}$$

Once the solution of the matrix G_D has been obtained from (3.68), the impulse response matrix $\mathbf{G}[d]$ of the ZF equalizer in (3.57) can be obtained by rearrangement of its elements.

In practice, the ZF equalizer can only be designed from the *estimated value* of the channel impulse response of the equivalent TI MIMO system. First, the length of the designed ZF equalizer has to satisfy (3.69). Second, $\hat{\Gamma}$ is constructed based on (3.65), using the estimated channel impulse responses $\hat{K}$ defined in (3.46). Then, $\hat{G}_D$ can be obtained via

$$\hat{G}_D = \Theta \hat{\Gamma}^\dagger, \tag{3.70}$$

which in turn leads to the impulse response $\hat{\mathbf{G}}[d]$ of the designed equalizer via (3.63). It will be convenient to express the latter in terms of its z-transform, i.e.

$$\hat{\mathbf{G}}(z) = \sum_{d=0}^{D} \hat{\mathbf{G}}[d] z^{-d}, \tag{3.71}$$

where $\hat{\mathbf{G}}[d]$ is the impulse response of the designed ZF equalizer. Hence, in the z-domain, the equalized received signal is characterized by

$$\hat{\mathbf{e}}(z) = \hat{\mathbf{G}}(z)^T \tilde{\mathbf{x}}(z), \tag{3.72}$$

where $\tilde{\mathbf{x}}(z)$ is z-transform of $\mathbf{x}[n]$ defined in (3.21). Finally, the equalized noise free signal of the equivalent TI MIMO system can be expressed as

$$\hat{\mathbf{e}}_t(z) = \hat{\mathbf{G}}(z)^T \mathbf{K}(z) \tilde{\mathbf{t}}(z), \tag{3.73}$$

where $\tilde{\mathbf{t}}(z) = \sum_n \mathbf{t}[n] z^{-n}$.

Chapter 4

Determination of Ambiguity Matrix

As explained in [18, 19, 25], the equivalent TI MIMO FIR channel in (3.21) can be identified up to an ambiguity matrix by using SOS. In the case of the TV SIMO system where the TV channels can be expressed in terms of complex exponential (i.e. Fourier) basis functions, an efficient technique has been developed in [18] to determine the ambiguity matrix up to a scalar, and therefore construct the blind equalizer. In this chapter, we extend this technique so that it can be applied to a TV SIMO system represented in terms of arbitrary basis functions as in [19]. Finally, a complete ZF equalization algorithm is proposed for this system model.

4.1 Reduction of the Ambiguity Matrix

It has been pointed out that not all of the TI MIMO FIR channels can be identified up to an ambiguity unitary matrix [25]. Several necessary and sufficient conditions that must be satisfied for this to be true are discussed in [18, 19, 25]. In this section, a set of three necessary and sufficient conditions will be considered. We first recall the definition of the transfer function $\mathbf{K}(z)$ associated to the FIR matrix $\mathbf{K}[m]$ from Section 3.2.2

$$\mathbf{K}(z) = \sum_{m=0}^{M} \mathbf{K}[m] z^{-m}, \tag{4.1}$$

where $\mathbf{K}[m]$ represents the impulse response of the equivalent TI MIMO channel, as defined in (3.23). We also note that in the following discussion, we assume $L > P$.

Condition I: $\mathbf{K}(z)$ *is irreducible* [18, 19, 25].

Here, $\mathbf{K}(z)$, in (4.1), is an $L \times P$ polynomial matrix. The irreducibility condition for $\mathbf{K}(z)$ indicates that there is no $P \times P$ polynomial matrix $\mathcal{D}(z)$ which has a *non-constant* determinant, such that $\mathbf{K}(z) = \hat{\mathbf{K}}(z)\mathcal{D}(z)$, where $\hat{\mathbf{K}}(z)$ is an $L \times P$ polynomial matrix. Also, it has been proven that $\mathbf{K}(z)$ is irreducible *if and only if* $\mathbf{K}(z)$ *is full rank for any* $z \in \mathbb{C}$ [26]. In this case, the entries $k_{l,p}(z)$ of $\mathbf{K}(z)$ are coprime, where $k_{l,p}(z)$ denotes the z-transfer functions of $k_{l,p}[m]$, and the rows of $K(z)$ can be considered as coprime for $1 \le p \le P$.

Condition II: $\mathbf{K}[M]$ *is full rank* [19].

Here, $\mathbf{K}[M]$ is an $L \times P$ matrix defined in (3.23) and $\text{rank}(\mathbf{K}[M]) = P$. Without loss of generality, we can represent the full rank polynomial matrix $\mathbf{K}(z)$ as [27]

$$\mathbf{K}(z) = \mathcal{N}(z)\mathcal{M}(z)^{-1}, \tag{4.2}$$

where $\mathcal{M}(z)$ is a full rank nonsingular minor of $\mathbf{K}(z)$ with size $P \times P$, and $\mathcal{N}(z)$ is the corresponding remainder. Because $\mathbf{K}(z)$ is irreducible, $\mathcal{M}(z)$ and $\mathcal{N}(z)$ have only right common unimodular factors [27]. Also, $\mathcal{M}(z)$ represents the z-domain transfer function of the $P \times P$ minor, denoted by $\mathcal{M}[m]$, of $\mathbf{K}[m]$ for $m \in \{0, \cdots, M\}$, and $\mathcal{N}(z)$ denotes the transfer function of a corresponding remainder of $\mathbf{K}[m]$. For later convenience, let $\mathcal{J}(z)$ denotes

$$\mathcal{J}(z) = \sum_{m=0}^{M} \mathcal{M}[m] z^m. \tag{4.3}$$

Note that $\mathcal{J}(z^{-1}) = \mathcal{M}(z)$. Therefore, the degree of $\det(\mathcal{J}(z))$ is expressed as

$$\deg(\det(\mathcal{J}(z))) = MP. \tag{4.4}$$

Based on a corollary from [27], the rank of the equivalent filtering matrix $\mathcal{K}$ in (3.27) can be obtained as

$$\begin{aligned} \operatorname{rank}(\mathcal{K}) &= NP + \deg(\det(\mathcal{J}(z))) \\ &= (N+M)P, \end{aligned} \tag{4.5}$$

where integer N has to satisfy $N \geq \frac{MP}{L-P}$ [27]. In short, if Condition II is true, $\mathcal{K}$ is a full column rank matrix if $N \geq \frac{MP}{L-P}$.

In conclusion, if the MIMO FIR channels satisfy the Conditions I and II, we consider that they can be identified by using the SOS.

Condition III: $\mathcal{R}(\mathcal{K}) = \mathcal{R}(\hat{\mathcal{K}})$ [13, 19].

Here, $\mathcal{R}(\cdot)$ denotes the range space of its matrix argument [23], and the full rank matrix $\hat{\mathcal{K}}$ is associated with the TI MIMO channel estimated value of $\mathbf{K}[m]$, i.e. $\hat{\mathbf{K}}[m]$. The Condition III corresponds to a situation where the noise subspace has been estimated exactly. In this case, $\mathcal{R}(\mathcal{K}) = \mathcal{R}(\hat{\mathcal{K}})$, which indicates that

$$\mathcal{K} = \hat{\mathcal{K}}\Psi, \tag{4.6}$$

where Ψ is a non-singular $(M+N)P \times (M+N)P$ matrix. Since the filtering matrix $\mathcal{K}$ and $\hat{\mathcal{K}}$ are full rank matrices based on the Condition II, Ψ can in turn be considered as a full rank matrix. Ψ can be partitioned into $(N+M)^2$ sub-matrices $\Psi_{i,j}$, each one being a $P \times P$ square matrix, for $i, j \in \{1, \cdots, M+N\}$.

