Remote-state estimation with packet drop

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Block diagram

\[ U_t = f_t(X_{t-1}, U_{t-1}), \quad t = 0, 1 \quad H_t \in \{\text{ON}, \text{OFF}\} \]

Markov process \( X_t \)
Transmitter \( U_t \)
Erasure channel \( Y_t \)
Receiver \( X_t = h(Y_t) \)
ACK/NACK

Difficulty

Decentralized optimization problem; non-classical information structure

Salient features

- Real-time transmission: size of data packet not important; sensing is cheap compared to transmission
- Known prior of packet size, TCP-like protocol

Model

- \(X, W_t \in \mathbb{Z}, W_t \sim \text{unimodal and symmetric pmf}
- \(Y_t = U_t X_t + (1 - U_t) K_t\)
- Pre-set errors: \(d(X_t - \hat{X}_t), d(0) = 0, d(x) = d(-x), d(x) \leq d(x + 1) \) for any \(x \in \mathbb{Z}\)

Performance metrics

\[
D_t(f, g) := (1 - \beta)E[f(X_t) \mid X_0 = 0]
\]

\[
N_t(f, g) := (1 - \beta)E[f(X_t) \mid X_0 = 0]
\]

Optimization problems

- Costly communication: \(C_t(f, g, \lambda) := \arg \min_{f, \lambda} D_t(f, g) = \arg \min_{f, \lambda} D_t(f, g) + \lambda N_t(f, g)\)
- Costly communication: \(\alpha \in (0, 1), D_t(\alpha) := \arg \min_{f, \lambda} D_t(f, g) + \lambda N_t(f, g)\)

Structure of optimal strategies

\[ Z_t = \begin{cases} X_t, & \text{if } Y_t \neq \epsilon, \quad X_t \neq \epsilon \quad Z_0 = 0. \\ X_t, & \text{if } Y_t = \epsilon, \quad X_t = \epsilon \end{cases} \]

Estimation strategy: \(X_t = g(Z_t) = \begin{cases} X_t, & \text{if } Y_t \neq \epsilon, \quad X_t \neq \epsilon \\ X_t, & \text{if } Y_t = \epsilon \end{cases} \)

Transmission strategy: \(U_t = \mathcal{U}_t(E_t) = \begin{cases} 1, & \text{if } |E_t| \geq k \\ 0, & \text{if } |E_t| < k \end{cases} \)

Majorization, AUS, Sizer convexity
- Beak state, POMDP
- \((E_t)_{t>0} \) is a regeneration process

Performance of threshold-based strategy

\[
\begin{align*}
L_t^{(k)}(n) &= \begin{cases} d(n) + \beta \sum_{m \geq k} n \cdot L_t^{(k)}(m), & \text{if } |n| \geq k \\ d(n) + \beta \sum_{m < k} n \cdot L_t^{(k)}(m), & \text{if } |n| < k \\ 
M_t^{(k)}(n) &= \begin{cases} 1 + \beta \sum_{m \geq k} n \cdot M_t^{(k)}(m), & \text{if } |n| \geq k \\ 1 + \beta \sum_{m < k} n \cdot M_t^{(k)}(m), & \text{if } |n| < k \\ 
\end{cases}
\end{align*}
\]

Figure: Optimal performances

Example: symmetric birth-death Markov chain

- \(p = 0.3, \beta = 0.99, \epsilon \in (0.03, 0.7)\). Constrained performance