HYBRID LINEAR/QUADRATIC TIME-FREQUENCY ATTRIBUTES

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ABSTRACT

We present an efficient method for robustly calculating time-frequency attributes of a signal, including instantaneous mean frequency, bandwidth, and kurtosis. Most current approaches involve a costly intermediate step of computing a (highly oversampled) 2-d bilinear time-frequency representation (TFR), which is then collapsed to the 1-d attribute. Using the principles of hybrid linear/bilinear time-frequency analysis, we propose computing attributes as nonlinear combinations of the (barely oversampled) linear Gabor transform of the signal. The method is both computationally efficient and accurate – it performs as well as the best bilinear techniques based on adaptive TFRs. To illustrate, we calculate an attribute of a seismic cross-section.

1. INTRODUCTION

Time-frequency attributes of signals have provided insights in fields as disparate as seismic analysis and neurophysiology. Such attributes include instantaneous frequency and bandwidth, kurtosis (normalised fourth-order instantaneous frequency moment), and time-frequency localised energy. Instantaneous frequency (IF) of a mono-component real signal is defined as the rate of change of the phase of the corresponding analytic signal at a given time. This is equivalent to the first moment in frequency of the Wigner distribution of the signal normalised by its instantaneous energy. When we try to estimate instantaneous frequency either directly from the phase of the analytic signal or from the Wigner distribution, we generate estimates with high variance, particularly in noisy environments [1].

Estimates based on the first moment in frequency of other bilinear TFRs (the spectrogram, for instance) reduce the variance at the expense of bias. The peak of the Wigner distribution or another suitable TFR offers an attractive alternative estimate. The variance of estimates based on TFR peaks is reduced compared to moment-based methods, especially in low SNR environments. In particular, iterative methods involving repeated calculation of TFRs have proven extremely effective [1, 2, 5].

There are instances, however, where the peak-based estimate is unsatisfactory. When multiple components are present in a signal, it is more desirable to calculate the moment-based estimate of instantaneous frequency. Such is the case in the seismic analysis presented in Section 5. We then need a high resolution, signal adaptive bilinear TFR to make the IF estimate accurate. Unfortunately, generating such TFRs is computationally expensive, especially when we will merely collapse the 2-d TFR back to the 1-d attribute.

In this paper, we present a very efficient method for estimating the time-frequency attributes that avoids the calculation of a bilinear TFR altogether. Our method makes use of hybrid linear/bilinear time-frequency representations [3, 4]; the generation of the estimates requires only a sparse linear decomposition of the signal. The paper presents a general procedure for calculating a range of attributes, but focuses in particular on the estimation of instantaneous frequency in order to illustrate the approach.

Section 2 reviews hybrid time-frequency representations, and Section 3 examines the IF and its estimation. Section 4 presents the new estimation technique, and Section 5 discusses its application to seismic analysis. We make some concluding remarks in Section 6.

2. HYBRID TIME-FREQUENCY REPRESENTATIONS

Qian and Chen [4] proposed a method for time-frequency analysis that utilises both a linear TFR (the Gabor transform) and a bilinear TFR (the Wigner distribution) to generate a signal adaptive bilinear TFR. The Gabor transform represents the signal $x(t)$ in terms of time-frequency atoms:

$$x(t) = \sum_{(m,n)} c_{m,n}\phi_{m,n}(t), \quad (m,n) \in \mathbb{Z}^2, \tag{1}$$

$$c_{m,n} = \left\langle x, \widetilde{\phi}_{m,n} \right\rangle = \int_t x(t)\widetilde{\phi}^*_{m,n}(t)\,dt. \tag{2}$$

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The synthesis atoms are generated by time-frequency shifting a prototype atom $g(t)$:

$$\phi_{m,n}(t) = e^{j2\pi m f_k t} g(t - nt_k), \qquad t_k f_k \leq 1. \quad (3)$$

A natural choice for the prototype is the Gaussian, because it has optimal concentration and localisation in time-frequency and a strictly positive Wigner distribution.

Using the Gabor representation of the signal, the Wigner distribution can be decomposed:

$$\begin{aligned} W_x(t,f) &= \sum_{(m,n)} |c_{m,n}|^2 W_{\phi_{m,n}}(t,f) + \\ &\sum_{(m,n)\neq(m',n')} c_{m,n}c^*_{m',n'} W_{\phi_{m,n},\phi_{m',n'}}(t,f). \quad (4) \end{aligned}$$

This expression identifies two contributions to the generation of the Wigner distribution. The first summation in (4) represents a linear sum of the strictly positive WDs of the time-frequency atoms; it provides an approximate description of the signal's time-frequency behaviour without capturing detail. The second summation arises due to the interaction between separate atoms in the Gabor decomposition. The terms in this summation may be positive or negative.

