NON-REDUNDANT, DIRECTIONALLY SELECTIVE, COMPLEX WAVELETS

Rutger van Spaendonck†*, Felix C. A. Fernandes, Mark Coates and C. Sidney Burrus*

Department of Electrical and Computer Engineering, Rice University, Houston, TX 77251-1892, USA
† Applied Earth Sciences, Delft University of Technology, Delft, Netherlands

ABSTRACT

Poor directional selectivity, a major disadvantage of the 2D separable discrete wavelet transform (DWT), has heretofore been circumvented either by using highly redundant, nonseparable wavelet transforms or by using restrictive designs to obtain a pair of wavelet trees. In this paper, we demonstrate that superior directional selectivity may be obtained with *no* redundancy in *any* separable wavelet transform. We achieve this by projecting the wavelet transform coefficients onto the Softy space of signals and decimating before processing. A novel reconstruction step guarantees perfect reconstruction within this critically-sampled framework.

1. INTRODUCTION

The separable 2D DWT is a powerful image-processing tool, but in some applications its poor directional selectivity is a serious disadvantage. The transform can only distinguish between three different orientations of spatial features. Nonseparable 2D DWTs can provide directional selectivity [1, 2], but engender complicated design problems and are more computationally expensive. Kingsbury's dual-tree complex discrete wavelet transform (CDWT) [3] is separable and has impressive directional selectivity due to an approximate quadrature relationship between its trees. However, Kingsbury's transform involves a redundancy of $2N$ in N dimensions, and neither tree in the dual-tree structure genuinely corresponds to a wavelet transform. Enforcing the quadrature relationship between the two trees is complicated, impeding the incorporation of other wavelet criteria in the design.

In this paper, we propose a separable wavelet transform that provides comparable directional selectivity to Kingsbury's transform, but introduces *no* redundancy. The method involves the application of projection filters to discriminate between positive and negative frequency components of 2D separable DWT wavelet coefficients. This results in complex-valued coefficients. Decimation of these coefficients preserves directional selectivity but makes the transform non-redundant. If the projection filters satisfy certain criteria, then we can achieve perfect reconstruction using a novel synthesis filter bank structure.

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Email: felixf@rice.edu, csb@rice.edu
Web: http://www.dsp.rice.edu

2. HARDY AND SOFTY SPACES

The Hardy space projection of a function $x \in L^2(R)$ is obtained by convolving x with a filter whose frequency response is the characteristic function of the half line $[0, \infty)$. If we were able to implement such a filter and apply it to the coefficients of a separable DWT, then we would achieve extremely good directional selectivity, and perfect reconstruction would be a simple matter. But little is easy in a complex world, and the ideal filter is unimplementable. Instead, we design projection filters whose frequency response approaches the ideal asymptotically, as the filter length increases. These projection filters can only suppress negative frequencies instead of excising them. The filters project onto a space of signals whose Fourier transforms have attenuated negative frequencies. We coined the term "Softy space S_h" to describe the space obtained by projecting $L^2(R)$ signals using a given projection filter h.

3. PERFECT RECONSTRUCTION

In this section, we demonstrate how to reconstruct the wavelet coefficients in a subband from their projection onto Softy space, since this is essential for perfect reconstruction of the original signal. We consider separately the cases for real- and complex-valued inputs to the projection filter since they lead to different synthesis filter-bank structures for non-redundant projection.

Theorem 1 *Let h^0 be the lowpass analysis filter of a two-band, real-coefficient, paraunitary filter bank such that h^0 has nonzero polyphase components h^0_e, h^0_o. Create a projection filter h^+ by frequency shifting the lowpass transform H^0 by $\pi/2$, so that $H^+(z) = H^0(-jz)$.*

(1) If $x \in \mathcal{L}^2(\mathcal{R} \to \mathcal{R})$ then $\hat{x} = x$ in Figure 1. (2) If $x \in \mathcal{L}^2(\mathcal{R} \to \mathcal{C})$ then the 2-band paraunitary filterbank (h^+, h^-, f^+, f^-) generated from h^+ perfectly reconstructs x.

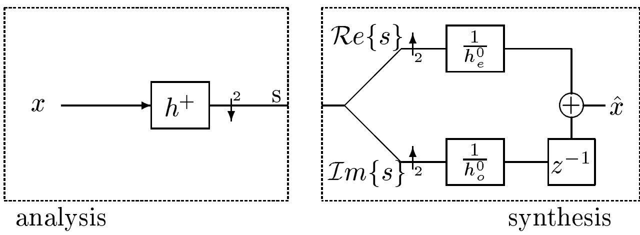


Figure 1: Reconstruction of real-valued input projected onto Softy space.

