On Privacy in Smart Metering Systems with Periodically Time-Varying Input Distribution

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***Abstract*—We consider a setup where a rechargeable battery is used to partially mask the load profile of a user from the utility provider in a smart-metered electrical system. Using mutual information as the privacy metric, a prior work by Li, Khisti and Mahajan has considered the case of i.i.d. input load and established the optimal battery charging policy as well as the associated leakage rate. In this work we consider the case when the input distribution is also sampled independently at each time, but from a periodically time-varying distribution. We propose upper and lower bounds on the optimal leakage rate by extending techniques developed by for the i.i.d. case. Numerical results suggest that the upper and lower bounds are close for examples involving binary and ternary inputs .**

I. Introduction

Smart meters are becoming a critical part of modern electrical grids. They deliver fine-grained household power usage measurements to utility providers. This information allows them to implement changes to improve the efficiency of the electrical grid. However, despite the promise of savings in energy and money, there is potentially a loss of privacy. Anyone with access to the load profile may employ data mining algorithms to infer details about the private activities of the user [1]–[6].

One possible solution to the privacy problem involves using a rechargeable battery. The user can distort the load profile generated by the appliances by charging and discharging the battery. Due to the proliferation of rechargeable batteries, energy harvesting devices and electric vehicles, the strategy of using these devices to partially obfuscate the user's load profile is becoming more feasible.

We consider a setup similar to [7]–[10] which consider using mutual information as a privacy metric. Reference [7] considers an instance of the problem with binary alphabets. The setup is extended in [8]–[10] where the multi-letter mutual information optimization problem is reformulated. A single letter solution for the case when the inputs are independent and identically distributed (i.i.d.) is presented in [10]. For other related works, see [11]–[15].

The present work extends previous results by considering the case of a periodically time-varying input distribution. For sake of convenience we limit our discussion to the case when the period equals two i.e., for all odd time-instants the input load distribution is sampled i.i.d. from a given distribution say $Q_1(\cdot)$ and for all even-time instants it is sampled i.i.d. from another distribution say $Q_2(\cdot)$. We show that this is a non-trivial extension of the i.i.d. case treated in [10] and develop new upper and lower bounds on the optimal leakage rate.

II. Problem definition

We consider a smart metering system as shown in Fig. 1 where at each time a residence generates an aggregate demand that must either be satisfied by charges in the battery or by drawing power from the grid. $\{X_t\}_{t\geq 1}$, $X_t \in \mathcal{X}$ where $\mathcal{X} := \{0, 1, 2, \ldots, m_x\}$ denotes the power demand process. We assume that X_t is sampled independently at each time and from a distribution $Q_1(\cdot)$ for odd values of t and from a distribution $Q_2(\cdot)$ for even values of t. The sequence $\{Y_t\}_{t\geq 1}$, $Y_t \in \mathcal{Y}$, denotes the energy consumed from the grid where $\mathcal{Y} := \{0, 1, 2, \ldots, m_y\}$ and $\{S_t\}_{t\geq 1}$, $S_t \in \mathcal{S}$ denotes the energy stored in the battery where $\mathcal{S} := \{0, 1, 2, \ldots, m_s\}$ and the initial charge S_1 of the battery is distributed according to probability mass function P_{S_1}.

We assume that $m_x \leq m_y$ so that the system is guaranteed to be able to satisfy the demand at any time by drawing solely from the grid i.e. $Y_t = X_t$, $\forall t$. While in general, the alphabets $\mathcal{X}$ and $\mathcal{Y}$ can be any finite subset of the integers – where negative values of X and Y would model a situation where energy (possibly generated from an alternative energy source) is sold back to the utility provider – it is more realistic to for them to be a contiguous interval. In this case, without further assumptions on the battery size, the alphabets would have to satisfy $\mathcal{X} \subset \mathcal{Y}$ in order to guarantee that energy is not wasted and the power demand can always be satisfied. Nonetheless, our results generalize to these cases.

