Continuous-Time ΔΣ Modulators with Noise-Transfer-Function Enhancement

Furrookh Ali and Anas A. Hamoui

Department of Electrical & Computer Engineering, McGill University, Montreal, Canada

furrookh.ali@mail.mcgill.ca; anas.hamoui@mcgill.ca

***Abstract* - A technique is presented to design high-order continuous-time (CT) ΔΣ modulators using noise-transfer-function (NTF) enhancement. This is achieved by coupling the quantization noise into the forward path of the CT ΔΣ modulator, using a passive CT filter. This introduces a real pole-zero pair into the NTF. Thus, the order of the NTF is increased, without affecting the signal transfer function (STF). The proposed NTF-enhancement technique is applied to a CT ΔΣ modulator with a feedforward architecture, where all feedforward paths are summed within the last integrator of the ΔΣ loop filter, thereby eliminating the need for an analog summation amplifier at the quantizer input. Behavioral simulation results confirm the improved noise-shaping and stability characteristics of the proposed feedforward CT ΔΣ modulator with NTF enhancement, compared to classical CT ΔΣ modulators.**

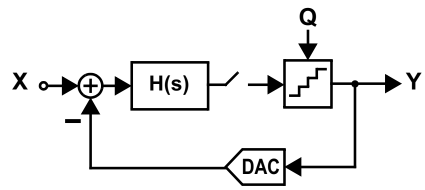

Fig. 1. Block diagram of a classical CT ΔΣ modulator.

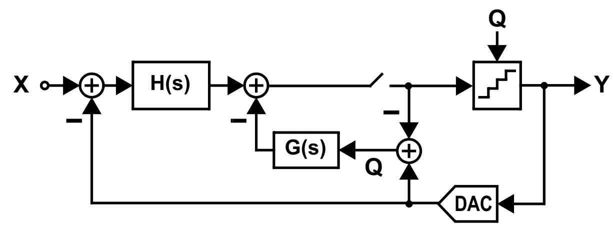

Fig. 2. A CT ΔΣ modulator with NTF enhancement.

I. INTRODUCTION

Continuous-time (CT) ΔΣ modulators (Fig. 1) are attractive architectures for low-power high-speed A/D conversion. Compared to their discrete-time (DT) counterparts, CT ΔΣ modulators provide the added benefit of inherent anti-alias filtering and sampling error suppression [1]. However, their principal disadvantage is their sensitivity to clock jitter [2].

Noise-transfer-function (NTF) enhancement is a technique for increasing the noise-shaping performance of a ΔΣ modulator, without affecting its signal transfer function (STF) and without adding integrators to its loop filter. The NTF enhancement is achieved by coupling the quantization noise into the forward path of the ΔΣ modulator via some filtering (Fig. 2). The advantage of this technique is that the noise-shaping performance of the ΔΣ modulator can be enhanced, while maintaining its original stability (before NTF enhancement). The modulator stability can be further improved by applying the NTF-enhancement technique to ΔΣ modulators with split architectures, in order to uncorrelate the coupled quantization noise from the quantization noise introduced by the quantizer in each path of the split ΔΣ modulator [3].

In this paper, the NTF-enhancement technique is developed for CT ΔΣ modulators. This is achieved by using passive CT filters, which have the potential for low-power design. In CT ΔΣ modulators, NTF enhancement can relax the requirements on the accuracy of the tuning methods, which are needed to ensure the modulator stability.

The CT loop-filter $H(s)$ in Fig. 1 can be implemented using a chain of integrators with either feedback or feedforward compensation [1]. The filter coefficients are implemented as RC time constants in the feedback topology, and as ratios of capacitors or resistors in the feedforward architecture. In standard digital CMOS technologies, capacitor and resistor ratios can be $\pm 0.1\%$ accurate, while RC products can vary by over $\pm 30\%$. Thus, feedforward architectures enable more accurate control over the modulator coefficients. Furthermore, feedforward architectures reduce the signal component at the output of the loop-filter integrators. Thus, the modulator sensitivity to integrator nonlinearities is reduced [4]. However, in order to sum the feedforward paths, a summation amplifier is required at the input of the quantizer in a ΔΣ modulator with feedforward architecture.

This paper presents a technique for designing CT ΔΣ modulators with a feedforward architecture where all feedforward paths are summed within the last integrator of the ΔΣ loop filter, thereby eliminating the need for an analog summation amplifier at the quantizer input. Furthermore, this paper integrates NTF enhancement and excess-loop-delay compensation into the proposed feedforward architecture for CT ΔΣ modulators.