Proof: We have $H_e^+(z) = H_e^0(-z)$ and $H_o^+(z) = jH_o^0(-z)$. Let $\downarrow h^+$ denote the linear transform that maps the input x to its projection s. Then $S(z) = H_e^+(z)X_e(z) + z^{-1}H_o^+(z)X_o(z)$ and the kernel of the transform is $\ker(\downarrow h^+) = \{X(z) : H_e^0(-z)X_e(z) = -jz^{-1}H_o^0(-z)X_o(z)\} = \{0\}$. Therefore, the mapping $\downarrow h^+$ is invertible. The inverse mapping to $\hat{x}$ is given by $\hat{x} = \mathcal{R}e\{S(z^2)\}/H_e^0(-z^2) + z^{-1}\mathcal{I}m\{S(z^2)\}/H_o^0(-z^2)$. This proves (1). Implication (2) follows from the definition of a paraunitary filterbank. □

For practical reasons, the design of H^0 must ensure that the IIR filters $1/H_e^+(z^2)$ and $1/H_o^+(z^2)$ are stable and have rapidly decaying impulse responses.

4. DIRECTIONAL SELECTIVITY

We first review the directional selectivity provided by a separable, real-valued DWT and then explain the improvement induced by Softy space projection.

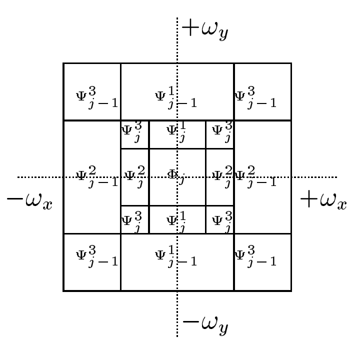


Figure 2: Frequency-domain energy localization of tensor wavelets in a two-level 2D DWT.

Figure 2 shows the Fourier-domain partitioning obtained using a separable, real-valued, 2-level, 2D DWT. The tensor wavelet $\psi_{j-1}^3(x,y)$ concentrates its energy in a region of the Fourier domain associated

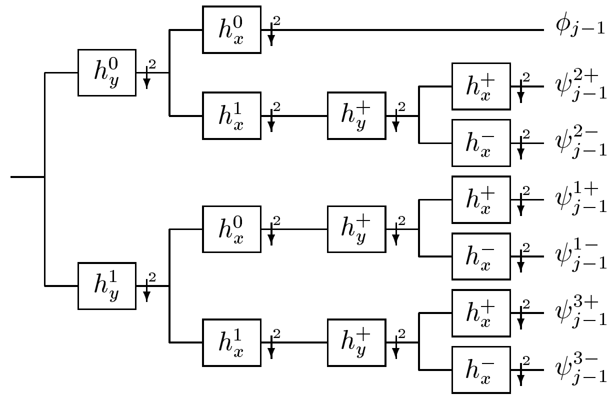


Figure 3: Analysis filter-bank structure for separable 2D DWT with Softy space projections. The y (x) subscripts indicate filtering along the columns (rows) of an image. The $+$ $(-)$ superscripts indicate filters that project onto the Softy space (or its orthogonal complement in $L^2(\mathcal{R})$).

with diagonally-oriented spatial features at scale $j-1$. Therefore high energy at the output of the filter in the subband associated with $\psi_{j-1}^3(x,y)$ indicates the presence of this class of features. The ψ_{j-1}^3 blocks in the upper-left and lower-right corners indicate features at -45 degrees, while those in the upper-right and lower-left corners indicate features at $+45$ degrees. Since all four blocks are associated with one filter, we cannot differentiate between these two orientations.

Subband projection onto Softy space provides improved directional selectivity by approximately decoupling the positively- and negatively-oriented blocks associated with each filter. Passing an input image through the filter bank in Figure 3 results in an approximation to the ideal partitioning of the Fourier plane displayed in Figure 4. We can now distinguish between features oriented in 6 directions at any particular scale. In addition, Softy space projection applied to the ϕ_{j-1} subband decouples it into ϕ_{j-1}^- and ϕ_{j-1}^+ blocks that discriminate between low-scale features oriented at -45 and +45 degrees.

5. RESULTS

To verify the improvement in directional selectivity, we used the zone-plate image displayed in Figure 5. It contains features oriented in all possible directions with low-scale features in the center and high scale features at the periphery. The separable, 2D, length-4, single-level Daubechies DWT of the image is displayed in the same figure. The wavelet coefficients for both the $+45$ and -45 degree orientations appear in the diagonal-orientation subband and cannot be distinguished. The transform also suffers from severe aliasing artifacts.