We assume an ideal battery that has no conversion losses or other inefficiencies. Therefore, the following conservation equation must be satisfied at all time instances:

$$S_{t+1} = S_t - X_t + Y_t. \tag{1}$$

The energy management system observes the power demand and battery charge and consumes energy from the grid according to a randomized *charging policy* $\mathbf{q} = (q_1, q_2, \ldots)$. In particular, at time t, given $(x^t,\ s^t, y^{t-1})$, the history of demand, battery charge, and past consumption, the battery policy chooses the level of current consumption Y_t to be

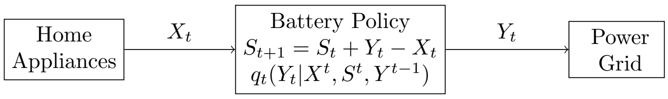

Fig. 1: System Diagram. The user demand is denoted by X_t, the grid consumption by Y_t, and the battery state by S_t. The battery policy is denoted by the conditional distribution $q(Y_t|X^t, S^t, Y^{t-1})$. The battery policy effectively defines a channel with memory from the residence to the utility provider.

y with probability $q_t(y \mid x^t, s^t, y^{t-1})$. For a randomized charging policy to be feasible, it must satisfy the conservation equation (1), so given the current power demand and battery charge (x_t, s_t), the feasible values of grid consumption are defined by

$$\mathcal{Y}_\circ(s_t - x_t) = \{y \in \mathcal{Y} : s_t - x_t + y \in \mathcal{S}\}.$$

Thus, we require that

$$\begin{aligned} &q_t(\mathcal{Y}_\circ(s_t - x_t) \mid x^t, s^t, y^{t-1}) \\ &:= \sum_{y \in \mathcal{Y}_\circ(s_t - x_t)} q_t(y \mid x^t, s^t, y^{t-1}) \\ &= 1. \end{aligned}$$

The set of all such feasible strategies is denoted by $\mathcal{Q}_A$. A battery policy effectively defines a channel with memory between a residence and the utility provider (as portrayed in Fig. 1).

The quality of a charging policy depends on the amount of information leaked under that policy. This notion is captured by mutual information $I^{\mathbf{q}}(S_1, X^T; Y^T)$ evaluated according to the joint probability distribution on (S^T, X^T, Y^T) induced by the sequence $\mathbf{q}$:

$$\begin{aligned} &\mathbb{P}^{\mathbf{q}}(S^T = s^T, X^T = x^T, Y^T = y^T) \\ &\quad = P_{S_1}(s_1) P_{X_1}(x_1) q_1(y_1 \mid x_1, s_1) \\ &\qquad \times \prod_{t=2}^{T} \Big[\mathbb{1}_{s_t}\{s_{t-1} - x_{t-1} + y_{t-1}\} \\ &\qquad\quad Q_t(x_t) q_t(y_t \mid x^t, s^t, y^{t-1}) \Big], \end{aligned} \tag{2}$$

where $Q_t \equiv Q_1$ for odd values of t and $Q_t \equiv Q_2$ for even values of t.

Given a policy $\mathbf{q} = (q_1, q_2, \dots) \in \mathcal{Q}_A$, we define the worst case information leakage *rate* as follows:

$$L_\infty(\mathbf{q}) := \limsup_{T \to \infty} \frac{1}{T} I^{\mathbf{q}}(S_1, X^T; Y^T). \tag{3}$$

Remark II.1. The random variable S_1 in the mutual information terms do not affect the asymptotic rate. However its inclusion often simplifies the analysis.

We are interested in the following optimization problem:

Problem A. *Given the alphabet $\mathcal{X}$ and distributions $\{Q_1, Q_2\}$ of the power demand, the alphabet $\mathcal{S}$ of the battery, and the alphabet $\mathcal{Y}$ of the demand: find a battery charging policy $\mathbf{q} = (q_1, q_2, \dots) \in \mathcal{Q}_A$ and the initial distribution P_{S_1} of the battery state, that minimizes the leakage rate $L_\infty(\mathbf{q})$ given by* (3).