This paper is structured as follows: Section II describes the concept and limitations of NTF enhancement in CT ΔΣ modulators. It then proposes a design technique for achieving NTF enhancement in CT ΔΣ modulators using passive filters. Section III develops a ΔΣ modulator with a feedforward architecture, where all feedforward paths are summed within the last integrator of the loop filter. It then demonstrates how the proposed NTF-enhancement technique, along with excess-loop-delay compensation, can be integrated into the developed feedforward architecture. Behavioral simulation results are presented in Section IV to confirm the improved noise-shaping and stability characteristics of the proposed CT ΔΣ modulator with NTF enhancement, compared to classical CT ΔΣ modulators.

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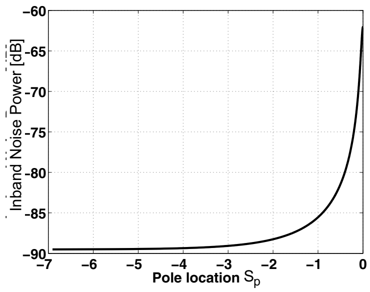

Fig. 3. Effect of pole location S_p on the inband quantization noise power for a normalized sampling time ($T_s = 1$).

II. NTF Enhancement in CT ΔΣ Modulators

A. Fundamentals of NTF Enhancement

Consider the linear model of a classical CT ΔΣ modulator in Fig. 1. The output of the modulator can be expressed as:

$$Y(z) = [X(s)H(s)]^{*} NTF(z) + NTF(z)Q(z) \quad (1)$$

where $$NTF(z) = \frac{1}{H(z)+1}$$

Here, $Q(z)$ is the quantization noise, $H(z)$ is the impulse-invariant transform of the loop filter $H(s)$ and [ ]* represents the sampling operation [5].

To increase the NTF order without modifying the STF, the quantization noise $Q(z)$ is coupled into the forward path of the ΔΣ modulator, as shown in Fig. 2. Thus, the output of the ΔΣ modulator with NTF enhancement can be expressed as:

$$Y(z) = [X(s)H(s)]^{*} NTF(z) + NTF_g(z)Q(z) \quad (2)$$

where $$NTF_g(z) = [1 - G(s)]^{*} NTF(z)$$

Assume that the ΔΣ modulator is designed with a finite-impulse-response (FIR) NTF:

$$NTF(z) = (1 - z^{-1})^{N} \quad (3)$$

Then, to increase the overall noise-shaping order by 1, define the enhancement factor in equation (2) as:

$$[1 - G(s)]^{*} = [1 - G(z)] = (1 - z^{-1}) \quad (4)$$

However, there is no CT representation of equation (4), based on the impulse-invariant transform. Therefore, a real pole P is introduced into the enhancement factor:

$$[1 - G(z)] = \frac{1 - z^{-1}}{1 - Pz^{-1}} \quad (5)$$

The impulse-invariant transform of equation (5) is

$$G(s) = \frac{A_g}{s - S_p} \quad (6)$$

where $$S_p = \ln(P)$$

$$A_g = \frac{(1-P)\ln(P)}{(P^{1-\alpha} - P^{1-\beta})}$$

Here, α and β define, respectively, the normalized start and end times of the rectangular pulses in the feedback digital-to-analog converter (DAC) [1]. The resulting enhancement filter $G(s)$ is simply a CT low-pass filter.

Observe that equation (4) results from (5) as P approaches 0. This implies that, ideally, pole S_p in equation (6) approaches $-\infty$.

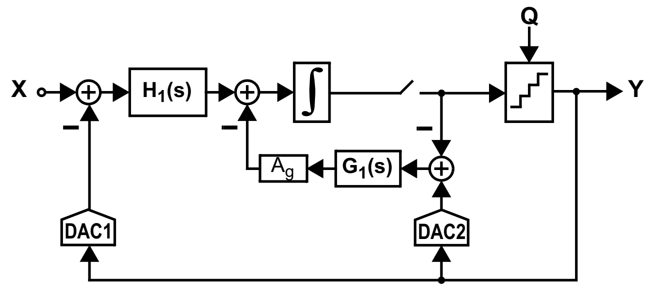

Fig. 4. Improved implementation of the NTF enhancement. Here, $\int \equiv 1/s$.

However, it is important to determine a practical range of values for S_p in $G(s)$ for proper design of the enhancement factor $[1 - G(z)]$. Figure 3 depicts the effect of the location of pole S_p on the inband quantization noise power. Accordingly, for values of $S_p < -2.5$, the signal-to-quantization noise ratio (SQNR) of the ΔΣ modulator will deviate by only about 1dB from its ideal level. Therefore, the actual value of S_p is not critical, provided $S_p < -2.5$.

