

Remote estimation over control area networks

Aditya Mahajan
 Electrical and Computer Engineering
 McGill University, Montreal, Canada
 email: aditya.mahajan@mcgill.ca

Abstract—The problem of assigning priorities for scheduling multiple sensor measurements over a control area network is considered. A dynamic priority allocation scheme is proposed, where priorities are assigned according to the value of information. The value of information is defined as the fee a receiver is willing to pay to access that information. It is shown that the value of information can be computed by solving two Fredholm integral equations. An efficient computation of the value of information is proposed. Numerical examples suggest that the proposed priority assignment scheme outperforms the existing schemes in the literature.

I. INTRODUCTION

The recent advances in autonomous vehicles are driven by sophisticated algorithms that rely on measurements from multiple sensors. As the number of sensors increases, the effectiveness and efficiency of the intra-vehicle communication between the sensors and the various electronic control units (such as engine control, lane following, cruise control, etc.) becomes critical. The communication between the sensors, controllers, and actuators takes place over a control area network (CAN) [1]. Scheduling sensor measurements over a CAN network is different from sensor scheduling over wireless networks because the contention resolution method used in CAN networks is different. CAN networks use a collision-free contention resolution protocol, in which each data-packet has a priority index and the network transmits the packet with the highest priority.

In this paper we consider the problem of assigning priorities for scheduling multiple sensor measurements over CAN networks. In particular, we consider a system (shown in Fig. 1) in which multiple sensors transmit their measurements to their respective remote estimators over a CAN network. At each time, each sensor takes a measurement and assigns a priority to its measurement. The network transmits the measurement from the sensor with the highest priority. All other sensors, simply discard their measurements rather than buffering them.¹ At the next time instant, all sensors take new measurements, assign priorities, and the above process is repeated. The performance of such a system depends on the scheme used to assign priorities.

Priority assignment in such a network may either be static (i.e., priorities do not change with time) or dynamic (i.e., priorities may change with time). Dynamic priority assignment may either be off-line (i.e., priorities depend on time but not

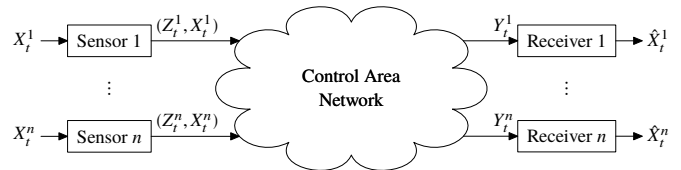


Fig. 1: Block diagram of a remote estimation system

on sensor measurements) or on-line (i.e., priorities depend on time as well as on sensor measurements).

The key conceptual difficulty in on-line priority assignment is that each sensor must determine its priority in a decentralized manner (i.e., based only on its local measurements). Various on-line priority assignment schemes have been proposed in the literature [2]–[5]. In [2], the priority is chosen to be the norm of the instantaneous estimation error; probabilistic variations where the probability of getting access is proportional to the norm of the instantaneous estimation error are considered in [3], [4]. In [5], the priority is chosen according to the difference in performance between transmitting the packet and not transmitting the packet. To compute this difference, it is assumed that future scheduling decisions are determined according to a baseline heuristic.

In this paper we propose an alternative priority assignment scheme that is based on the economic concept of value of information. Instead of the networked problem, suppose there is a single sensor with a dedicated link. However, the sensor has to pay a cost to access the link. We say that the value of information at a particular state (and, therefore, the priority at that state) is equal to the communication cost for which the sensor is indifferent between transmitting or not transmitting its current state. This definition of value of information is inspired by the multi-armed bandit literature [6], [7], where the Gittins index has a similar interpretation.

II. MODEL AND PROBLEM FORMULATION

A. System model

Consider a system consisting of n sensor-estimator pairs that are connected over a CAN-like network (see Fig. 1). Each block of this system is described below.