To obtain the relationship between $\mathbf{K}[m]$ and $\hat{\mathbf{K}}[m]$ from the Condition III, first we will analyze (4.6) column-wise from the right to the left. The last P columns of $\mathcal{K}$ in (4.6) can be expressed as

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ K[M] \end{bmatrix} = \left[\begin{array}{c|c} \varsigma_1 & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \hline \varsigma_2 & \hat{K}[M] \end{array}\right] \begin{bmatrix} \Psi_{1,M+N} \\ \Psi_{2,M+N} \\ \vdots \\ \Psi_{M+N,M+N} \end{bmatrix}, \tag{4.7}$$

where ς_i, for $i = 1, 2$, are sub-matrices of $\hat{\mathcal{K}}$. Since $\hat{\mathcal{K}}$ is a full rank matrix, ς_1 can be considered as a full rank matrix. Therefore, comparing the left-hand side with the product on the right-hand side of (4.7), we obtain

$$\Psi_{i,M+N} = 0 \quad \text{for} \quad i < M + N, \tag{4.8}$$

$$\mathbf{K}[M] = \hat{\mathbf{K}}[M]\Psi_{M+N,M+N}. \tag{4.9}$$

Then, each subset of P adjacent columns sub-matrix of matrix $\mathcal{K}$ in (4.6) is analyzed by using the above method from right to left, up to the first P columns of $\mathcal{K}$. In this way we show that

$$\Psi_{i,j} = 0 \quad \text{for all} \quad i < j, \tag{4.10}$$

$$\mathbf{K}[m] = \hat{\mathbf{K}}[m]\Psi_{m+N,m+N}. \tag{4.11}$$

Next, we apply the same approach but beginning with the first P columns up to the last P columns of matrix $\mathcal{K}$, i.e. from the left to the right. Proceeding in this way, we obtain

$$\Psi_{i,j} = 0 \quad \text{for all} \quad i > j. \tag{4.12}$$

Therefore, from(4.10) and (4.12), we can conclude that

$$\Psi_{i,j} \neq 0 \quad \text{if and only if} \quad i = j. \tag{4.13}$$

Also, due to the special structure of matrix $\hat{\mathcal{K}}$, $\mathcal{K}$, and Ψ, we can define a full rank matrix $\mathcal{A}$, such that

$$\Psi_{i,i} = \mathcal{A} \quad i \in \{1, \cdots, M+N\}. \tag{4.14}$$

Now, we can express (4.6) as

$$\mathcal{K} = \hat{\mathcal{K}}(\mathcal{A} \otimes I_{M+N}). \tag{4.15}$$

where $\otimes$ denotes a Kronecker product [23].

If we further assume that $\hat{\mathcal{K}}$ satisfies the normalization condition for noise-free SOS approach, that is

$$\mathcal{K}\mathcal{K}^H = \hat{\mathcal{K}}\hat{\mathcal{K}}^H, \tag{4.16}$$

then, we can have

$$I_{(M+N)P} = (\mathcal{A} \otimes I_{M+N})(\mathcal{A} \otimes I_{M+N})^H, \tag{4.17}$$

which in turn implies

$$\mathcal{A}\mathcal{A}^H = I_P. \tag{4.18}$$

Therefore, the full rank matrix $\mathcal{A}$, so-called the ambiguity matrix, is proven to be an unitary matrix and we may write

$$\mathbf{K}(z) = \hat{\mathbf{K}}(z)\mathcal{A}, \tag{4.19}$$

$$\hat{\mathbf{K}}(z) = \mathbf{K}(z)\mathcal{A}^{-1}. \tag{4.20}$$

In short, Condition III is true *if and only if* $\mathbf{K}[m]$ and $\hat{\mathbf{K}}[m]$ are proportional [13, 19].

Consequently, if the MIMO FIR channel satisfies the above three conditions, it can be identified up to an ambiguity unitary matrix. In this chapter, the TI MIMO system is assumed to follow the Conditions I, II, and III.

4.2 Approach for the Determination of the Ambiguity Matrix

Generally, if the ambiguity matrix $\mathcal{A}$ is known, the source signal $\mathbf{t}[n]$ in (3.21), for the equivalent TI MIMO system, can be constructed without using higher order statistics. Furthermore, the source signal $s[n]$ of the TV SIMO system would be recovered by inverting (3.22).

In the equivalent TI MIMO system, the "ambiguous" equalizer $\hat{\mathbf{G}}[d]$ is obtained based on the channel estimate $\hat{\mathbf{K}}[m]$, so that $\hat{\mathbf{G}}(z)$ and $\hat{\mathbf{K}}(z)$ satisfy the relation (3.60). Therefore, we can have

$$\hat{\mathbf{G}}(z)^T\hat{\mathbf{K}}(z) = z^{-n_o}I_P. \tag{4.21}$$

Since Condition III ensures that the ambiguity matrix $\mathcal{A}$ is unitary, we may further manipulate (3.73) as follows

$$\begin{aligned}\hat{\mathbf{e}}_t(z) =&\hat{\mathbf{G}}^T(z)\mathbf{K}(z)\tilde{\mathbf{t}}(z)\\ =&\hat{\mathbf{G}}^T(z)\mathbf{K}(z)\mathcal{A}^{-1}\mathcal{A}\tilde{\mathbf{t}}(z)\\ =&\hat{\mathbf{G}}(z)^T\hat{\mathbf{K}}(z)\mathcal{A}\tilde{\mathbf{t}}(z)\\ =&z^{-n_o}\mathcal{A}\tilde{\mathbf{t}}(z).\end{aligned} \tag{4.22}$$

Now, multiplying both sides of (4.22) by $\mathcal{A}^{-1}$, we obtain

$$\mathcal{A}^{-1}\hat{\mathbf{G}}^T(z)\mathbf{K}(z)\mathbf{t}(z) = z^{-n_o}\tilde{\mathbf{t}}(z). \tag{4.23}$$

Considering (3.60) and (4.22), we have the following relation between the ambiguous equalizer $\hat{\mathbf{G}}[d]$ and the true, i.e. desired equalizer $\mathbf{G}[d]$

$$\mathbf{G}(z) = \hat{\mathbf{G}}(z)\mathcal{A}^{-T}. \tag{4.24}$$

Based on the results of (4.22) and (3.22), and setting $n_o = 0$ for simplicity, we can express the equalized signal of the TI MIMO system as

$$\hat{\mathbf{e}}_t[n] = \mathcal{A}\Phi[n]s[n], \tag{4.25}$$

where $s[n]$ is the desired source signal of the TV SIMO system. The $P \times P$ ambiguity matrix $\mathcal{A}$ can be expressed as

$$\mathcal{A} = [\mathcal{A}_1 \cdots \mathcal{A}_P], \tag{4.26}$$

where $\mathcal{A}_p$, for $1 \leq p \leq P$, is a $P \times 1$ vector defined as

$$\mathcal{A}_p = [\mathcal{A}_{1p} \cdots \mathcal{A}_{Pp}]^T. \tag{4.27}$$

To recover $s[n]$, we may construct a linear system

$$s[n]^{-1}\hat{\mathbf{e}}_t[n] - \mathcal{A}\Phi[n] = 0. \tag{4.28}$$

For more convenience, we expand (4.28) as

$$s[n]^{-1}\begin{bmatrix} \hat{e}_{t1}[n] \\ \vdots \\ \hat{e}_{tP}[n] \end{bmatrix} - \begin{bmatrix} \mathcal{A}_{11}\phi_1[n] + \cdots + \mathcal{A}_{1P}\phi_P[n] \\ \vdots \\ \mathcal{A}_{P1}\phi_1[n] + \cdots + \mathcal{A}_{PP}\phi_P[n] \end{bmatrix} = 0, \tag{4.29}$$

