Structural results for two-user interactive communication

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Motivating example: self-driven cars

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- Operating in a **common environment** - evolving, Markovian
- Access to different information
- Goal: avoid collision
- Control strategy: involving **zero-delay, sequential** communication.

The model

- Two users; sequentially observe two correlated sources.

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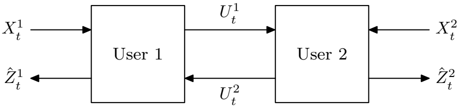

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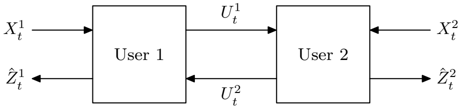

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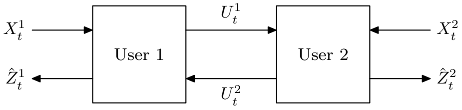

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- Action (encoder):
  $U_t^1 = f_t^1(X_{1:t}^1, U_{1:t-1}^1, U_{1:t-1}^2), \quad U_t^2 = f_t^2(X_{1:t}^2, U_{1:t}^1, U_{1:t-1}^2)$

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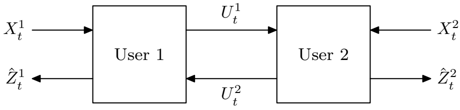

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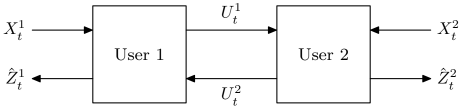

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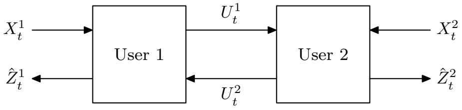

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- Distortion function: $d_t^i : \mathcal{Z} \times \mathcal{Z} \to \mathbb{R}_{\geq 0}$

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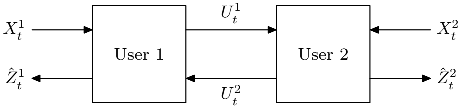
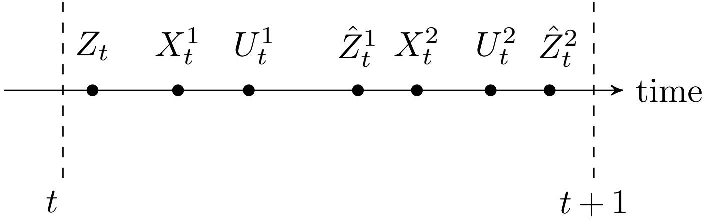


X_t^1
$\hat{Z}_t^1$
User 1
U_t^1
U_t^2
User 2
X_t^2
$\hat{Z}_t^2$



Z_t
X_t^1
U_t^1
$\hat{Z}_t^1$
X_t^2
U_t^2
$\hat{Z}_t^2$
time
t
$t+1$

The optimization problem

- Encoding strategy: $\mathbf{f}^i := (f_1^i, \cdots, f_T^i)$, $i \in \{1, 2\}$
- Decoding strategy: $\mathbf{g}^i := (g_1^i, \cdots, g_T^i)$, $i \in \{1, 2\}$
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Finite horizon optimization problem

Choose a communication strategy $(\mathbf{f}^1, \mathbf{f}^2, \mathbf{g}^1, \mathbf{g}^2)$ that minimizes

$$J(\mathbf{f}^1, \mathbf{f}^2, \mathbf{g}^1, \mathbf{g}^2) = \mathbb{E}\Big[\sum_{t=1}^{T}\sum_{i=1}^{2}\big[c^i(U_t^i) + d_t^i(Z_t, \hat{Z}_t^i)\big]\Big].$$

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- **This paper**: Constant Z.

Literature overview: real-time communication

- Point-to-point case
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  - **Asnani and Weissman**, 2013; source-coding with finite lookahead
- Multi-terminal case
  - **Nayyar and Teneketzis**, 2011; source-coding without feedback
  - **Yuksel**, 2013; source-coding with feedback

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So, what is new?

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Our paper:

- Real-time communication in tandem with interactive communication

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So, what is new?

Our paper:

- Real-time communication in tandem with interactive communication
- Noiseless channel
- Finite-horizon optimization problem
- Structure of optimal strategies

Team-theoretic approach

We use Team Theory to **characterize the qualitative properties** of the solution

Team-theoretic approach

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- Multiple players,

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- Access to different information,

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Our model is a team.