1) *Sensors*: There are n sensors. Sensor i , $i \in N := \{1, \dots, n\}$, observes a first-order autoregressive process $\{X_t^i\}_{t \geq 0}$, $X_t^i \in \mathbb{R}$. In particular, the initial state X_0^i is distributed according to known distribution, and for $t \geq 0$,

$$X_{t+1}^i = a^i X_t^i + W_t^i, \quad (1)$$

¹This approach is sometimes called “try once and discard” [2].

where $a^i \in \mathbb{R}$ is a known parameter and $\{W_t^i\}_{t \geq 0}$ is an i.i.d. noise process and W_t^i is distributed according to probability density function $\varphi^i(\cdot)$.

The observation processes are assumed to satisfy the following:

- 1) The observation processes across sensors are independent.
- 2) The noise process at sensor is independent across time and independent of the initial state.
- 3) The density φ^i of the noise process is even and unimodal (e.g., a Gaussian distribution satisfies these properties). An immediate implication is that W_t^i is zero-mean.

Assumptions 1) and 2) imply that all the primitive random variables $(X_0^1, \dots, X_0^n, \{W_t^1\}_{t \geq 0}, \dots, \{W_t^n\}_{t \geq 0})$ are independent.

At each time, sensor i assigns a priority Z_t^i to its observation. The priority index Z_t^i is transmitted along with the data-packet X_t^i .

2) *Network*: The network uses the CAN method for contention resolution and transmits the packet with the highest index. Let $Y_t^i, i \in N$, denote the packet received by receiver i . Then,

$$Y_t^i = \begin{cases} X_t^i, & \text{if sensor } i \text{ has the highest priority} \\ \mathfrak{E}, & \text{otherwise} \end{cases} \quad (2)$$

where \mathfrak{E} denotes that no packet was received.

3) *Receivers*: There is a receiver associated with each sensor. Receiver $i, i \in N$, observes $\{Y_t^i\}_{t \geq 0}$ and sequentially generates estimates $\{\hat{X}_t^i\}_{t \geq 0}, \hat{X}_t^i \in \mathbb{R}$. If receiver i receives a packet, then the estimate \hat{X}_t^i equals the observation X_t^i ; if the receiver does not receive a packet, then it needs to estimate based on observations received in the past. We assume that the receiver uses a minimum mean squared estimate, which is given by $a^i \hat{X}_{t-1}^i$. Thus,

$$\hat{X}_t^i = \begin{cases} Y_t^i, & \text{if } Y_t^i \neq \mathfrak{E} \\ a^i \hat{X}_{t-1}^i, & \text{if } Y_t^i = \mathfrak{E} \end{cases} \quad (3)$$

The sensor-receiver pair $i, i \in N$, incurs an estimation error $d^i(X_t^i - \hat{X}_t^i)$ at time t , where $d^i: \mathbb{R} \rightarrow \mathbb{R}$ is the error function. (For example, $d^i(x - \hat{x}) = (x - \hat{x})^2$ corresponds to the mean squared error function.) We assume that, for every $i \in N$, d^i is continuous, even and quasi-convex.

B. The optimization problem

Let $S_t \in N$ denote the sensor with the highest priority at time t . We assume that S_t is observed by all sensors. Thus, before assigning priority at time t , sensor i has access to $(X_{1:t}^i, S_{1:t-1})$, where $X_{1:t}^i$ is a short-hand for (X_1^i, \dots, X_t^i) and a similar interpretation holds for $S_{1:t-1}$. Sensor i chooses priority Z_t^i according to a *priority rule*

$$g_t^i: (X_{1:t}^i, S_{1:t-1}) \mapsto Z_t^i. \quad (4)$$

Let S_t denote the sensor with the highest priority, i.e.,

$$S_t = \arg \max_{i \in N} Z_t^i.$$

The network transmits the measurement of the sensor S_t . The receivers choose an estimate according to (3) and the system incurs a total error²