The key observation is this: interactions between closely spaced atoms generally serve to refine the auto-components of the signal in the representation, whereas interaction between distant atoms gives rise to global interference terms.

Qian and Chen generate a "hybrid TFR" by retaining all of the terms in the first summation, but only those terms arising from closely spaced atoms in the second. Due to the elliptical or circular symmetry of the Gaussian, a Euclidean distance metric is the most natural measure of atom separation; an l_∞ metric (Manhattan distance) $d[(m,n),(m',n')] = |m-m'| + |n-n'|$ is a good approximation. A threshold distance δ is chosen, and terms in the second summation are only included in the final representation if the distance between the interacting atoms is less than the threshold. The hybrid method thus produces a δ-parameterised class of TFRs:

$$\begin{aligned} \widetilde{W_x}^{(\delta)}(t,f) = \sum_{(m,n)} |c_{m,n}|^2 W_{\phi_{m,n}}(t,f) + \\ \sum_{0<d[(m,n),(m',n')]<\delta} c_{m,n}c^*_{m',n'} W_{\phi_{m,n},\phi_{m',n'}}(t,f). \quad (5) \end{aligned}$$

The parameter δ controls the trade-off between component resolution and interference. By a suitable selection of δ, high resolution TFRs are generated; moreover they can be determined at a much lower computational expense than the signal adaptive bilinear TFR approaches that display similar performance. The Wigner distributions in (5) are signal-independent and can be stored in memory; the computational demands of the Gabor transform are much less than those of bilinear TFRs. By replacing the Gabor decomposition with a wavelet transform, we can generate high-resolution, low interference time-scale representations [3].

3. INSTANTANEOUS FREQUENCY

The instantaneous frequency (IF) $\nu(t)$ and instantaneous amplitude $a(t)$ of a real-valued signal $s(t)$ can be defined as a unique pair by reference to the analytic signal $z_s(t)$:

$$\begin{aligned} a(t) &= |z_s(t)| \\ \nu(t) &= \frac{1}{2\pi}\frac{d}{dt}\arg z_s(t). \quad (6) \end{aligned}$$

One means of estimating the IF of a discrete signal is immediately clear: (6) can be applied directly. Unfortunately, such an estimate is extremely susceptible to noise.

More robust estimates can be formed by considering the relationship between the Wigner distribution and instantaneous frequency

$$\nu(t) = \frac{\int_{-\infty}^{\infty} f W(t,f)\, df}{\int_{-\infty}^{\infty} W(t,f)\, df}. \quad (7)$$

Direct utilisation of this formula again leads to high-variance estimates in noisy environments, but bias can be traded against variance by formulating the estimated IF using smoother TFRs than the Wigner distribution. To obtain a good trade-off and robustly track the instantaneous frequency of signals whose time-frequency behaviour is unknown and variable, we generally have to determine extremely computationally intensive signal-adaptive TFRS. Whatever TFR is chosen, computational effort is wasted because we generate a highly-redundant two-dimensional description of the time-frequency nature of the signal before collapsing back to a one-dimensional quantity.

4. HYBRID INSTANTANEOUS FREQUENCY CALCULATION

In this section we detail how the hybrid TFR approach of Section 2 can be adopted to robustly estimate instantaneous frequency. The approach requires the much smaller computational expense of determining a linear, mildly oversampled representation of the signal (the Gabor expansion).

Substituting the Gabor expansion of the signal, we can rewrite the numerator of (7) as

$$\sum_{(m,n,m',n')} c_{m,n}c^*_{m',n'} \int f W_{\phi_{m,n}}(t,f)\, df.$$

When we use a Gaussian of variance σ^2 as the Gabor synthesis atom, closed form expressions can be developed for the integrals in the summation. Applying the same expansion to the denominator, we can express (7) as

$$\nu(t) = \frac{\sum_{m,n,m',n'} c_{m,n}c^*_{m',n'} B_{m,n,m',n'}(t)}{\sum_{m,n,m',n'} c_{m,n}c^*_{m',n'} A_{m,n,m',n'}(t)}$$

where

$$A_{m,n,m',n'}(t) = \frac{1}{2\pi}\exp\left(-j2\pi(n-n')f_k t\right) \times \exp\left(-\frac{(t-mt_k)^2+(t-m't_k)^2}{2\sigma^2}\right)$$

and

$$B_{m,n,m',n'}(t) = \left[\frac{-j(m-m')t_k}{4\pi\sigma^2} - \frac{(n+n')f_k}{2}\right] A_{m,n,m',n'}(t).$$