Solution methodology for team problems

Mahajan, 2013 - Control sharing
Nayyar, Mahajan and Teneketzis, 2013 - Partial history sharing

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- Step 1: *Person-by-person* approach: sufficient statistic for $\mathbf{X}^{\mathbf{i}}_{\mathbf{1:t}}$

Solution methodology for team problems

Mahajan, 2013 - Control sharing

Nayyar, Mahajan and Teneketzis, 2013 - Partial history sharing

- Step 1: *Person-by-person* approach: sufficient statistic for $\mathbf{X^i_{1:t}}$
- Step 2: *Common information* approach: sufficient statistic for $(\mathbf{U^1_{1:t-1}}, \mathbf{U^2_{1:t-1}})$ for user 1 and for $(\mathbf{U^1_{1:t}}, \mathbf{U^2_{1:t-1}})$ for user 2 and a suitable dynamic program

Solution methodology for team problems

Person-by-person approach

- Arbitrarily fix the strategy of one user and search for the best response strategy of the other.
- Identify a sufficient statistic $\xi^i_{t|t-1}$ of $x^i_{1:t}$.
- No loss of optimality:
  $U^1_t = \hat{f}^1_t(\Xi^1_{t|t-1}, U^1_{1:t-1}, U^2_{1:t-1}), U^2_t = \hat{f}^2_t(\Xi^2_{t|t-1}, U^1_{1:t}, U^2_{1:t-1})$.
- Similar structure for the decoder.

Solution methodology for team problems

Common information approach

- Based on common information available to both users, identify a sufficient statistic: π_t^1 of $(u^1_{1:t-1}, u^2_{1:t-1})$ at user 1 and a sufficient statistic π_t^2 of $(u^1_{1:t}, u^2_{1:t-1})$ at user 2
- No loss of optimality: $U_t^1 = \tilde{f}_t^1(\Xi^1_{t|t-1}, \Pi^1_t)$, $\quad U_t^2 = \tilde{f}_t^2(\Xi^2_{t|t-1}, \Pi^2_t)$

Step 1: person-by person approach

Key Lemma: conditional independence

$$\mathbb{P}(x^1_{1:t}, x^2_{1:t} \mid z, u^1_{1:t}, u^2_{1:t}) = \prod_{i \in \{1,2\}} \mathbb{P}(x^i_{1:t} \mid z, u^1_{1:t}, u^2_{1:t})$$

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Similar results:

- CEO problem - V. Prabhakaran, Ramchandran and Tse; Allerton, 2004
- Control sharing - Mahajan; IEEE TAC, 2013
- Secret key agreement - Tyagi and Watanabe; IEEE TIT, 2015

Step 1: person-by person approach

The belief states

$$\xi^i_{t|t-1}(z) = \mathbb{P}(Z = z \mid X^i_{1:t} = x^i_{1:t}, U_{1:t-1} = u_{1:t-1}),$$
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Lemma: update of $\xi^i_{t|t-1}$ and $\xi^i_{t|t}$

There exist functions $F^i_{t|t}$, $F^i_{t+1|t}$, $i \in \{1, 2\}$, such that

$$\xi^i_{t|t} = F^i_{t|t}(\xi^i_{t|t-1}, u_{1:t}, \mathbf{f}^{-i}), \; \xi^i_{t+1|t} = F^i_{t+1|t}(\xi^i_{t|t}, u_{1:t}, x^i_{t+1}).$$

Step 1: P-by-P approach - structure of optimal strategies

Decoder is solving a filtering problem.

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Optimal decoding strategy

$$\hat{Z}_t^i = \hat{g}^i(\Xi_{t|t}^i), \quad i \in \{1, 2\}, \tag{1}$$

$$\hat{g}^i(\xi^i) = \arg\min_{\hat{z}^i \in \mathcal{Z}} \sum_{z \in \mathcal{Z}} d^i(z, \hat{z}^i)\xi^i(z).$$

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To find **best performing** encoder at user 1:

- Fix $\mathbf{g}^1, \mathbf{g}^2$ as (1), $\mathbf{f}^2$ arbitrarily

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- Find **sufficient statistic** for the encoder at user 1

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Lemma: sufficient statistic for encoder at user 1

$R_t^1 = (\Xi^i_{t|t-1}, U_{1:t-1})$ is an information state for the encoder at user 1.

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Lemma: sufficient statistic for encoder at user 1

$R^1_t = (\Xi^i_{t|t-1}, U_{1:t-1})$ is an information state for the encoder at user 1.

- R^1_t is a function of $(X^1_{1:t}, U_{1:t-1})$, available at user 1.
- $\mathbb{P}(R^1_{t+1} \mid X^1_{1:t}, U_{1:t-1}, U^1_t) = \mathbb{P}(R^1_{t+1} \mid R^1_t, U^1_t)$
-

$$\mathbb{E}\Big[\sum_{i\in\{1,2\}} (c^i(U^i_t) + d^i(Z, \hat{Z}^i_t)) \mid X^1_{1:t}, U_{1:t-1}, U^1_t\Big]$$
$$= \mathbb{E}\Big[\sum_{i\in\{1,2\}} (c^i(U^i_t) + d^i(Z, \hat{Z}^i_t)) \mid R^1_t, U^1_t\Big]$$

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Step 2: Common information approach

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Remaining information - Local information: $L_t^i = \xi^i_{t|t-1}$

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Common information?