where there are P equations and P^2+1 unknown parameters, which are the entries of matrix $\mathcal{A}$ and source signal symbol $s[n]$. At a specific discrete-time n, $P < P^2 + 1$ and (4.29) lead to an underdetermined system, for which an unique solution can not be obtained. However, a linear overdetermined system can be constructed by considering a group of multiple equalized symbols, so that unique solution can be found. Specifically, we consider N_e consecutive equalized symbols with time index $\{n, n+1, \cdots, n+(N_e-1)\}$. We may then construct an overdetermined system, that is

$$\begin{bmatrix} \phi_1[n]I_p & \cdots & \phi_P[n]I_P & \hat{\mathbf{e}}_t[n] & 0 & \cdots & 0 \\ \phi_1[n+1]I_p & \cdots & \phi_P[n+1]I_P & 0 & \hat{\mathbf{e}}_t[n+1] & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & & 0 \\ \phi_1[n+(N_e-1)]I_p & \cdots & \phi_P[n+(N_e-1)]I_P & 0 & \cdots & 0 & \hat{\mathbf{e}}_t[n+(N_e-1)] \end{bmatrix} \cdot \begin{bmatrix} \mathcal{A}_1 \\ \vdots \\ \mathcal{A}_P \\ -s^{-1}[n] \\ \vdots \\ -s^{-1}[n+(N_e-1)] \end{bmatrix} = 0, \tag{4.30}$$

where there are now PN_e equations and $P^2 + N_e$ unknown parameters. To keep (4.30) as an overdetermined system, we restrict $N_e \geq \frac{P^2}{P-1}$. In practice, the least squares method is introduced to approximate $\mathcal{A}$. If the bases function are exponential, the ambiguity is determined up to a scalar [18].

4.3 Summary of the Proposed Subspace-based Blind Equalization Algorithm

Up to this point, we have studied the subspace-based blind channel identification for TV SIMO system under the SOS framework, designed the corresponding direct ZF equalizer, and proposed an approach for determining the remaining ambiguity matrix. In this section, our proposed algorithm for subspace-based blind channel equalization of TV SIMO system is summarized.

In Section 3.1, it has been proposed that the TV SIMO system can be described by an equivalent TI MIMO system in (3.21). To this end, TV channels were expanded as linear combinations of a set of unknown coefficients and known basis functions in (3.13). Furthermore, the subspace method was exploited to identify the channel impulse responses of the equivalent TI MIMO system, under the assumptions that Conditions I and II are satisfied. In the equivalent TI MIMO system, the vector $X[n]$ consists of N successive received signal vectors as in (3.5). The matrices that span the signal and noise subspaces are obtained by using the eigenvalue decomposition of the autocorrelation matrix of $X[n]$. Then, the TI MIMO channels are identified in(3.40)-(3.45) by using the orthogonality between the signal and noise subspaces. Finally, the TV SIMO channel impulse responses can be identified with the help of (3.13). This subspace-based blind channel identification algorithm for TV SIMO system is summarized in Table 4.1. In the model for the proposed algorithm, L denotes the number of receivers, P denotes the the number of transmitters in equivalent MIMO system, which is equal to the number of the basis functions in the SIMO system, M denotes the number of delays, and $i \in \{P(N+M)+1, \cdots, LN\}$. The label $\leftarrow$ means that the left-hand side expression is the result of deriving or manipulating the right-hand side expression.

l

Table 4.1 Subspace-based blind channel identification algorithm for TV SIMO FIR channels

Initialize the algorithm by setting	
$L > P$	
$LN > (N+M)$	
Conditions I and II	
$X[n] \leftarrow \mathbf{x}[\mathbf{n}]$	(3.5)
$\hat{R}_X(N_a) = \frac{1}{N_a}\sum_{n=0}^{N_a-1} X[n]X[n]^H$	(3.30)
$\hat{U}_i \leftarrow$ EVD of $\hat{R}_X(N_a)$	(3.37)
$\hat{\mathcal{U}}_i \leftarrow \hat{U}_i$	(3.43)
$\mathcal{P} = \sum_i \hat{\mathcal{U}}_i \hat{\mathcal{U}}_i^H$	(3.41)
$\hat{\mathbf{K}}[m] \leftarrow$ EVD of $\mathcal{P}$	(3.45)
$\hat{\mathbf{h}}[n,m] \leftarrow \hat{\mathbf{K}}[m]$	(3.47)

Based on the result of the subspace-based blind channel identification, a ZF equalizer is proposed for the TI MIMO system. To design this filter, the length of the equalizer is decided first from (3.69). After that, matrix $\hat{\Gamma}$ is constructed as (3.65) based on the results of the channel estimation, i.e. using the estimation matrix $\hat{\mathbf{K}}[m]$ resulting from the application of the algorithm in Table 4.1, and then matrix Θ is constructed as in (3.67). Finally, the impulse response of the ZF equalizer, an FIR filer, is obtained from (3.71). The ZF equalization algorithm for the equivalent TI MIMO system is summarized in Table 4.2.

Table 4.2 Summary of ZF equalizer

Initialize the algorithm by setting:	
$\hat{\mathbf{K}}[m]$ = estimated expansion coefficients	
$D+1 \geq \frac{PM}{L-P}$	
$\hat{\Gamma} = [\hat{K}_0 \cdots \hat{K}_{M+D}]$	(3.65)
$\Theta = [0 \cdots I_p \cdots 0]$	(3.67)
$\hat{G}_D = \Theta \hat{\Gamma}^{\dagger}$	(3.70)
$\hat{\mathbf{G}}[n] \leftarrow \hat{G}_D$	(3.71)

Based on previous works, the MIMO FIR channel can be identified up to an ambiguity matrix if Condition III is satisfied. To determine the ambiguity matrix for the TV SIMO system, the equalized signal is obtained by filtering the received signal with the designed FIR filter, which is the ZF equalizer obtained from Table 4.2. Then, we may construct the overdetermined system in (4.30), so that the least squares method is exploited to solve for the columns of ambiguity matrix $\mathcal{A}$. This approach for the determination of the ambiguity matrix is summarized in Table 4.3. Note that N_e is the numbers of equations in the overdetermined system.

Table 4.3 Summary of ambiguity matrix approach

Initialize the algorithm by setting:	
$\hat{\mathbf{G}}[n]$	
$N_e \geq \frac{P^2}{P-1}$	
Condition III	
$\hat{\mathbf{e}}[n] = \hat{\mathbf{G}}[n] * \mathbf{x}[n]$	(3.72)
Using least squares technique to solve	
$s[n]^{-1}\hat{\mathbf{e}}[n] - \mathcal{A}\Phi[n] = 0,\ 0 \leq n < N_e$	
for the ambiguity matrix $\mathcal{A}$	

Chapter 5

Simulation Study

In the previous chapters, we have presented the overall approach and analysis for developing a blind subspace-based channel equalization algorithm for a SIMO system with arbitrary TV FIR channels. This includes building an equivalent TI MIMO system for the TV SIMO system, applying a blind subspace-based channel estimation algorithm, developing a ZF equalizer and determining ambiguity matrix of the whole system. In this chapter, the results of numerical simulation experiments will be presented to illustrate the performance of the TV SIMO channel estimation algorithm, as well as the proposed equalization algorithm and ambiguity matrix determination approach.

5.1 Methodology

To measure the performance of the proposed algorithm, a testing platform is designed first for the purpose of Monte Carlo simulation. The source signal $s[n]$ is a sequence of 16 QAM symbols which is created by constellating independent discrete complex random variables. We shape $s[n]$ to be zero mean and with variance σ_s^2. The additive noise sequences for each of the channels are obtained from independent complex Gaussian random variables with zero mean and variance σ_n^2.