B. Implementing NTF Enhancement using Passive CT Filters

Implementing the NTF enhancement as shown in Fig. 2 is not attractive, because of the additional summation amplifier needed at the input of the quantizer. For ease of implementation, the CT ΔΣ modulator with NTF enhancement in Fig. 2 can be designed as shown in Fig. 4. Then, the output of the ΔΣ modulator can be expressed as:

$$Y(z) = \left[X(s)H_1(s)\frac{1}{s}\right]^{*} NTF(z) + \left[1 - G_1(s)\frac{A_g}{s}\right]^{*} NTF(z)Q(z) \quad (7)$$

where $$G_1(s) = \frac{s}{s - S_p}$$

Here, $G_1(s)$ is a passive high-pass CT filter. Therefore, the only additional hardware required for implementing the NTF enhancement is some passive elements to implement the enhancement filter $G_1(s)$. Furthermore, the summation at the input of the enhancement filter $G_1(s)$ can be incorporated into the filter implementation itself and, hence, an explicit summation block is not needed, as described in Section III. Thus, implementing the NTF enhancement dissipates no power.

C. Higher-Order Enhancement of NTF

As presented above, enhancing the order of an FIR NTF by 1 (i.e., 1st-order enhancement) is attractive, as it can be implemented using only passive filters and, hence, does not dissipate power.

Enhancing the order of an FIR NTF by more than 1 (i.e., higher-order enhancement) is also feasible. However, it requires positioning multiple zeros at DC (for high-pass FIR NTFs), which cannot be implemented using passive filters only. Hence, higher-order enhancement of the NTF is not attractive, because it requires additional active filters and therefore increases the power dissipation.

III. Implementation of NTF Enhancement

A. Classical Feedforward Architecture

Figure 5 depicts a classical feedforward ΔΣ modulator architecture. This corresponds to an N-order CT ΔΣ modulator with a loop filter:

$$H(s) = \frac{k_N s^{N-1} + \ldots + k_2 s + k_1}{s^N} \quad (8)$$

where a normalized sampling frequency is assumed.

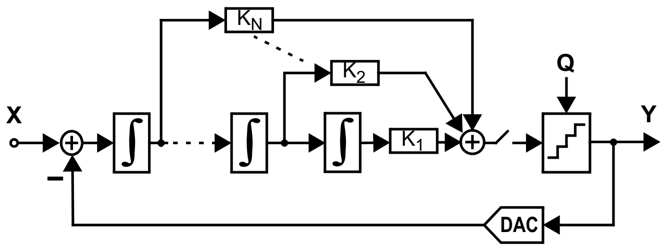

Fig. 5. An *N*-order CT ΔΣ modulator with feedforward compensation.

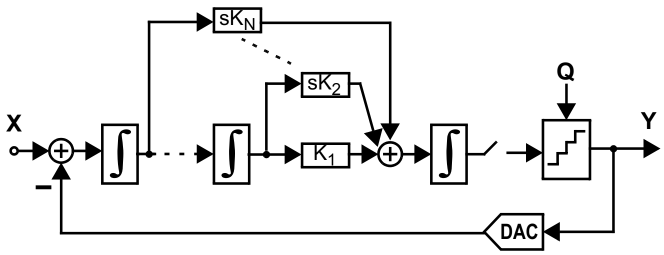

Fig. 6. Proposed feedforward CT ΔΣ modulator.

The impulse-invariant transform of $H(s)$ in equation (8) has the general form:

$$H(z) = \frac{1}{NTF(z)} - 1 = \frac{a_N}{(z-1)^N} + \ldots + \frac{a_2}{(z-1)^2} + \frac{a_1}{(z-1)} \quad (9)$$

Therefore, the architecture in Fig. 5 can be utilized to implement an N-order FIR NTF:

$$NTF(z) = \frac{1}{1 + H(z)} = (1 - z^{-1})^N \quad (10)$$

B. Proposed Feedforward Architecture

In [6], it is proposed to design a feedforward ΔΣ modulator architecture, where the summation of the signals in the feedforward paths is performed within the last integrator stage of the loop filter, rather than at the input of the quantizer. Thus, no summation amplifier is required at the quantizer input, thereby reducing the circuit complexity and saving power.

Figure 6 extends the concept in [6] to CT ΔΣ modulators, by using capacitive feedforward paths [7]. As depicted in Fig. 7, the feedforward coefficients sK_i $(i = 2, \ldots, N)$ in Fig. 6 can be implemented as ratios of capacitors and, hence, their values are accurately controlled (as compared to *RC* time constants).