III. State Invariant Battery Policies

A. State Invariant Policy: I.I.D. Inputs

We first briefly summarize the results in [10] for i.i.d. inputs i..e, when $Q_1(x) = Q_2(x) = Q_X(x)$. The simplest class of policies are stationary and memoryless, conditioning only on the current battery state and power demand:

$$q(y|x, s). \tag{4}$$

As such evaluating the leakage rate (3) even for this simplified class of policies requires numerical approaches, see e.g., [7], [13]. Instead if one imposes a certain invariance condition we can obtain a closed form expression for the leakage rate. Our proposed class preserves the following property:

$$\mathbb{P}(S_2 = s_2 | Y_1 = y_1) = \mathbb{P}(S_1 = s_2),\ \forall s_2 \in \mathcal{S}, y_1 \in \hat{\mathcal{Y}} \tag{5}$$

where $\hat{\mathcal{Y}} := \{y : P_{Y_1}(y_1) > 0\}$ for some initial battery state distribution $\mathbb{P}_{S_1}$. This invariance condition implies that $S_t \perp Y_{t-1}$ and also that $\mathbb{P}_{S_t} = \mathbb{P}_{S_1},\ \forall t$. By exploiting this property, we can obtain single-letter achievable leakage rates as follows [10]:

Lemma III.1. *([10]) Given an instance of Problem A with i.i.d. power demand $Q_1(x) = Q_2(x) = Q_X(x)$ and initial battery state distribution $\mathbb{P}_{S_1}$, if the stationary memoryless policy $\mathbf{q} = (q, q, \dots) \in \mathcal{Q}_A$ satisfies the invariance property* (5)*, then*

$$L_\infty(\mathbf{q}) = I(S_1, X_1; Y_1),$$

where $(S_1, X_1, Y_1) \sim \mathbb{P}_{S_1}(s_1) Q(x_1) q(y_1|x_1, s_1)$.

While the restriction to the invariance class may appear restrictive, interestingly for the case of i.i.d. inputs it was shown in [10] that the optimal policy belongs to this class. In particular the optimal policy was shown to be of the following form

$$q^*(y|x, s) = \begin{cases} \frac{Q_X(y) P_{S_1}(y+s-x)}{P_{W_1}(s-x)} & \text{if } y \in \mathcal{X} \cap \mathcal{Y}_\circ(s-x) \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where recall that $Q_X(\cdot) = Q_1(\cdot) = Q_2(\cdot)$ for the case of i.i.d. inputs, and $W_1 = S_1 - X_1$. Furthermore the optimal choice for $P_{S_1}(\cdot)$ was also characterized in [10] as follows

$$P^\star_{S_1} = \arg\min_{P_{S_1}} I(S_1 - X_1; X_1) \tag{7}$$

where the term on the right hand side corresponds to the minimum achievable leakage rate associated with the policy in (6).

B. State Invariant Policy: Periodic Case

We propose a simple extension of the state invariant policy to the case when the input is sampled from distribution $Q_1(x)$ at the odd times and $Q_2(x)$ at the even times. In this extension we assume two stationary memoryless battery policies: $q_1(y_t|x_t, s_t)$ for odd times and $q_2(y_t|x_t, s_t)$ at even times. Thus the overall policy is of the form $\mathbf{q} = (q_1, q_2, q_1, q_2, \ldots)$. We further require that the choice of q_1 and q_2 satisfy the following conditions:

$$\begin{aligned} P(S_2 = s_2|Y_1 = y_1) = P(S_1 = s_2), \forall(s_2, y_1) \in \mathcal{S} \times \hat{\mathcal{Y}} \\ P(S_3 = s_3|Y_2 = y_2) = P(S_1 = s_3), \forall(s_3, y_2) \in \mathcal{S} \times \hat{\mathcal{Y}} \end{aligned} \tag{8}$$