The TV SIMO system is comprised of $L = 4$ receivers. The number of path delays for the SIMO channel is 5, which means that $M = 4$. The impulse response of each channel is represented as a linear combination of the two scalar basis functions $\phi_1[n]$ and $\phi_2[n]$ with corresponding coefficients expansion vectors $\mathbf{k}_1[m]$ and $\mathbf{k}_2[m] \in \mathbb{C}^4$. That is, with reference to model equation (3.13), we have $P = 2$ and

$$\mathbf{h}[n, m] = \mathbf{k}_1[m]\phi_1[n] + \mathbf{k}_2[m]\phi_2[n]. \tag{5.1}$$

In our experiments, we consider linearly varying channel impulse responses and the basis functions are therefore chosen as

$$\phi_1[n] = 1, \tag{5.2}$$

$$\phi_2[n] = \alpha(n - \frac{N_a - 1}{2}), \tag{5.3}$$

for $0 \leq n < N_a$, where real parameter α controls how fast the channel would change. For each simulation run, the predetermined expansion coefficients vectors $\mathbf{k}_1[m]$ and $\mathbf{k}_2[m]$ are generated as independent complex Gaussian random variables with zero mean, properly sealed such that $\|K\| = 1$, where the definition of matrix K is given in (3.24). For a predetermined TV SIMO system, the SNR with respect to the given choice of K is defined to be the ratio of expected received signal power over expected noise power, further averaged over the integration time N_a. That is

$$\text{SNR} = \frac{1}{N_a} \sum_{n=0}^{N_a - 1} \frac{E[\|\mathbf{x}[n] - \mathbf{w}[n]\|]^2}{E\|\mathbf{w}[n]\|^2} = \frac{\sigma_s^2}{L\sigma_n^2} \text{tr}[K \Upsilon K^H], \tag{5.4}$$

where tr denotes the trace of a matrix, and

$$\Upsilon = \frac{1}{N_a} \sum_{n=0}^{N_a - 1} \Phi[n]\Phi[n]^H. \tag{5.5}$$

In practice, we adjust the parameters σ_s^2 to control the SNR value in the simulation process.

To measure the performance of the blind subspace channel estimation, the mean square error (MSE) for the TV SIMO system is defined as

$$\mathrm{MSE_{dB}} = 10\log_{10}\frac{1}{L(M+1)}\mathrm{tr}(\|\hat{\mathbf{h}}[n,m]-\mathbf{h}[n,m]\|\Upsilon\|\hat{\mathbf{h}}[n,m]-\mathbf{h}[n,m]\|^{H}), \tag{5.6}$$

where $\hat{\mathbf{h}}[n,m]$ denotes the estimated channel impulse response of the TV SIMO system based on the estimated coefficient matrix $\hat{K}$.

To measure the performance of the ZF equalizer, we chose the length of the equalizer based on equation (3.69) to be $D+1 \geq \frac{PM}{L-P} = 4$, i.e. we set $D = 3$. Mathematically, the overall impulse response of the cascaded TI MIMO system and equalizer can be written as

$$y[n] = \sum_{d=0}^{D}\mathbf{G}[d]^{T}\mathbf{K}[n-d]. \tag{5.7}$$

Therefore, the ISI measure can be defined for the TV SIMO system as

$$\mathrm{ISI} = \frac{\sum_{n=0}^{N_a-1}\|y[n]\|_F^2 - \max_{i,j,n}\{|y_{i,j}[n]|^2\}}{\max_{i,j,n}\{|y_{i,j}[n]|^2\}}, \tag{5.8}$$

where where $y_{i,j}[n]$ denotes the ij^{th} entry of matrix $y[n]$. Also, to measure the performance of the overall algorithm, specially ambiguity matrix approach, the symbol error rate (SER) is defined as the ratio of the number of error symbols with N_a.

5.2 Results

5.2.1 Channel Estimation

In both theory and simulation works, the estimation of channel expansion coefficients of the TV SIMO system plays a crucial role. These expansion coefficients, which are used to define the impulse response matrix of the equivalent TI MIMO system (3.21), can be exploited to construct the channel impulse response of the TV SIMO system via (3.13). Fig.5.1

shows that the true and estimated values of expansion coefficients of the 2^{nd} basis function associated with the 3^{rd} receiver, that is $k_{l,p}[m]$ with $p = 2$, $l = 3$ and $m \in \{0, \cdots, 4\}$. The main parameters values here are SNR=20dB $N_a = 1000$, and $\alpha = 0.001$. It can been seen that the estimated values of the expansion coefficients remain very close to the true values.

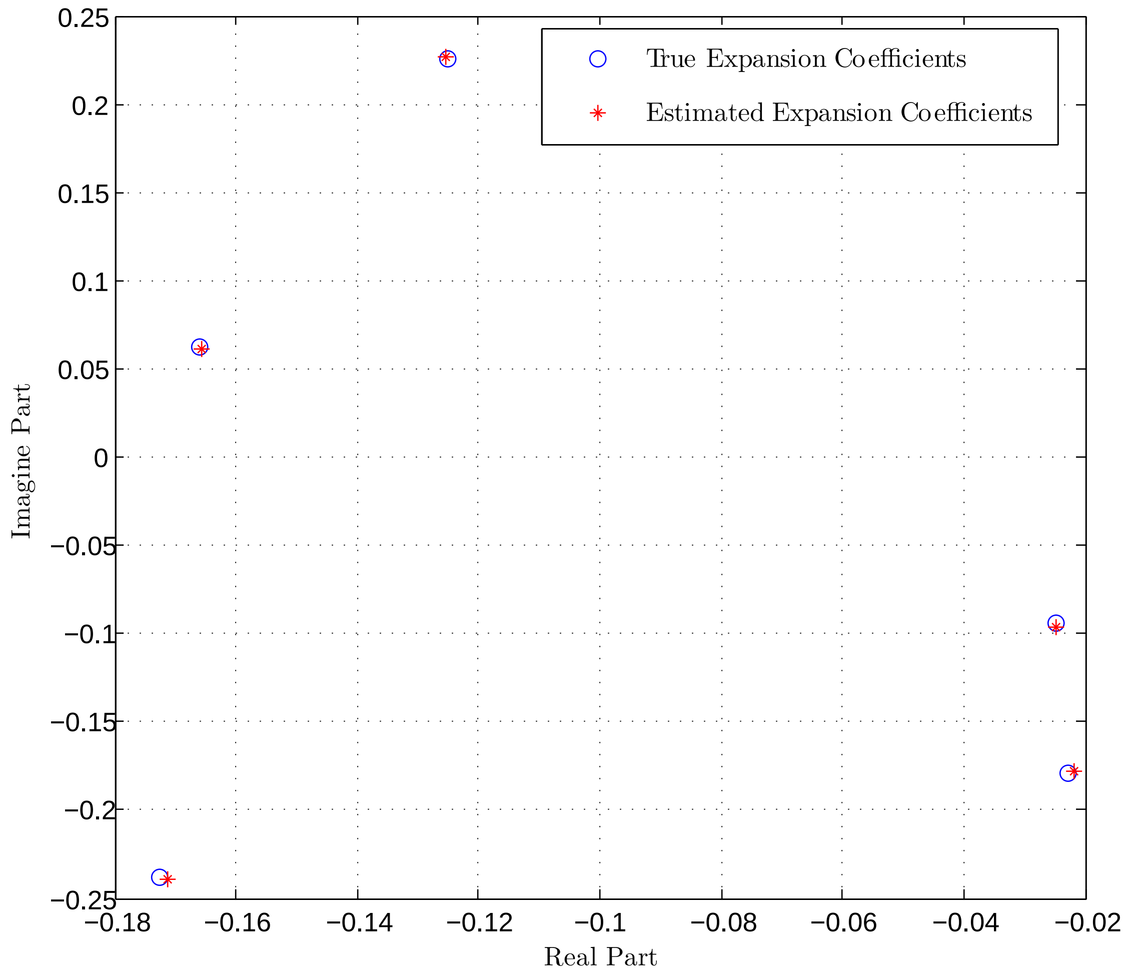

Fig. 5.1 True and estimated values of complex expansion coefficients for the 2^{nd} basis function associated with the 3^{rd} receiver (SNR=20dB).