$$\sum_{\substack{i \in N \\ i \neq S_t}} d^i(X_t^i - \hat{X}_t^i).$$

The collection $g^i := \{g_t^i\}_{t \geq 0}$ is called the *priority strategy* of sensor i and $g := (g^1, \dots, g^n)$ is called the *priority strategy profile*. The performance of any priority strategy profile is measured by the average expected error over time, which is given by

$$J(g) = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}^g \left[\sum_{t=0}^{T-1} \sum_{\substack{i \in N \\ i \neq S_t}} d^i(X_t^i - \hat{X}_t^i) \right]. \quad (5)$$

where \hat{X}_t^i is chosen according to (3), S_t is the sensor with the highest priority, and the expectation is taken with respect to the joint measure induced on all system variables due to the choice of g .

We are interested in the following optimization problem.

Problem 1 *In the model described above, choose a priority strategy profile g that minimizes $J(g)$ given by (5), where the minimization is over all history dependent priority rules of the form (4).*

In the model described above, there are n decision makers (the n sensors) that have different information, yet they have to cooperate to minimize a common system-wide objective given by (5). Therefore, the system is a *dynamic team* or a *decentralized stochastic control* problem [8]. Finding the optimal solution of such problems is notoriously difficult. For this reason, most existing approaches do not try to find an optimal priority assignment and use heuristic priority assignment instead. We follow the same general approach and propose to assign priorities based on the value of information. Our notion of value of information, which we explain in the next section, is closely related to the notion of calibration in multi-armed bandits [6], [7].

III. VALUE OF INFORMATION BASED PRIORITY ASSIGNMENT

A. A change of variables

For the purpose of our analysis, it is more convenient to work with an “error process” rather than the original state process $\{X_t^i\}_{t \geq 0}$. We define the error process $\{E_t^i\}_{t \geq 0}$ as follows. The initial state E_0^i has the same distribution as X_0^i and for $t \geq 0$, we have

$$E_{t+1}^i = \begin{cases} W_t^i, & \text{if } S_t = i \\ aE_t^i + W_t^i, & \text{if } S_t \neq i. \end{cases} \quad (6)$$

When $S_t = i$ (i.e., sensor i transmits), then $\hat{X}_t^i = X_t^i$ and the estimation error for sensor i is zero; when $S_t \neq i$, then

²Recall that the estimation error of sensor S_t is zero.

$X_t^i - \hat{X}_t^i = E_t$ and the estimation error for sensor i is $d^i(E_t^i)$. Thus, Eq. (5) can be written as

$$J(g) = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}^g \left[\sum_{t=0}^{T-1} \sum_{\substack{i \in N, \\ i \neq S_t}} d^i(E_t^i) \right]. \quad (7)$$

B. Value of information

From the point of view of economics, the value of information equals the amount of money someone is willing to pay to access that information. We capture this intuition using the following model. Suppose there is a single sensor, say i , and a dedicated channel is available to the sensor but the sensor has to pay an access fee of λ^i each time it uses the channel to transmit its measurement. If the sensor does not use the channel, then the receiver generates an estimate according to (3) and a estimation error $d^i(X_t^i - \hat{X}_t^i) = d^i(E_t^i)$ is incurred.

Thus, at each time, the sensor decides whether or not to transmit. Let $U_t^i \in \{0, 1\}$ denote the decision of the sensor, where $U_t^i = 1$ denotes that the sensor transmits and pays the access fee λ^i ; $U_t^i = 0$ denotes that the sensor does not transmit and incurs the estimation error $d^i(E_t^i)$. The controlled dynamics of the error process are given by

$$E_{t+1}^i = \begin{cases} W_t^i, & \text{if } U_t^i = 1 \\ aE_t^i + W_t^i, & \text{if } U_t^i = 0. \end{cases} \quad (8)$$