Note that condition (8) and the choice of $\mathbf{q}$ immediately lead to $\mathbb{P}(S_t = s) = \mathbb{P}(S_1 = s)$ for all $t \geq 1$ as required by the state invariance condition. Furthermore notice that S_t is independent of Y_{t-1}, i.e., $S_t \perp Y_{t-1}$, which can be further generalized to show that:

$$Y_1^{t-1} \perp (S_t, Y_t, X_t), \quad \forall t > 1. \tag{9}$$

As a specific example of a policy that achieves the invariance condition in (8) consider a natural generalization of (6) as follows:

$$q_t^*(y|x, s) = \begin{cases} \frac{Q_t(y) P_{S_1}(y+s-x)}{P_{S_1 - X_t}(s-x)} & \text{if } y \in \mathcal{X} \cap \mathcal{Y}_\circ(s-x) \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where recall that $X_t \sim Q_t(\cdot)$, $Q_t(\cdot) = Q_1(\cdot)$ for odd values of t and $Q_t(\cdot) = Q_2(\cdot)$ for even values of t and $P_{S_1}(\cdot)$ is the desired distribution on the state. It is straightforward to verify that the policy in (10) satisfies the invariance conditions (8) and furthermore the output distribution is memoryless and satisfies $\mathbb{P}(Y_t = y) = Q_t(y)$ for each t i.e., the output sequence has the same distribution as the input sequence. Furthermore the policy in (10) achieves the following leakage rate.

Lemma III.2. *Given a fixed $\mathbb{P}_{S_1}$ the leakage rate achieved by the policy in* (10) *is given by:*

$$L_1 = \frac{1}{2} I(S_1 - X_1; X_1) + \frac{1}{2} I(S_1 - X_2; X_2) \tag{11}$$

where $(S_1, X_t) \sim \mathbb{P}_{S_1}(s_1) Q_t(x_t)$ for $t = 1, 2$.

Proof. Consider the following:

$$\begin{aligned} &\frac{1}{T} I(S_1, X^T; Y^T) \\ &\overset{(a)}{=} \sum_{t=1}^{T} \frac{1}{T} I(S^t, X^T; Y_t | Y^{t-1}) \\ &\overset{(b)}{=} \sum_{t=1}^{T} \frac{1}{T} I(S_t, X_t; Y_t | Y^{t-1}) \\ &\overset{(c)}{=} \sum_{t=1}^{T} \frac{1}{T} I(S_t, X_t; Y_t) \\ &= \sum_{t=1, t:\text{odd}}^{T} \frac{1}{T} I(S_t, X_t; Y_t) + I(S_{t+1}, X_{t+1}; Y_{t+1}) \\ &\overset{(d)}{=} \frac{1}{2} I(S_1, X_1; Y_1) + \frac{1}{2} I(S_1, X_2; Y_2) \end{aligned}$$

where (a) follows from the fact that S_t is a deterministic function of S_1, X_1^t and Y_1^{t-1} based on the battery update equation, (b) follows is due to the memoryless structure of the battery policy in (10), (c) is from (9), (d) follows from the state invariance condition and the fact that $P_{S_t} = P_{S_1}$ for all $t \geq 1$.

Finally for the choice of the policy in (10) the equivalence between the expression in (d) and (11) can be established by following the analysis in [10]. We skip the steps due to space constraints. $\square$

While Lemma (III.2) recovers the optimal leakage rate for the I.I.D. case, it is not the best possible that can be achieved in the time-varying setup. The constraint of state invariance turns out to be too restrictive. We next describe one generalization that can lead to lower leakage rate.

C. Alternating State Invariance

In this section we propose a more relaxed condition on state invariance which can lead to lower leakage rates than (11). Instead of imposing the same marginal distribution of the state at each time, we allow the state distribution to alternate at odd and even times.