If all (m,n,m',n') combinations are included in the estimate then it is equivalent to the (high-variance) Wigner distribution estimate. By adopting the same approach as in the generation of the hybrid TFR, i.e., retaining only a subset of the terms when $(m,n) \neq (m',n')$, we can reduce the variance with the introduction of some bias. The performance is of a similar level to that obtained when using adaptive bilinear TFRs; however, the computational expense is much reduced, because the $A_{m,n,m',n'}(t)$ and $B_{m,n,m',n'}(t)$ are signal-independent and can be precomputed and stored in memory. The primary running cost of the algorithm is then that of computing the (barely oversampled) Gabor transform.

5. APPLICATION TO SEISMIC ANALYSIS

Seismic imagery of the subsurface is critical to all aspects of the oil and gas exploration and production process – from the location of reserves to their appraisal and subsequent monitoring. In oil and gas exploration, seismic cross-sections of the earth's subsurface are scrutinized by interpreters who search for features that indicate possible hydrocarbon reservoirs. While previously interpreters dealt with large plots of 2D cross-sections, they now work on computers with 3D volumes comprising gigabytes of data.

Seismic attributes aid the interpretation of seismic data by bringing forward the salient characteristics from the signal. Traditionally complex-trace analysis is used for attribute extraction [7]. The standard seismic attributes are the instantaneous amplitude, phase, and frequency of the complex signal. Changes in interference patterns that are associated with changes in subsurface stratification become apparent in the instantaneous frequency cross-sections. In some cases, the presence of oil is associated with an anomalously low reflection frequency, which may be detected as a shift in the instantaneous frequency of the seismic data trace.

When estimated using time-frequency peak methods, the instantaneous frequency estimate does not display sufficient continuity in space to aid interpretation of the data. The instantaneous frequency estimated using the first moment of a TFR provides a much more informative portrayal of the data [6].

Figure 1 shows a comparison of instantaneous frequency estimation from three different TFRs of a seismic signal. The IF estimate that is obtained by the fast hybrid TFR approach is close to the result that is obtained using the AOK TFR [6]. The time resolution of the IF estimate using the spectrogram is significantly poorer than that of the other two methods; moreover, a substantial smoothing is noticeable where rapid changes occur, e.g., at $t = 0.4$ s.

Figure 2 shows a seismic section, and the extracted IF estimates plotted in columns as grey-level intensities. Closely spaced seismic reflectors are indicated by relatively high instantaneous frequency. Lateral changes in reflector spacing and frequency are more easily detected in the instantaneous frequency cross-section than in the raw data.

6. CONCLUSIONS

We have presented a robust and efficient method for estimating the time-frequency attributes of a signal. Using hybrid linear/bilinear time-frequency concepts avoids the expensive calculation of a bilinear TFR. While we have focused on estimating instantaneous frequency, closed form expressions analogous to (7) can be developed for many other attributes, including instantaneous bandwidth, kurtosis, and so on. Our results have particular relevance for seismic data analysis, where vast volumes of data require processing, and it is critical that efficient algorithms be used.

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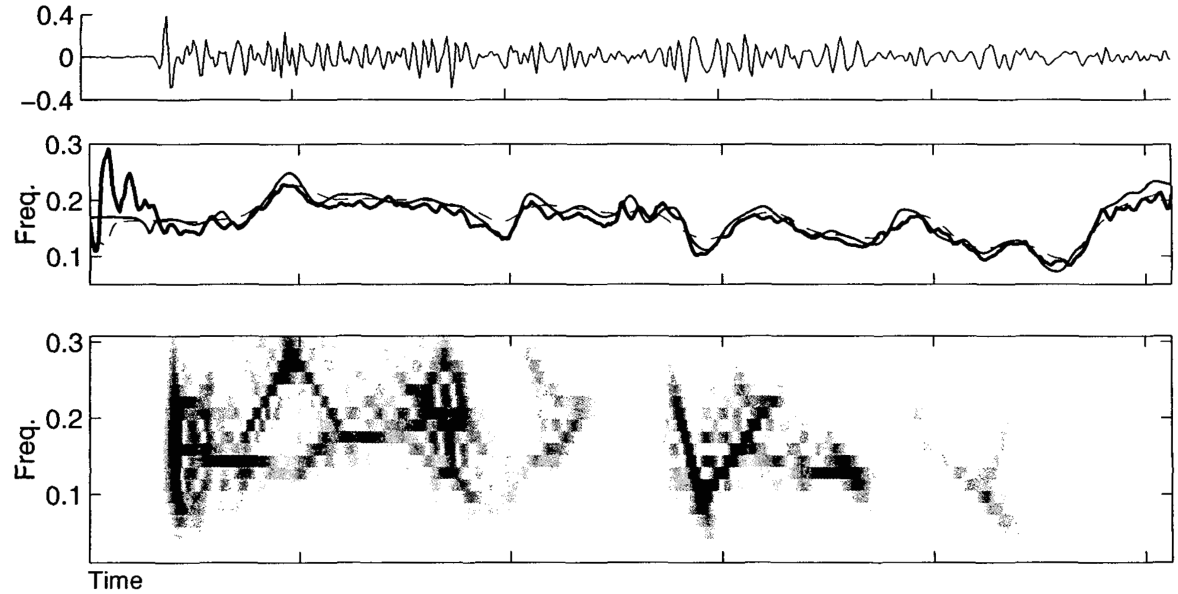