The objective is to choose a scheduling strategy $f^i = (f_0^i, f_1^i, \dots)$ where $U_t^i = f_t^i(E_t^i)$ to minimize

$$J^i(f^i) = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T-1} \left[\lambda^i U_t^i + (1 - U_t^i) d^i(E_t^i) \right] \right]. \quad (9)$$

Problem (9) is a single agent Markov decision process [9] and the optimal solution is given by the solution to the following dynamic programming equation. Suppose there exists a constant h^i and a function $v^i: \mathbb{R} \rightarrow \mathbb{R}$ that satisfy the following system of equations: for any $e \in \mathbb{R}$

$$h^i + v^i(e) = \min \left\{ \lambda^i + \int_{\mathbb{R}} \varphi^i(w) v^i(w) dw, \right. \\ \left. d^i(e) + \int_{\mathbb{R}} \varphi^i(w) v^i(ae + w) dw \right\}. \quad (10)$$

It can be shown that the model defined above satisfies the SEN conditions of [9], [10] (using argument similar to those given in [11]).

Define a function $f_*^i: \mathbb{R} \rightarrow \{0, 1\}$ such that for any $e \in \mathbb{R}$, $f_*^i(e)$ is 1 if the first term in the right hand side of (10) is smaller than the second term; otherwise $f_*^i(e)$ is 0.

Then, from Markov decision theory [9], we get that the time-homogeneous sensor scheduling strategy $f_*^{i, \infty}$ given by (f_*^i, f_*^i, \dots) is optimal for Problem (9) and optimal performance $J^i(f_*^{i, \infty})$ equals h^i .

The dynamic program of (10) is also useful in identifying qualitative properties of the optimal strategy. In particular, since the per-step cost $d^i(\cdot)$ is even and quasi-convex and the noise distribution $\varphi^i(\cdot)$ is symmetric and unimodal. Therefore,

from [12, Theorem 1], we get that there exists a threshold $k^i(\lambda^i)$ such that the optimal strategy is of the form:

$$f_*^i(e) = \begin{cases} 1, & \text{if } |e| < k^i(\lambda^i) \\ 0, & \text{otherwise,} \end{cases} \quad (11)$$

Furthermore, since the per-step cost $d^i(\cdot)$ is continuous, it can be shown that the function v^i is also continuous. Therefore, at the threshold $e_o^i = k^i(\lambda^i)$, the two alternatives in (10) are equal, i.e.,

$$\lambda^i + \int_{\mathbb{R}} \varphi^i(w) v^i(w) dw = d^i(e_o^i) + \int_{\mathbb{R}} \varphi^i(w) v^i(ae_o^i + w) dw.$$

Therefore, at state e_o^i , the sensor is indifferent between transmitting or not transmitting. (Since d^i and φ^i are even, we also get that the sensor is indifferent between transmitting and not transmitting at state $-e_o^i$.)

Using the above property, we define the value of information at state e to be the smallest value of the access fee for which the sensor is indifferent between transmitting and not transmitting when the state is $|e|$, i.e.,

$$\text{VOI}^i(e) = \inf \{ \lambda^i \in \mathbb{R}_{\geq 0} : k^i(\lambda^i) = |e| \}$$

As per the economic interpretation stated earlier, $\text{VOI}^i(e)$ is the amount of money that receiver is willing to pay to get the measurement e .

Let f_k^i denote a threshold strategy of the form (11) for sensor i with threshold k . Define D_k^i and N_k^i to be the average expected distortion and average number of transmissions under strategy f_k^i , i.e.,

$$D_k^i = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}^{f_k^i} \left[\sum_{t=0}^{T-1} d^i(E_t^i) (1 - U_t^i) \right],$$

$$N_k^i = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}^{f_k^i} \left[\sum_{t=0}^{T-1} U_t^i \right].$$