In Fig.5.2, we show the MSE of blind subspace channel estimation as a function of the SNR for different rate of change of the time varying channels, as specified by $\alpha \in$

$\{0.001, 0.01, 0.1\}$. Again, the integration time is set to $N_a = 1000$. The results show that the blind subspace estimation algorithm has a better performance as value of α decreases. That is, when the rate of change of TV SIMO is slow, a better performance can be expected from the estimation algorithm.

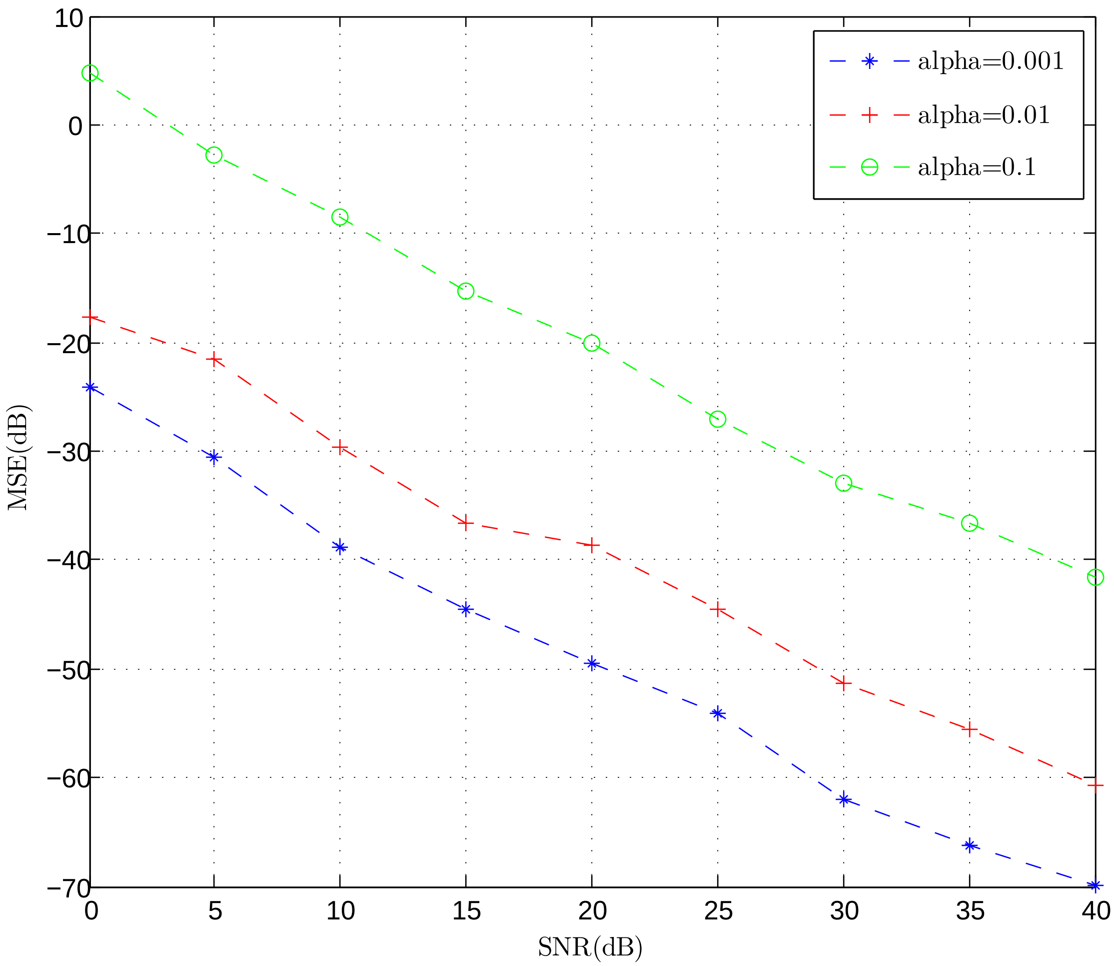

Fig. 5.2 Channel estimation MSE versus SNR for different channel rate of change α=0.1, 0.01, and 0.001.

In Fig.5.3, we show the MSE as a function of the integration time N_a for given SNR=10dB, 20dB, and 30dB. Here, the results show that the use of longer integration time can lead to a better performance for the blind subspace-based estimation algorithm.

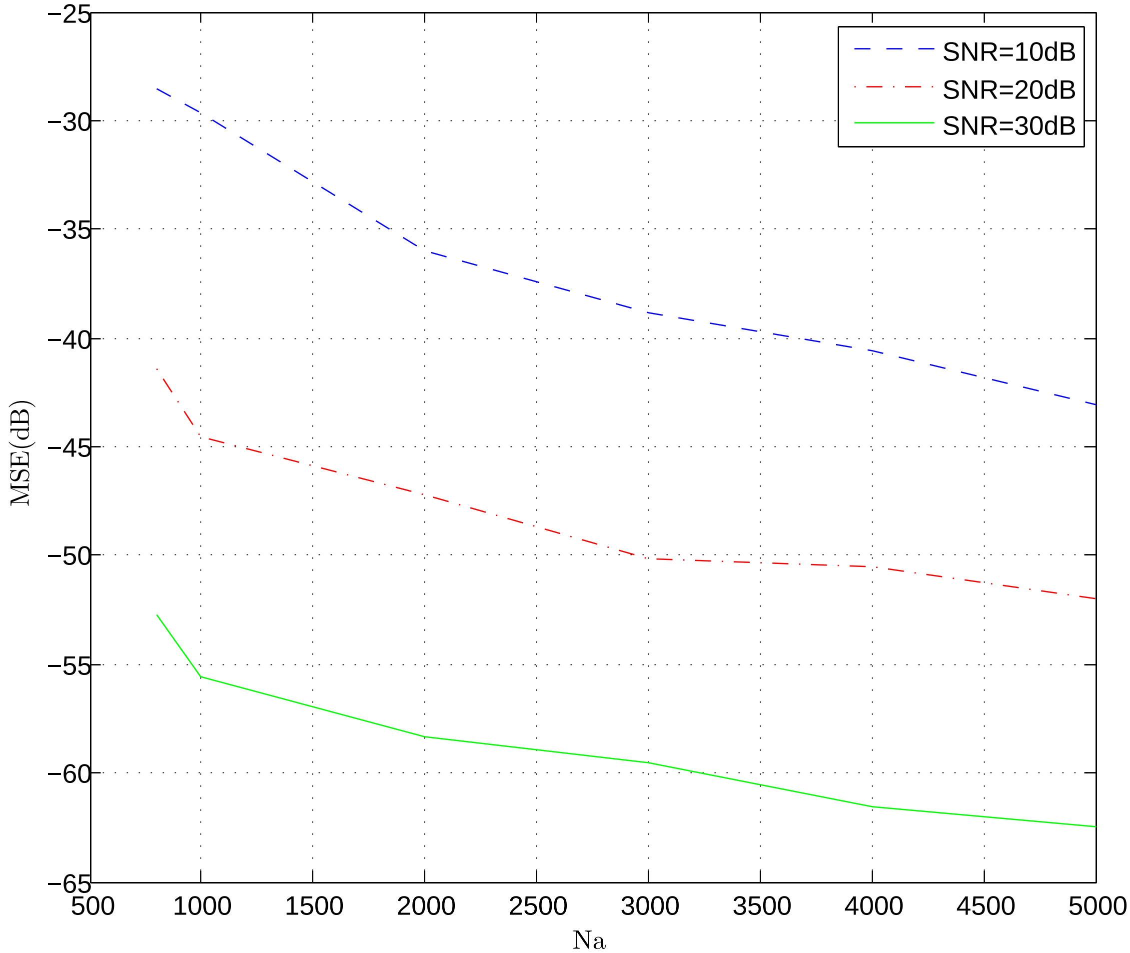

Fig. 5.3 Channel estimation MSE versus integration time N_a for different SNR=10dB, 20dB and 30dB.