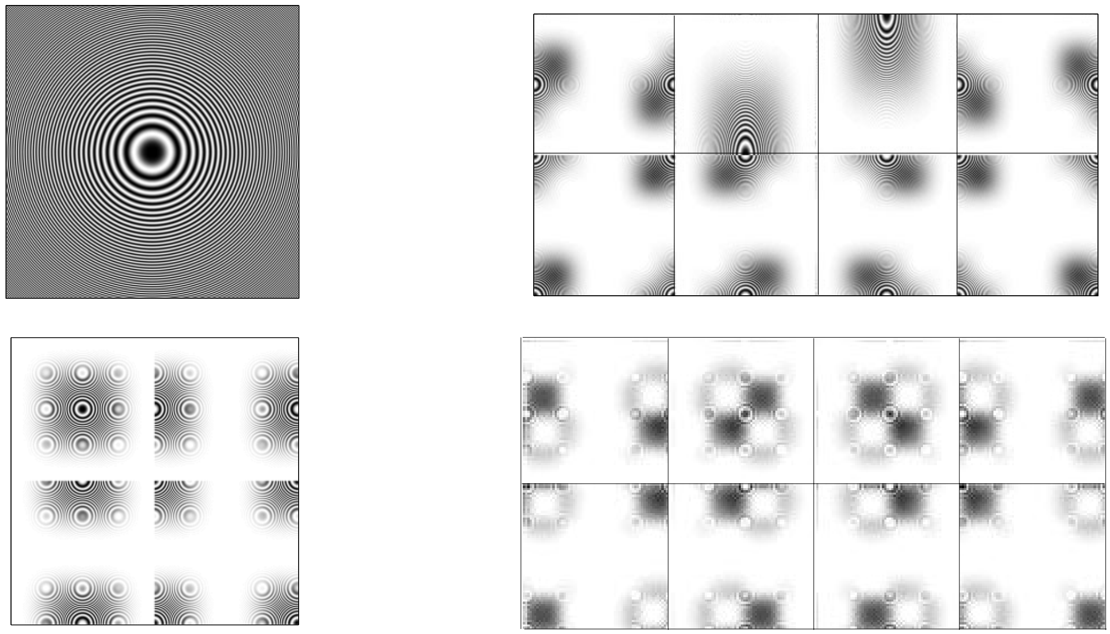


Figure 5: **Upper Left:** Zone-plate Image. **Lower Left:** 2D Daubechies DWT. Clockwise from upper-right block: vertical-, diagonal-, horizontal-orientation subbands and lowpass subband. **Upper Right:** Dual-tree DWT. Clockwise from upper-rightmost block: -75, +45, -15, +15, -45, +75 degree highpass subband, upper half of lowpass subband, lower half of lowpass subband. **Lower Right:** DWT with Softy space projection. Clockwise from upper-rightmost block: -75, +45, -15, +15, -45, +75 degree highpass subbands, -45, +45 degree lowpass subbands. Length-4 Daubechies filters are used for the wavelet decomposition and as the basis for the projection filter, since these filters meet the criteria of Section 3.