C. NTF Enhancement

Figure 8 depicts how the passive enhancement filter $G_1(s)$ in Fig. 4 can be integrated into the proposed feedforward architecture in Fig. 6. Here, the input of the last integrator is utilized as a summation node. An additional DAC (DAC3 in Fig. 8) is needed to implement the NTF enhancement. However, it can have a simple implementation employing switches and resistors, with the resistors shared with the implementation of the passive CT filter $G_1(s)$.

D. Excess-Loop-Delay Compensation

Excess loop delay is a timing error in the feedback DAC pulse, which arises in CT ΔΣ modulators due to the finite time required for the quantizer to resolve its input and for the DAC to respond to this input. This nonideality can cause the DAC pulse to be shifted into the next clock cycle. This effectively increases the order of the ΔΣ loop filter, potentially de-stabilizing the ΔΣ modulator and degrading its noise-shaping performance [8].

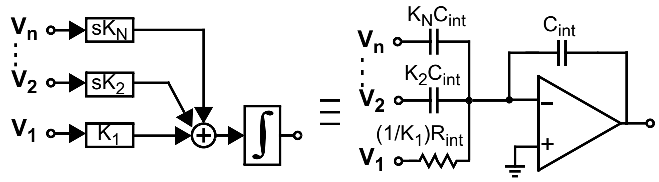

Fig. 7. Single-ended circuit-level representation of the last integrator in Fig. 6

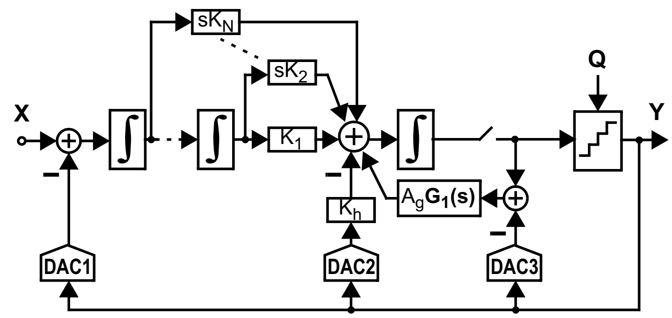

Fig. 8. Proposed CT ΔΣ modulator with NTF enhancement and excess-loop-delay compensation.

In the proposed CT ΔΣ modulator with NTF enhancement, excess loop delay can be compensated for by modifying the feedforward coefficients and introducing an additional feedback term from the modulator output to the input of the last integrator, as shown in Fig. 8. Here, DAC2 needs to have a different pulse shape than DAC1. However, it can have a simple implementation using DAC1 as a building block [9]. A methodology for determining the coefficients K_i and the additional feedback coefficient K_h is presented in [8]. Thus, the additional path required for excess loop delay compensation can be easily integrated into the proposed feedforward architecture with NTF enhancement.

IV. SIMULATION RESULTS

Behavioral simulations were performed in SIMULINK to compare the performance of a classical feedforward CT ΔΣ modulator with one that employs the proposed NTF-enhancement technique. The ΔΣ modulators were implemented with an FIR NTF, a 5-bit quantizer and an ideal DAC with non-return-to-zero pulses. The pole of the enhancement filter S_p was set to $-3f_S$, where f_S is the sampling frequency. This is approximately the worst case scenario for FIR NTFs. An oversampling ratio of OSR = 8 was assumed.

A. Noise-Shaping Improvement

The NTF characteristics of 2nd-order, 3rd-order, and enhanced 2nd-order CT ΔΣ modulators are compared in Fig. 9. The NTFs of the 3rd-order and enhanced 2nd-order CT ΔΣ modulators are almost indistinguishable, thereby confirming the achievable NTF enhancement using the proposed technique.

B. Tolerance to Quantizer Overload

The quantizer overload ratio is defined as $A_{OL} = V_{in}/V_{ref}$, where V_{in} is the modulator input voltage and V_{ref} is the quantizer reference voltage. Figure 10 compares the signal-to-noise ratio (SNR) of the classical and the NTF-enhanced CT ΔΣ modulators versus A_{OL}. Accordingly, when comparing stability with respect to A_{OL}, the behavior of the classical and the NTF-enhanced ΔΣ modulators is almost identical. This is expected, since this stability depends on the maximum gain of the NTF and the number of bits in the quantizer [6]. Since these two parameters are almost identical for both the classical 3rd-order and the enhanced 2nd-order ΔΣ modulators, no improvement in stability is expected.