Then, the performance of strategy f_k^i can be written as

$$J^i(f_k^i) = D_k^i + \lambda^i N_k^i.$$

Now, a necessary condition for the threshold based strategy f_k^i to be optimal when the access fee is λ^i is that

$$\partial_k D_k^i + \lambda^i \partial_k N_k^i = 0,$$

where ∂_k denotes the partial derivative with respect to k . Therefore, the value of information at state $k \in \mathbb{R}_{\geq 0}$ is given by

$$\text{VOI}^i(k) = - \frac{\partial_k D_k^i}{\partial_k N_k^i}. \quad (12)$$

C. Computing the value of information

In order to compute the value of information (12), we first derive computable expressions for D_k^i and N_k^i and then derive computable expressions for their derivatives.

Consider the error process at sensor i that starts in state x and follows threshold strategy f_k^i . Let $L_k^i(x)$ and $M_k^i(x)$

denote the expected estimation error and the expected time until the first transmission, i.e.,

$$L_k^i(x) = \mathbb{E} \left[\sum_{t=0}^{\tau-1} d(E_k^i) \mid E_0^i = x \right]$$

$$M_k^i(x) = \mathbb{E}[\tau \mid E_0^i = x]$$

where τ denote the stopping time until the first transmission.

Note that at each transmission, the error process resets to a random variable with distribution φ^i . Thus, the error process is a regenerative process [13]. and satisfies renewal relationships. In particular,

$$D_k^i = \frac{L_k^i(0)}{M_k^i(0)} \quad \text{and} \quad N_k^i = \frac{1}{M_k^i(0)}. \quad (13)$$

Taking the derivative of Eq. (13) and substituting back in (12), we get

$$\text{VOI}^i(k) = M_k^i(0) \frac{\partial_k L_k^i(0)}{\partial_k M_k^i(0)} - L_k^i(0). \quad (14)$$

Below we explain how to compute $L_k^i(0)$, $M_k^i(0)$ and their derivatives.

1) *Computing $L_k^i(0)$ and $M_k^i(0)$* : From balance equation for absorbing Markov chains, we have that for all $x \in [-k, k]$

$$L_k^i(x) = d^i(x) + \int_{-k}^k \varphi^i(y - ax) L_k^i(y) dy, \quad (15)$$

$$M_k^i(x) = 1 + \int_{-k}^k \varphi^i(y - ax) M_k^i(y) dy. \quad (16)$$

Eqs. (15) and (16) are Fredholm integral equations of the second kind [14]. These can be solved efficiently by discretizing the integral equation using quadrature methods [15], [16]. In particular, consider a quadrature rule (e.g., Simpson's rule or a Gauss quadrature rule such as Gauss-Legendre, Gauss-Chebyshev, or Gauss-Kronrod; see [16]) over the interval $[-k, k]$ with $2m + 1$ points. Let $(w_{-m}, w_{-m+1}, \dots, w_m)$ and $(x_{-m}, x_{-m+1}, \dots, x_m)$ be the weights and abscissas of the quadrature rule.³ Then, for any $p \in \{-m, -m+1, \dots, m\}$, Eq. (15) can be approximated as

$$L_k^i(x_p) \approx d^i(x_p) + \sum_{q=-m}^m w_q \varphi^i(x_p - ax_q) L_k^i(x_q),$$

and a similar approximation holds for (16). Let \mathbf{L}^i , \mathbf{M}^i , and \mathbf{d}^i denote $(2m + 1)$ -dimensional vectors given by $\mathbf{L}_p^i = L_k^i(x_p)$, $\mathbf{M}_p^i = M_k^i(x_p)$, $\mathbf{d}_p^i = d^i(x_p)$ and $\mathbf{\Phi}^i$ denote a $(2m + 1) \times (2m + 1)$ matrix given by $\Phi_{pq}^i = w_q \varphi^i(x_p - ax_q)$. Then the above equation can be written as $\mathbf{L}^i = \mathbf{d}^i + \mathbf{\Phi}^i \mathbf{L}^i$, or equivalently

$$\mathbf{L}^i = (\mathbf{I} - \mathbf{\Phi}^i)^{-1} \mathbf{d}^i,$$

where \mathbf{I} denotes the $(2m + 1)$ -dimensional identity matrix. By a similar argument, we can write

$$\mathbf{M}^i = (\mathbf{I} - \mathbf{\Phi}^i)^{-1} \mathbf{1},$$

where $\mathbf{1}$ is a $(2m + 1)$ -dimensional vector of ones.