5.2.2 Equalization

In these experiments, the source signal is recovered by passing the received signal through the designed FIR ZF equalizer. Fig. 5.4 displays the received signal samples before and after equalization. The simulation parameters are set as follows: $N_a = 1000$, SNR=20dB and $\alpha = 0.01$. We notes that despite the time-varying nature of the channel, the designed ZF equalizer is quite efficient in restoring the original 16-QAM symbol constellation.

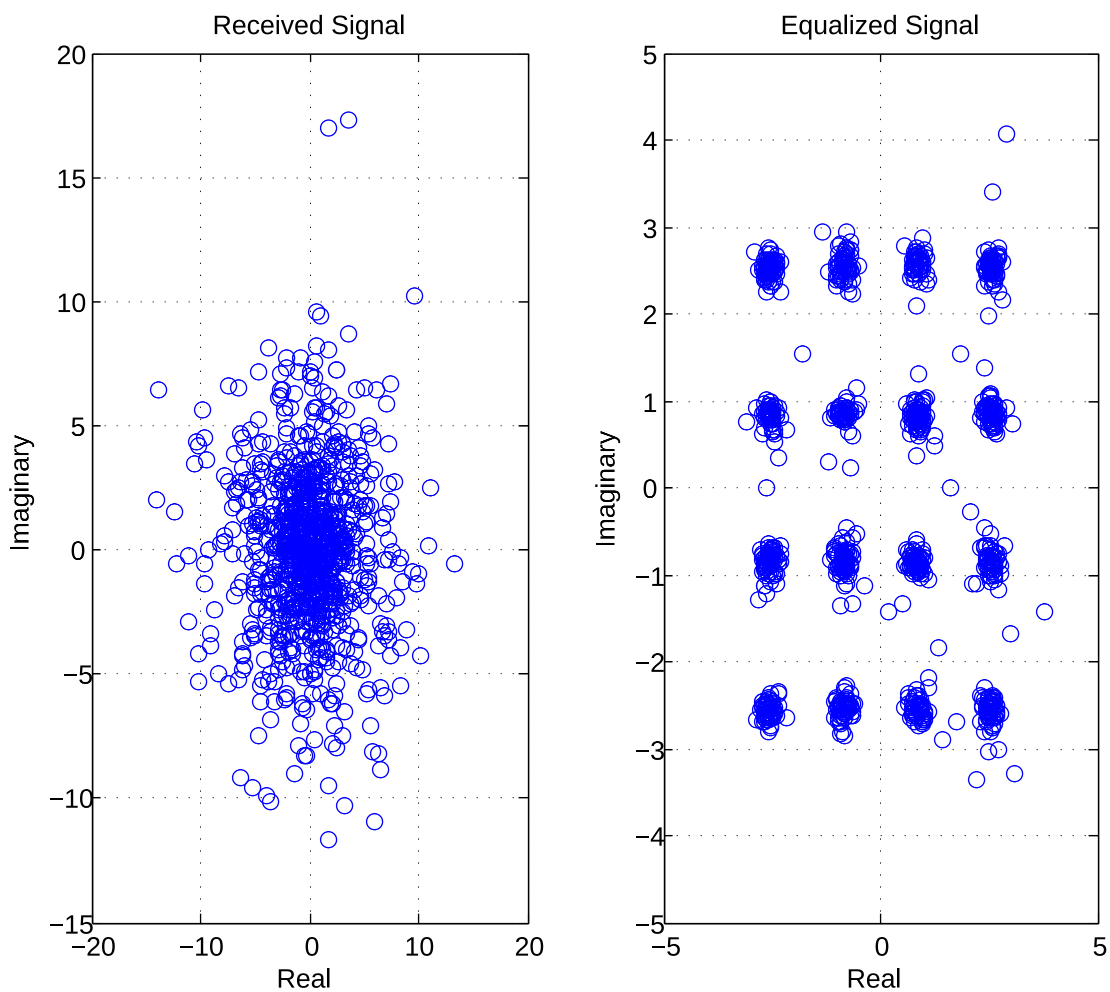

Fig. 5.4 Distribution of the received and equalized signal samples ($N_a = 1000$, $\alpha = 0.01$, and SNR=20dB).

In Fig. 5.5, SER is plotted as the function of integration time N_a for different SNR, i.e. 10dB, 20dB and 30dB and for $\alpha = 0.01$. In Fig.5.6, the ISI measure is investigated

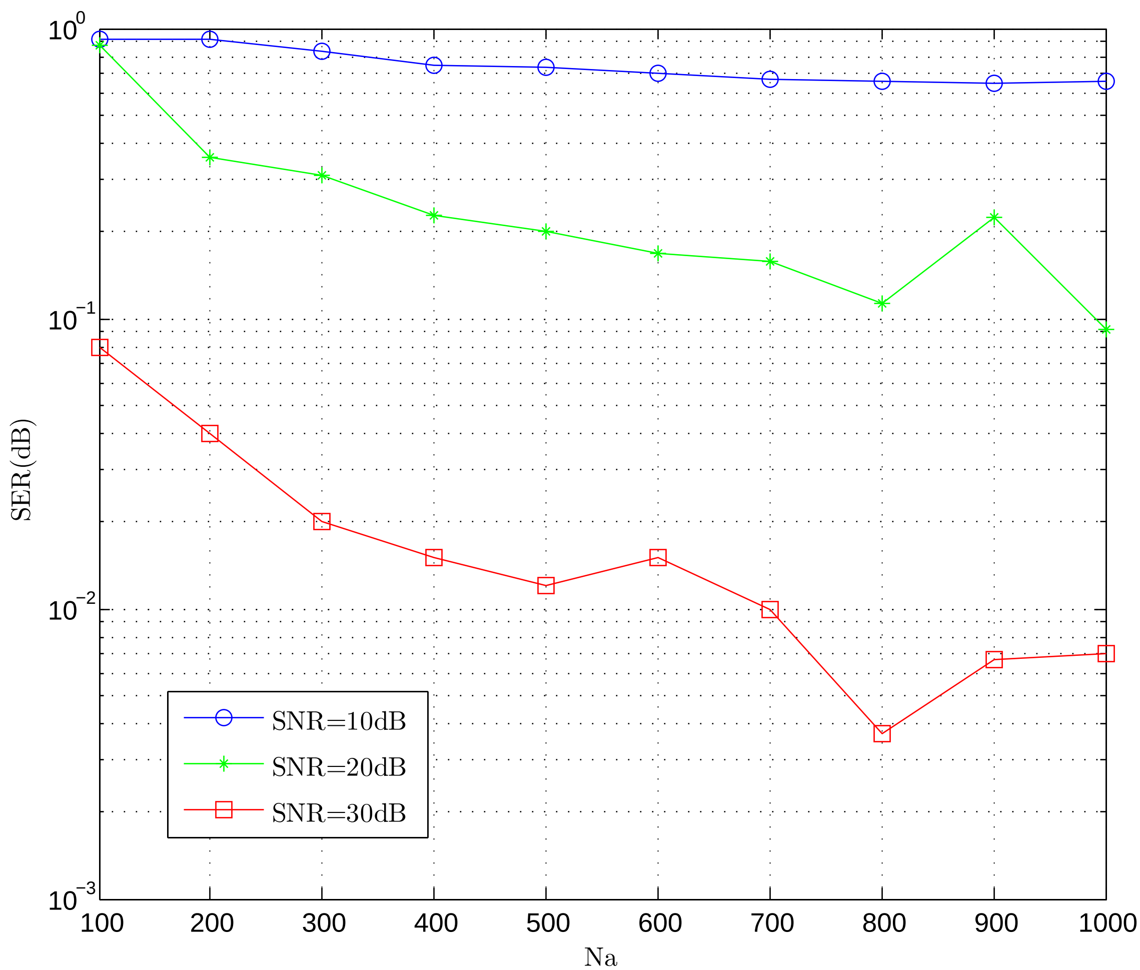

Fig. 5.5 SER versus N_a of the blind subspace equalization.

as a function of SNR for different integration times, i.e. N_a= 500, 1000 and 2000, and for $\alpha = 0.01$. Here, we note SER and ISI of the equalized symbols can be improved by using longer integration time, or choosing a strong signal strength, i.e. higher SNR.