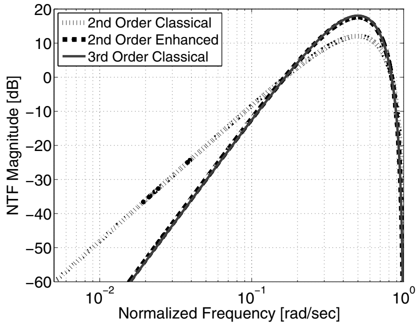

Fig. 9. Magnitude of NTF vs. frequency for classical and NTF-enhanced CT ΔΣ modulators.

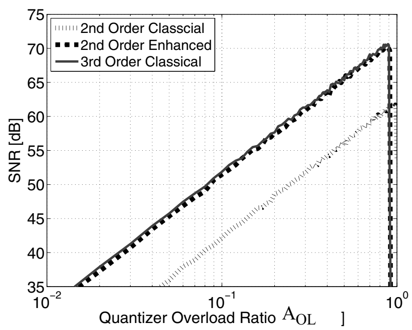

Fig. 10. SNR vs. quantizer overload ratio ($A_{OL} = V_{in}/V_{ref}$) for classical and NTF-enhanced CT ΔΣ modulators.

C. Tolerance to Integrator Coefficient Errors

Figure 11 compares the SNR versus the error in the *RC* time constant (τ_{RC}) of the integrators in the classical and the NTF-enhanced ΔΣ modulators. Accordingly, while the classical 3rd-order ΔΣ modulator is only stable up to 10% variation in τ_{RC}, the enhanced 2nd-order ΔΣ modulator is stable up to 20% variations. Similarly, while the classical 4th-order ΔΣ modulator is only stable up to 2.5% variations in τ_{RC}, the enhanced 3rd-order ΔΣ modulator is stable up to 6% variations. This confirms that a significant improvement in the tolerance of the ΔΣ modulator to τ_{RC} variations can be achieved by using the proposed technique for NTF enhancement. Thus, although coefficient tuning to compensate for τ_{RC} errors in high-order CT ΔΣ modulators may still be required, the constraints on the accuracy of the tuning methods are significantly relaxed.

Note that, in Fig. 8, the NTF enhancement is implemented in a ΔΣ modulator with a feedforward single-loop architecture and, hence, the coupled quantization noise is partially correlated to the quantization noise introduced by the quantizer. A further improvement in stability can be achieved if the coupled quantization noise and the noise introduced by the quantizer were uncorrelated.

This can be accomplished by applying the NTF-enhancement technique to the ΔΣ modulators with split architecture [3]. In this case, the NTF-enhancement ΔΣ modulator will maintain the stability of the original ΔΣ modulator (i.e., before NTF-enhancement).

V. Conclusion

In this paper, a new design methodology was presented for CT ΔΣ modulators. It is based on a feedforward architecture and includes NTF enhancement. This enhancement, demonstrated for an FIR NTF, increases the noise-shaping order of the ΔΣ modulator by one.

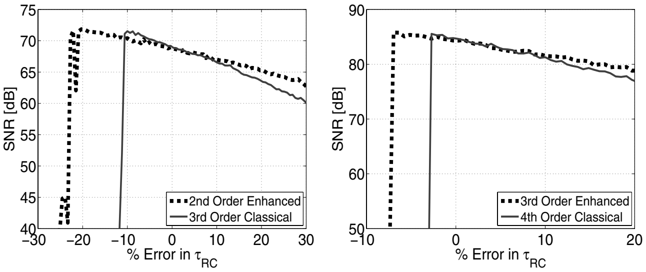

Fig. 11. SNR vs. percentage error in the *RC* time constant τ_{RC} in the integrators of the classical and NTF-enhanced ΔΣ modulators.

Furthermore, all feedforward paths of the ΔΣ modulator are summed within the last integrator of its loop filter and, thus, no summation amplifier is required at its quantizer input. Moreover, excess-loop-delay compensation is integrated into the proposed architecture. Behavioral simulations demonstrate that, compared to a classical ΔΣ modulator with an order higher by one, the NTF-enhanced ΔΣ modulator has the same noise-shaping performance and sensitivity to quantizer overload, while achieving an improved stability with respect to variations in the *RC* time constants of the integrators in the ΔΣ loop filter. This relaxes the requirements on the accuracy of the tuning methods needed to ensure the stability of the CT ΔΣ modulator. This stability can be further improved through the use of split architectures to decorrelate the coupled quantization noise in the NTF-enhanced ΔΣ modulator from the quantization noise introduced by the quantizer.

VI. References

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