³Since we are considering a quadrature rule with $2m + 1$ points over the interval $[-k, k]$, the middle abscissas x_0 is equal to 0.

2) *Computing $\partial_k L_k^i(0)$ and $\partial_k M_k^i(0)$* : Differentiate both sides of (15) and (16) with respect to k and use Leibniz rule to get

$$\partial_k L_k^i(x) = \varphi^i(x - ak) L_k^i(k) + \varphi^i(x + ak) L_k^i(-k) + \int_{-k}^k \varphi^i(x - ay) \partial_k L_k^i(y) dy, \quad (17)$$

$$\partial_k M_k^i(x) = \varphi^i(x - ak) M_k^i(k) + \varphi^i(x + ak) M_k^i(-k) + \int_{-k}^k \varphi^i(x - ay) \partial_k M_k^i(y) dy. \quad (18)$$

Eqs. (17) and (18) are also Fredholm integral equations of the second kind. Since d^i and φ^i are even, we can use (15) and (16) to show that L_k^i and M_k^i are also even. In particular, $L_k^i(k) = L_k^i(-k)$ and $M_k^i(k) = M_k^i(-k)$. Now notice that (18) is simply a scaled version of (17). Thus, we have that

$$\frac{\partial_k L_k^i(0)}{\partial_k M_k^i(0)} = \frac{L_k^i(k)}{M_k^i(k)} = \frac{\mathbf{L}_m^i}{\mathbf{M}_m^i}.$$

Substituting this back in (14), we get that the value of information can be computed as

$$\text{VOI}^i(k) = \mathbf{M}_0^i \frac{\mathbf{L}_m^i}{\mathbf{M}_m^i} - \mathbf{L}_0^i. \quad (19)$$

D. An example: Gauss-Markov process

Suppose $\{W_t\}_{t \geq 0}$ is an i.i.d. Gaussian process with mean zero and variance σ^2 . Therefore, the process $\{X_t\}_{t \geq 0}$ is a Gauss-Markov process. Fig. 2 shows the values of information for various choices of parameters (a, σ) .

For Gauss-Markov processes, we only need to compute the value of information for $\sigma = 1$ for the following reason. First, for ease of notation, we use $L_{k,\sigma}^i$ and $M_{k,\sigma}^i$ to show the dependence on σ . Then, by a change of variables, we can show that

$$L_{k,\sigma}^i(x) = \sigma^2 L_{k/\sigma,1}^i(x/\sigma) \quad \text{and} \quad M_{k,\sigma}^i(x) = L_{k/\sigma,1}^i(x/\sigma).$$

Substituting this in (19), we get that

$$\text{VOI}_\sigma^i(k) = \sigma^2 \text{VOI}_1^i(k/\sigma).$$

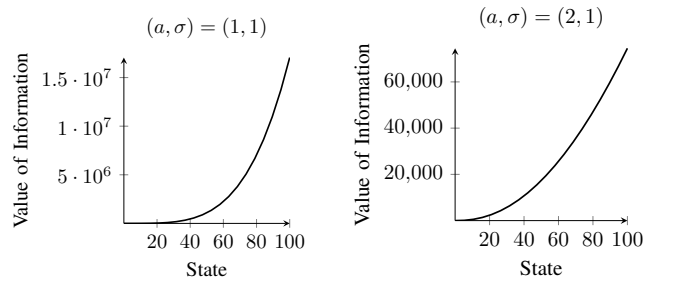


Fig. 2: The value of information for a Gauss-Markov process for different values of the parameters (a, σ) .