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$$U^i_t = f^i_t(L^i_t, C^i_t).$$

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The coordinated system

- From the point of view of a **virtual decision maker**

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The coordinated system

- From the point of view of a **virtual decision maker**
- Observes C_t^i

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The coordinated system

- From the point of view of a **virtual decision maker**
- Observes C_t^i
- Chooses prescriptions $\phi_t^i : L_t^i \mapsto U_t^i$
- Encoder i uses ϕ_t^i and its local information to generate U_t^i.

The coordinated system

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- Centralized system

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- Tools from Markov Decision Theory

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Lemma: update of $\xi^i_{t|t-1}$

$$\xi^i_{t|t} = F^i_{t|t}(\xi^i_{t|t-1}, u^{-i}_t, \phi^{-i}_t).$$

Recall: $\xi^i_{t|t} = F^i_{t|t}(\xi^i_{t|t-1}, u_{1:t}, \mathbf{f}^{-i})$

The coordinated system

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The belief states

$$\pi_t^1(\xi^1, \xi^2) = \mathbb{P}(\Xi^1_{t|t-1} = \xi^1, \Xi^2_{t|t-1} = \xi^2 \mid U_{1:t-1} = u_{1:t-1}),$$
$$\pi_t^2(\xi^1, \xi^2) = \mathbb{P}(\Xi^1_{t|t-1} = \xi^1, \Xi^2_{t|t-1} = \xi^2 \mid U_{1:t-1} = u_{1:t-1}, U_t^1 = u_t^1),$$

Update similar to $\xi^i_{t|t-1}$

The coordinated system

Theorem: Sufficient statistic

No loss of optimality

$$U_t^1 = \tilde{f}_t^1(\xi_{t|t-1}^1, \Pi_t^1), \quad U_t^2 = \tilde{f}_t^2(\xi_{t|t-1}^2, \Pi_t^2).$$

The coordinated system

Theorem: Dynamic program

- $D^i_t(\xi^i_{t|t}) \coloneqq \sum_{z \in \mathcal{Z}} d^i_t(z, \hat{g}^i(\xi^i_{t|t}))\xi^i_{t|t}(z)$, $i \in \{1, 2\}$

- $V^2_{T+1}(\pi^2) = 0$. For $t = T,\ T-1, \dots, 1$

$$V^2_t(\pi^2) = \min_{\phi^2_t \colon \Delta(\mathcal{Z}) \to \mathcal{U}^2} \mathbb{E}[c^2(U^2_t) + D^2_t(\Xi^2_{t|t}) + V^1_{t+1}(\Pi^1_{t+1}) \mid \Pi^2_t = \pi^2, U^2_t = \phi^2_t(\Xi^2_{t|t-1})],$$

$$V^1_t(\pi^1) = \min_{\phi^1_t \colon \Delta(\mathcal{Z}) \to \mathcal{U}^1} \mathbb{E}[c^1(U^1_t) + D^1_t(\Xi^1_{t|t}) + V^2_t(\Pi^2_t) \mid \Pi^1_t = \pi^1, U^1_t = \phi^1_t(\Xi^1_{t|t-1})].$$

Discussion

Contributions? Limitations?

Discussion

Contributions

- Identify a sufficient statistic at the encoder and the decoder; the domain of which does not depend on time
- The DP: identify optimal strategies
- The search complexity increases linearly with time horizon (rather than double exponentially, as for brute force search)
- Natural extension to Infinite horizon setup; time-homogeneous optimal strategies
- Extension to multi-terminal setup: n users; sequentially broadcasts their own action to all users and generates its own estimate and so on.

Discussion

Computational issues

- Computationally formidable !
- Special case: Finite $\mathcal{Z}$ (say, cardinality n). Then, $\Delta(\mathcal{Z}) = \mathbb{R}^{n-1}$; $\Delta(\Delta(\mathcal{Z}) \times \Delta(\mathcal{Z})) = \mathbb{R}^{2n-2}$.

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- DP similar to POMDP.

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Discussion

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- DP similar to POMDP. Minimization over functional space.
  - Point-based algorithms for continuous-state POMDPs
  - Discretization based algorithms developed for real-time communication - Wood, Linder and Yuksel, ISIT 2015

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✗

Future scope

- Study of some special structure of Markovian Z_t so that the belief states do not grow with time
- Finite state machines - DP decomposition
- Lipsa and Martins, IEEE TAC, 2011: Optimality of threshold-based strategies for Gaussian setup and the transmitter may transmit or not

Thank you !