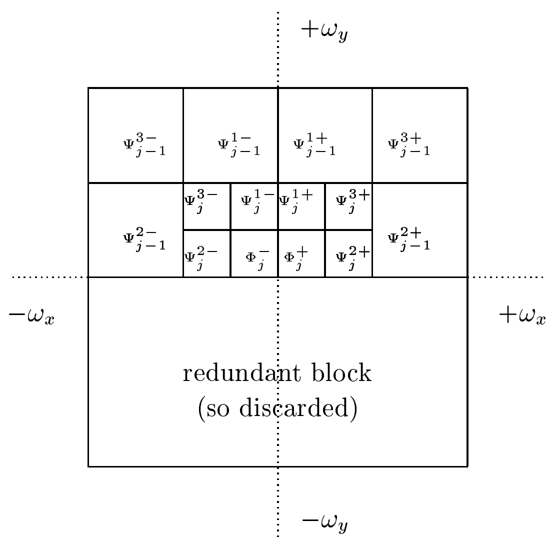


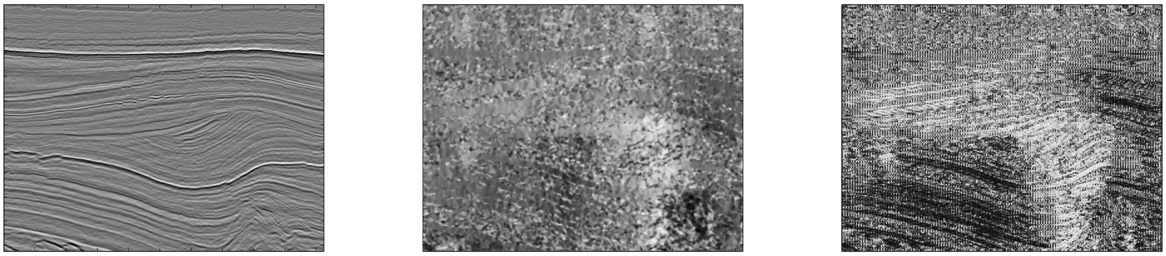
Figure 4: Fourier domain partitioning after Hardy space projection.

Figure 5 also shows the 2D, single-level DWT obtained using Kingsbury's dual-tree CDWT and presents the 2D, single-level DWT obtained using the filter-bank structure of Figure 3. Both transforms provide six highpass directional subbands. In addition, our transform discriminates between features oriented at +45 and -45 degrees in the lowpass subband. Although our CDWT uses length-4 filters and the dual-tree CDWT uses length-10 filters, the features are better localized in each subband of the former. We emphasize that the dual-tree transform has a redundancy of four whereas our transform is critically sampled.

6. SEISMIC ATTRIBUTE ANALYSIS

Seismic imagery of the earth's subsurface plays a critical role in all aspects of oil and gas exploration and production — from the location of reserves to their appraisal and subsequent monitoring. In oil and gas exploration, seismic cross-sections are scrutinized by interpreters who search for features that indicate possible hydrocarbon reservoirs. Previously, interpreters dealt with large plots of 2D cross sections; they now work on computers with 3D volumes comprising gigabytes of data. Local signal attributes aid the interpretation of seismic data, elucidating its salient characteristics.

A particularly useful attribute is the local angle (dip) of the reflecting surface. Dip representations enable 3D interpretation of *structures* using seismic depth-slices. Channels and faults appear as dip varia-



(a) (b) (c)

Figure 6: (a) Seismic section showing moderate angle variations (b) Angle analysis results using complex steerable pyramid. (c) Angle analysis results using nonredundant complex wavelet transform. In (b) and (c), intensity represents angle, black (-90 degrees) through white (+90 degrees).

tions, but are often barely visible in amplitude slices.

Previously, we used the complex steerable pyramid (CSP [1]) to develop attribute representations that provide very accurate angle indications for 2D cross-sections [4]. However, the overwhelming redundancy of the pyramid representation prohibits its application to 3D seismic volumes because of computational expense.

We now outline a method for developing local angle representations for 2D seismic images using the *nonredundant* CDWT (see [5] for details). Firstly, we apply the nonredundant transform to the seismic image, using different length filters in the vertical and horizontal directions. A short Daubechies-4 wavelet in the vertical direction permits perfect reconstruction using filters that satisfy the constraints of Section 3. In the horizontal direction, both positive and negative frequencies are retained (see Figure 3), so longer (Daubechies-20) wavelets can be used. The longer wavelets suppress aliasing and improve differentiation between angles by enhancing the distinction between positive and negative lateral frequencies.

In the second stage of the method, we perform six separate reconstructions by considering each angle in isolation. Each reconstructed image reflects the extent to which the seismic cross-section is oriented in the corresponding direction. We estimate the local dip as the weighted average of the six angles.

Figure 6 compares the angle representations generated for a seismic cross-section. The geometry in the section (Figure 6(a)) shows a structure with moderate angle variations. The CSP angle representation (6(b)) provides an accurate and smooth representation of local dip. However, the nonredundant transform generates an almost equally informative representation with a finer resolution (although slightly noisier and less smooth) and requires *14 times* less redundancy.

7. CONCLUSION

This paper demonstrates that projection onto Softy space significantly improves the directional selectivity of *any* DWT. Our method partitions highpass coefficients into six directional subbands at each scale, and lowpass coefficients into two directional subbands. A novel synthesis filter bank structure guarantees perfect reconstruction in a non-redundant framework.

We have demonstrated the value of the nonredundant CDWT in seismic attribute analysis. In this paper we only outline a procedure for analysing 2D seismic images. The absence of redundancy in the transform, however, permits development of computationally tractable algorithms that analyse the vast 3D data volumes commonly encountered in seismic imagery.

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