E. The priority assignment scheme

In this section, we summarize the priority assignment scheme. We assume that the parameters (a^i, φ^i) for the observation processes of the n sensors are given. Before the system starts running, each sensor computes the value of information function $\text{VOI}^i(\cdot)$ using (19). Either the discretized approximation or a polynomial approximation of this function is stored at each sensor. When the system is running, at each time t , sensor i observes the state X_t^i , computes E_t^i using (8), and sets the priority to be $\text{VOI}^i(E_t^i)$. The sensor with the highest priority, which is picked using a CAN-like contention resolution scheme, transmits its packet.

Note that although we started with the assumption that all sensors at time t know $S_{1:t-1}$, this information is not needed to implement the proposed priority assignment scheme. To compute the value of information, sensor i , $i \in N$, only needs to know the value of E_t^i which evolves according to (8) (or equivalently, (6)). Thus, sensor i only needs to know the events $\{S_{t'} = i\}_{t' < t}$, i.e., the time instances when it transmitted in the past. No information about other sensors is needed.

IV. NUMERICAL EXAMPLE

We consider a system with n sensors, each observing a Gauss-Markov process. We compare the performance of three schemes: a static TDMA (time division multiple access) priority scheme that alternates between all sensors one-by-one; a dynamic priority allocation scheme that sets the priority equal to e^2 (this corresponds to the scheme proposed in [2]); and a dynamic priority allocation scheme that sets the priority according to the value of information. We refer to these schemes as TDMA, ERR, and VOI, respectively. For VOI, we approximate the integration using a Gauss-Legendre quadrature of order $m = 256$ (i.e., with $2m + 1 = 513$ points). We compare these schemes by running Monte Carlo simulations for $T = 100\,000$ time steps.

We use the following three scenarios to compare these schemes. Each scenario consists of 50 sensors, but they vary in the heterogeneity of sensors.

- **Scenario A** consists of 50 homogeneous sensors, each with parameters $(a^i, \sigma^i) = (1, 1)$.
- **Scenario B** consists of 25 sensors with parameters $(a^i, \sigma^i) = (1, 1)$ and 25 sensors with parameters $(a^i, \sigma^i) = (1, 5)$.
- **Scenario C** consists of 20 sensors with parameters $(a^i, \sigma^i) = (1, 1)$, 15 sensors with parameters $(a^i, \sigma^i) = (1, 5)$, and 15 sensors with parameters $(a^i, \sigma^i) = (1, 10)$.

The average expected distortion of the three schemes for the three scenarios are shown in Table I. Note that in Scenario A, ERR and VOI have identical performance. This is because the priority assignment of ERR and VOI are even and quasi-convex functions. Since all sensors are identical, the sensor with the maximum priority is equal to the sensor with the highest absolute value. Therefore, ERR and VOI make identical scheduling decisions and, therefore, have identical performance.

These results show that both dynamic priority allocation schemes outperform a time division multiplexing scheme.

TABLE I: Performance of TDMA, ERR, and VOI on three different scenarios.

Scenarios	TDMA	ERR	VOI
Scenario A	24.35	8.47	8.47
Scenario B	315.79	92.47	76.45
Scenario C	921.14	255.35	207.45

When the sensors are heterogeneous, then the proposed scheme of assigning priorities based on value of information outperforms the baseline scheme of assigning priorities based on instantaneous estimation error.

V. CONCLUSION

We consider the problem of assigning priorities for scheduling multiple sensor measurements over CAN-like networks. We propose a dynamic priority allocation scheme, where the priority is assigned according the value of information. We show that the value of information can be computed by solving two Fredholm integral equations. Numerical examples suggest that the proposed priority assignment scheme outperforms the existing schemes in the literature.

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