Fig.5.7 shows distribution of equalized symbols with and without ambiguity matrix determination. The system parameters are $N_a = 1000$, SNR=20dB and $\alpha = 0.01$. We note the use of the ambiguity matrix obtained with the algorithm in Table 4.3 revises the distribution of the equalized signal. In short, the performances of estimation and

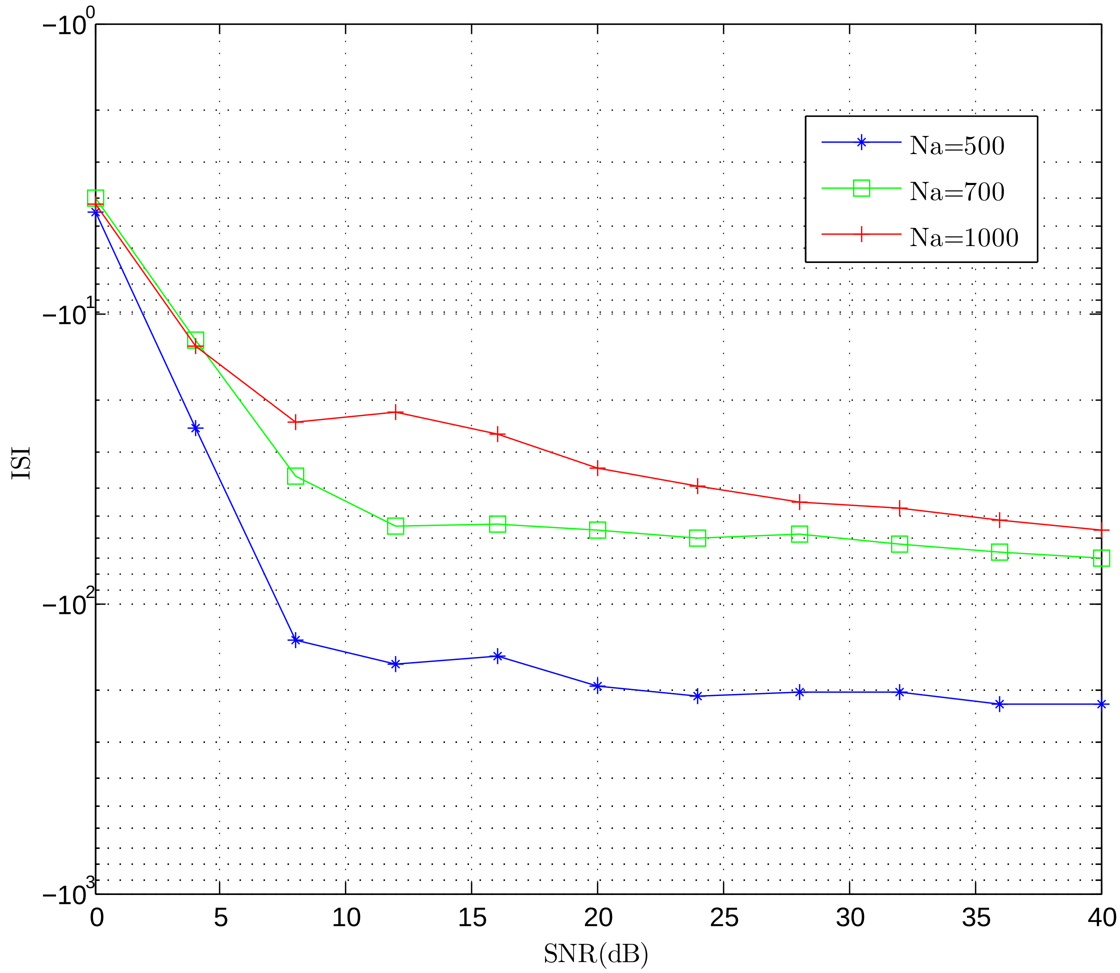

Fig. 5.6 ISI versus SRN of the blind subspace equalization.

equalization are improved as well as the longer integration time.

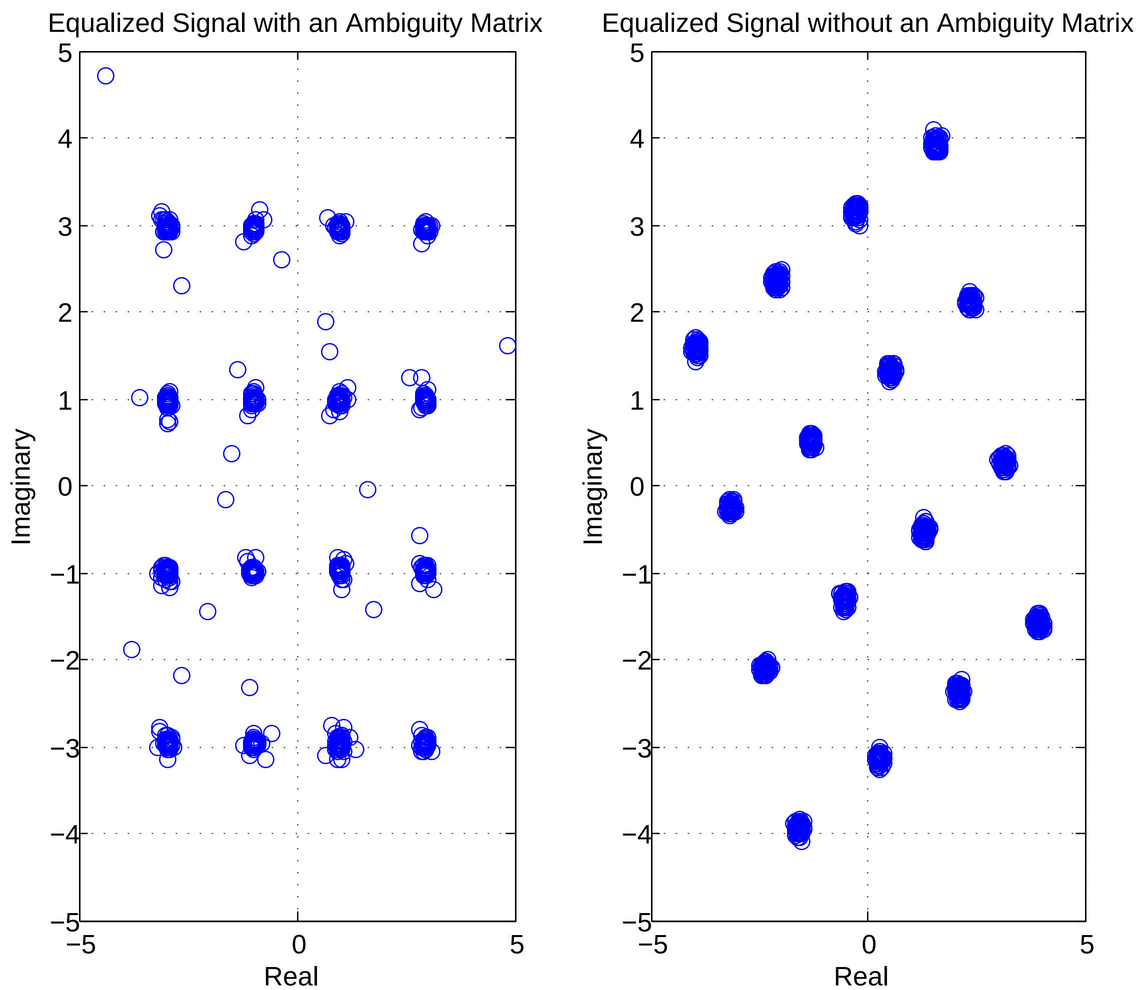

Fig. 5.7 Distribution of the equalized symbols with and without an ambiguity matrix.