Figure 1: (top) Seismic recording, (middle) normalized instantaneous frequency. Thin line is hybrid IF estimate, thick line adaptive TFR estimate [6], and dashed line is spectogram estimate. (bottom) Adaptive TFR of the seismic trace.

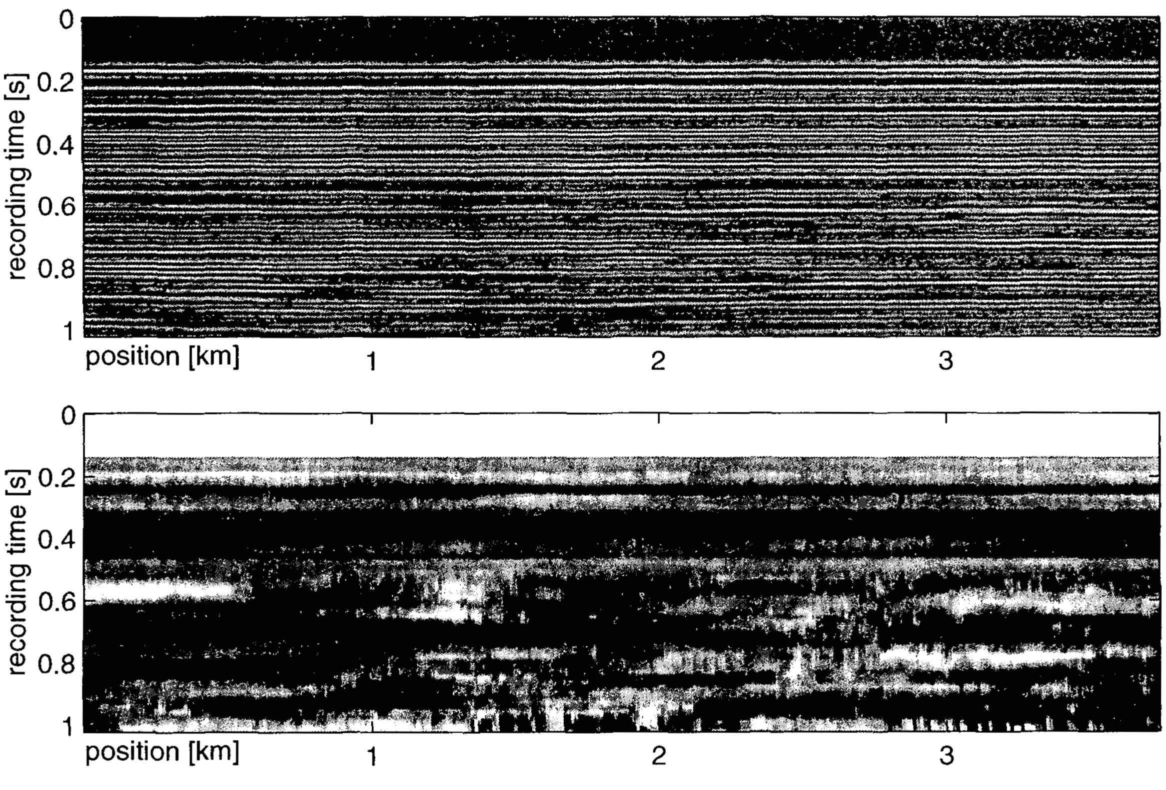

Figure 2: (top) Seismic cross-section, (bottom) hybrid TFR IF estimate