Consider a set of memoryless battery policies $q_1(y_t|x_t, s_t)$ for odd times and $q_2(y_t|x_t, s_t)$ for even times. These policies are said to satisfy alternating state invariance condition if the following holds. Given distributions $P_{S_1}(\cdot)$ and $P_{S_2}(\cdot)$ on $\mathcal{S}$:

$$\begin{aligned} \sum_{s_1, x_1, y_1} P_{S_1}(s_1) Q_1(x_1) q_1(y_1|x_1, s_1) \mathbb{I}_{s_2 = s_1 - x_1 + y_1} = P_{S_2}(s_2), \\ \sum_{s_2, x_2, y_2} P_{S_2}(s_2) Q_2(x_2) q_2(y_2|x_2, s_2) \mathbb{I}_{s_3 = s_2 - x_2 + y_2} = P_{S_1}(s_3) \end{aligned} \tag{12}$$

for all $s_2, s_3 \in \mathcal{S}$ where $\mathbb{I}_{x=y}$ is the indicator function which equals 1 is $x = y$ and zero otherwise. The left hand side expressions in (12) are the marginal distributions on state induced by the associated battery policies. The right hand side guarantees that these distributions are consistent with the pre-specified choice.

Note that the state invariance constraint (8) enforces $P_{S_1}(\cdot) = P_{S_2}(\cdot)$ and $S_t \perp Y_{t-1}$. While it belongs to the class of policies in (12), the class of policies in (12) is more general than (8) leads to a lower leakage rate, as will be confirmed in the numerical results section. We next provide an expression for the leakage rate for the policies in (12)

Lemma III.3. *Given a fixed $\mathbb{P}_{S_1}$ and $\Pr_{S_2}$ the leakage rate achieved by any policy satisfying* (12) *is upper bounded by:*

$$L_2 = \frac{1}{2} I(S_1, X_1; Y_1) + \frac{1}{2} I(S_2, X_2; Y_2) \tag{13}$$

where $(S_t, X_t) \sim \mathbb{P}_{S_t}(s_1) Q_t(x_t)$ for $t = 1, 2$.

Proof. Consider the following:

$$\begin{aligned}\frac{1}{T}I(S_1, X^T; Y^T) &\overset{(a)}{=} \sum_{t=1}^{T} \frac{1}{T} I(S^t, X^T; Y_t|Y^{t-1}) \\ &\overset{(b)}{=} \sum_{t=1}^{T} \frac{1}{T} I(S_t, X_t; Y_t|Y^{t-1}) \\ &\overset{(c)}{\leq} \sum_{t=1}^{T} \frac{1}{T} I(S_t, X_t; Y_t) \quad \overset{(d)}{=} L_2.\end{aligned}$$

where (a) is due to the chain rule of mutual information and the fact that S^t is a deterministic function of (S_1, X^{t-1}, Y^{t-1}) given by the battery update equation (1), (b) and (c) are due to the memoryless policy, and (d) is due to the alternating invariance condition in (12). □

Remark III.1. The advantage of Lemma III.3 over Lemma III.2 is that we have a larger set of state distributions to minimize over, which can lead to a lower leakage rate in general.

IV. Lower Bound

We now provide a lower bound on the leakage rate that applies to any feasible battery charging policy . Consider the following inequalities: for any admissible policy $\mathbf{q} \in \mathcal{Q}_A$ and defining $W_t = S_t - X_t$, we have

$$\begin{aligned}&I(S_1, X^T; Y^T) \geq \sum_{t=1}^{T} I(S_t, X_t; Y_t|Y^{t-1}) \geq \sum_{t=1}^{T} I(W_t; Y_t|Y^{t-1}) \\ &= H(W_1) - H(W_1|Y_1) + H(W_2|Y_1) - H(W_2|Y^2) + \cdots \\ &\overset{(a)}{=} H(W_1) - H(S_2|Y_1) + H(S_2 - X_2|Y_1) - H(S_3|Y^2) + \cdots \\ &= H(W_1) + \sum_{t=2}^{T} I(W_t; X_t|Y^{t-1}) - H(W_T|Y^T)\end{aligned}$$

where (a) is because S_{t+1} is an invertible function of W_t given Y_t. Now, taking the limit $T \to \infty$ to obtain a lower bound to the leakage rate we have