Finally, the SER versus SNR is shown in Fig. 5.8. It is seen that the use of the properly determined ambiguity matrix makes it possible to decode the transmitted source symbols.

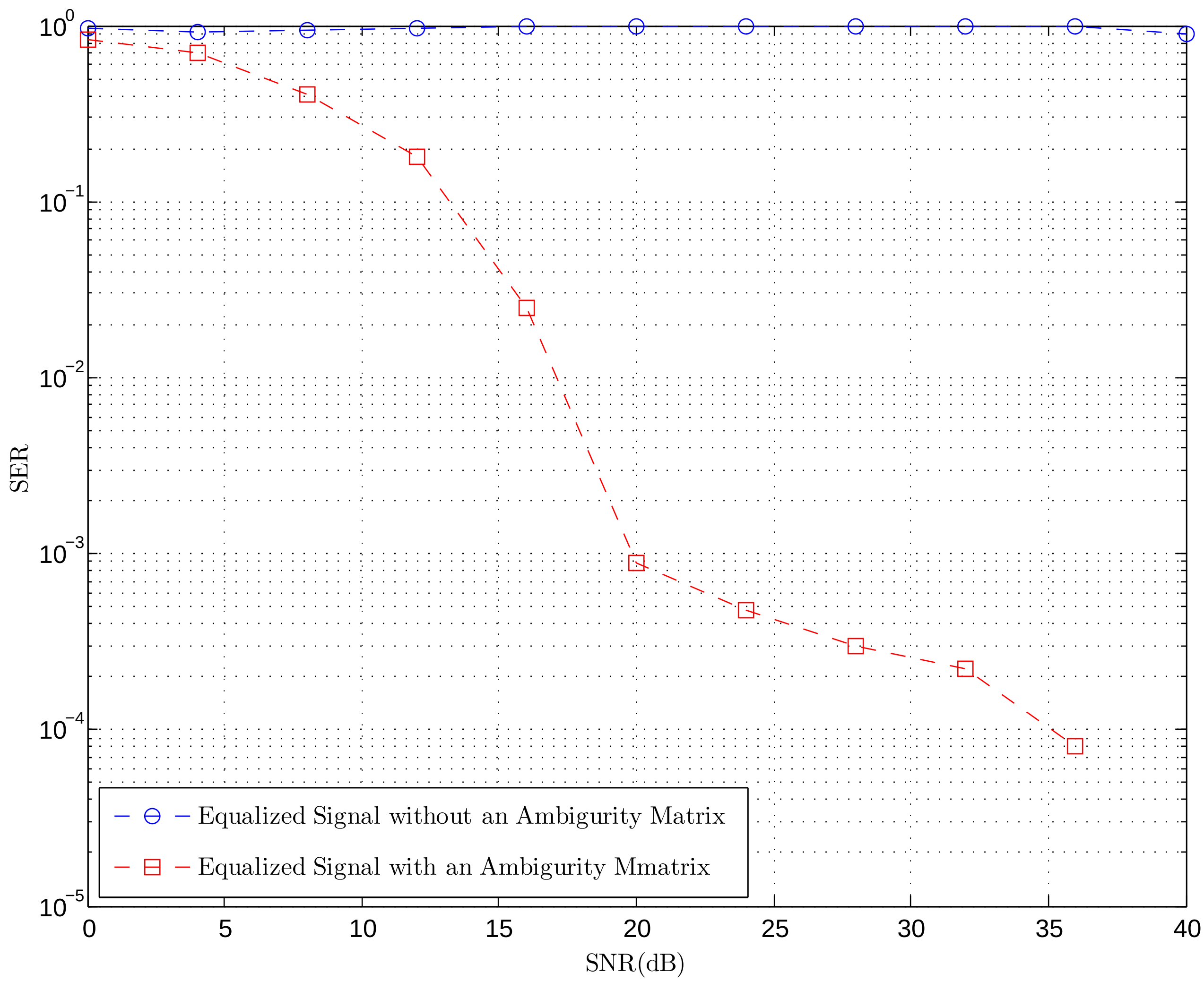

Fig. 5.8 SER versus SNR of the blind subspace equalization.

Chapter 6

Summary and Conclusion

In this thesis, we have studied the blind subspace channel estimation of multiple TV SIMO channels without *a priori* estimation of frequency parameters. This was made possible by the use of a linear expansion in terms of arbitrary, albeit slowly-varying basis functions. Based on its results, we have developed a ZF equalizer for the TV SIMO system, and a novel approach was presented to determine the ambiguity matrix, the later being used to control the phase shift and scaling distortion in the detected symbol constellation. Here, we summarize the main ideas and results obtained in this thesis and present our conclusions.

6.1 Summary of Thesis

In Chapter 2, the classical subspace approach for the blind estimation of multi-path (FIR) TI SIMO channel was reviewed. In this approach, the orthogonality between the signal and noise spaces is exploited to identify the channel impulse response under the SOS framework. The basic principles of blind channel equalization for both SISO and MIMO systems were then exposed.

In Chapter 3, based on the work [19], an equivalent TI MIMO system was introduced as a substitute to the TV SIMO system, by expanding the TV channels as linear combinations of arbitrary basis functions with expansion coefficient vectors associated to the different multi-path lags. The relationship between the TV SIMO system and the equivalent TI MIMO system was investigated. The unknown expansion coefficient vectors were identified by the application of a subspace method to the equivalent TI MIMO system. Then, a indirect ZF equalization approach was proposed to recover the received signal symbols based on the identified TV channel impulse responses.

In Chapter 4, the necessary and sufficient conditions were investigated so that the TI MIMO FIR channel can be identified up to an ambiguity matrix under the SOS framework. Following this, an approach for determining the ambiguity matrix was proposed by solving an overdetermined system using the LS method.

In Chapter 5, the simulation methodology was introduced. Then, the simulation results were presented and analyzed for three tasks of blind channel identification, ZF channel equalization and ambiguity matrix determination. The results so obtained provided good support for the theoretical algorithm derivation.

6.2 Conclusion

To deal with the time variability of impulse response in SIMO system, the TV vectors channel was represented as linear combination of known, but arbitrary basis functions associated with time varying property of channels, with the corresponding unknown expansion coefficient vectors associated with different multi-path delays. The indirect ZF equalization was developed from the SOS-based expansion coefficients identification. The difference between the new algorithm and other proposed linear equalization algorithms is that in the latter, the basis functions must be of the complex type exponential, while here this restriction does not apply, and more general types of TV models for the channel impulse response can be used.

The simulation results show that the proposed blind channel equalization ZF algorithm can effectively recover the transmitted source signal symbols. In particular, the proposed approach for the determination of the ambiguity matrix has been shown to be effective in removing phase shift and other distortion, and restoring the original signal constellation in the equalized output. The use of the proposed blind ZF equalization with ambiguity matrix determination helps to significantly improve the SER of the wireless system, so that adequate link quality can be maintained over TV SIMO channels.

Future possible work includes the analytical characterization of the detection performance of the equalized output and the extension of the proposed indirect equalization approach into a direct one, i.e. in which the TV SIMO channel parameters need not to be estimated explicitly.

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