$$\begin{aligned}&L_\infty(\mathbf{q}) = \lim_{T\to\infty} \frac{1}{T} I(S_1, X^T; Y^T) \\ &\geq \lim_{T\to\infty} \frac{1}{T}\left[H(W_1) + \sum_{t=2}^{T} I(W_t; X_t|Y^{t-1}) - H(W_T|Y^T)\right] \\ &\overset{(a)}{=} \lim_{T\to\infty} \frac{1}{T}\left[\sum_{t=2}^{T} I(W_t; X_t|Y^{t-1})\right] \\ &\overset{(b)}{\geq} \min_{P_{S_1}\in\mathcal{P}_S} \frac{1}{2} I(S_1 - X_1; X_1) + \min_{P_{S_2}\in\mathcal{P}_S} \frac{1}{2} I(S_2 - X_2; X_2).\end{aligned}$$

Note that (a) is because the entropy of any discrete random variable is bounded and (b) follows from the observation that every term in the summation is only a function of the posterior $P(S_t|Y^{t-1})$. Therefore, minimizing each term over a $P_S \in \mathcal{P}_S$ results in a lower bound to the optimal leakage rate. Furthermore since X_t is sampled from the stated periodically time-varying distribution the result follows.

V. Numerical Results

A. Binary Case

We consider the case when $\mathcal{X} = \mathcal{Y} = \{0, 1\}$ and the state alphabet $\mathcal{S} = \{0, 1, \ldots, C\}$ where $C \geq 2$ denotes the battery capacity. We assume $Q_1(x = 1) = 0.3$ and $Q_2(x = 1) = 0.7$. We numerically computed the achievable leakage rates for the state invariance policy (policy 1) in section III-B and the alternating state policy (policy 2) in section III-C. We also evaluate the lower bound presented in section IV. We vary $C \in [1, 6]$ and the results are presented Table II.

TABLE I: Leakage Rates for binary case

Capacity	1	2	3	4	5	6
Policy 1	0.4406	0.2682	0.1812	0.1309	0.099	0.0776
Policy 2	0.4391	0.2675	0.1809	0.1307	0.0989	0.0775
Lower Bound	0.4354	0.2646	0.1788	0.1292	0.0979	0.0767

We note that the extra flexibility provided by the alternating state invariance policy (12) does in fact lead to lower leakage rate as shown in Table II. Thus the straightforward extension of the policy for i.i.d. inputs stated in (10) is no longer optimal. Also interestingly the lower bound presented in Section IV appears very close to the proposed scheme. The gap between the bounds is largest when $C = 1$ and seems to vanish quickly as C increases.

B. Ternary Case

We also consider the ternary case when $\mathcal{X} = \mathcal{Y} = \{0, 1, 2\}$. We assume that $Q_1(x) = 1/3$ for $x \in \{0, 1, 2\}$ and $Q_2(0) = Q_2(2) = 1/4$ while $Q_2(1) = 1/2$. Numerically we obtained the following upper and lower bounds:

TABLE II: Leakage Rates for ternary case

Capacity	2	3	4	5	6
Policy 2	0.5602	0.3950	0.2943	0.2281	0.1822
Lower Bound	0.5597	0.3947	0.2941	0.2280	0.1821

We again note that the upper and lower bounds are very close with the largest gap when $C = 2$ and the gap decreases monotonically.

VI. Conclusions

We study information theoretic privacy in a smart metering system with a rechargeable battery. We extend previous results on the case of i.i.d. inputs to the case of periodically time-varying inputs. Although we only consider the case when the period equals two, the techniques presented here can be easily extended to arbitrary periods. We show that a straightforward extension of the state invariant policy for i.i.d. inputs is not optimal and propose another scheme, alternating state invariance, that results in a lower leakage rate. We also develop an information theoretic lower bound that appears to be very close to the proposed scheme in numerical results. Future work will focus on analytically characterizing the battery policies that optimize the alternating state invariance scheme and establishing the gap between upper and lower bounds.

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