INVITED PAPER

Measurement Scheduling for Soil Moisture Sensing: From Physical Models to Optimal Control

Scheduling and estimation methods are compared in this paper, and a method that sharply reduces energy consumption is presented; the paper is grounded in the physics of soil moisture.

By David I Shuman, *Member IEEE*, Ashutosh Nayyar, *Student Member IEEE*, Aditya Mahajan, *Member IEEE*, Yuriy Goykhman, *Student Member IEEE*, Ke Li, Mingyan Liu, *Member IEEE*, Demosthenis Teneketzis, *Fellow IEEE*, Mahta Moghaddam, *Fellow IEEE*, and Dara Entekhabi, *Senior Member IEEE*

ABSTRACT | In this paper, we consider the problem of monitoring soil moisture evolution using a wireless network of in situ sensors. Continuously sampling moisture levels with these sensors incurs high-maintenance and energy consumption costs, which are particularly undesirable for wireless networks. Our main hypothesis is that a sparser set of measurements can meet the monitoring objectives in an energy-efficient manner. The underlying idea is that we can trade off some inaccuracy in estimating soil moisture evolution for a significant reduction in energy consumption. We investigate how to dynamically schedule the sensor measurements so as to balance this tradeoff. Unlike many prior studies on sensor scheduling that make generic assumptions on the statistics of the observed phenomenon, we obtain statistics of soil moisture evolution from a physical model. We formulate the optimal measurement scheduling and estimation problem as a partially observable Markov decision problem (POMDP). We then utilize special features of the problem to approximate the POMDP by a computationally simpler finite-state Markov decision problem (MDP). The result is a scalable, implementable technology that we have tested and validated numerically and in the field.

KEYWORDS | Dynamic programming; energy conservation; Markov decision process; scheduling; soil moisture sensing; stochastic optimal control; wireless sensor networks

Manuscript received November 20, 2009; revised April 8, 2010; accepted May 21, 2010. Date of publication August 30, 2010; date of current version October 20, 2010. This work was carried out at the University of Michigan and the Massachusetts Institute of Technology through a grant from the National Aeronautics and Space Administration (NASA) Earth Science Technology Office, Advanced Information Systems Technologies program.

D. I Shuman is with the Institute of Electrical Engineering, École Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland (e-mail: david.shuman@epfl.ch).

A. Nayyar, **Y. Goykhman**, **M. Liu**, **D. Teneketzis**, and **M. Moghaddam** are with the Electrical Engineering and Computer Science Department, University of Michigan, Ann Arbor, MI 48109 USA (e-mail: anayyar@umich.edu; yuriy@umich.edu; mingyan@umich.edu; teneket@umich.edu; mmoghadd@umich.edu).

A. Mahajan is with the Department of Electrical and Computer Engineering, McGill University, Montreal, QC H3A 2T5 Canada (e-mail: adityam@ieee.org).

K. Li is with the State Key Laboratory of Precision Management Technology and Instruments, Tsinghua University, Beijing 100084, China and also with the Electrical Engineering and Computer Science Department, University of Michigan, Ann Arbor, MI 48109 USA (e-mail: ericlike@eecs.umich.edu).

D. Entekhabi is with the Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA 02138 USA (e-mail: darae@mit.edu).

Digital Object Identifier: 10.1109/JPROC.2010.2052532

I. INTRODUCTION

Spatial and temporal variations of soil moisture are key factors in several scientific areas. For example, soil moisture is a measurement need in four (climate, carbon, weather, and water) out of the six National Aeronautics and Space Administration (NASA) Earth Science strategic focus areas [1]. It is used in all land surface models, water and energy balance models, weather prediction models, general circulation models, and ecosystem process simulation models. Therefore, it is important to develop technologies to enable the monitoring of soil moisture evolution.

A number of current systems achieve this by deploying dense sensor networks over the region of interest, and using the sensors to continuously sample soil moisture levels (see, e.g., [2] and [3]). However, deploying a large number of sensors that sample continuously incurs high installation, maintenance, and energy consumption costs. Energy efficiency is particularly important in *wireless* sensor networks, which are often expected to run for long periods of time without human intervention, while relying on limited battery power or renewable energy resources. A wireless sensor node's radio is the biggest consumer of energy. For example, the current draw for the XBee PRO ZB module by Digi International, Minnetonka, MN, is on the order of 100 mA when the radio is transmitting and 10 mA when the radio is receiving or idling. When the radio is powered off in sleep mode, however, the current draw is less than 10 μA [4].

Our main hypothesis in this paper is that a sparser set of measurements can meet the monitoring objectives in an energy-efficient manner. The underlying idea is that we can trade off some inaccuracy in estimating soil moisture evolution for a significant reduction in energy consumption. The main research question we address is how to dynamically schedule the sensor measurements so as to balance this tradeoff. The premise is that an intelligent scheduling strategy should utilize statistical models of soil moisture evolution and sensor noise, as well as the outcomes of past sensor measurements, in order to schedule measurements when they are expected to yield the most information.

This problem belongs to the larger class of sensor scheduling problems. In some sensor scheduling problems, such as those introduced in [5] and [6], the underlying system being monitored is *time invariant*. Because soil moisture evolves with time, techniques developed for sensor scheduling problems with a time-invariant underlying system are not directly applicable to our problem. Techniques developed to monitor time-varying linear Gaussian systems under a quadratic estimation criterion (e.g., [7]–[11]) are also not directly applicable, as empirical data show that such a linear Gaussian model is not appropriate for soil moisture evolution (see, e.g., [12]).

Instead, we model soil moisture evolution as a discrete-time Markov process and use Markov decision theory to determine the sensor scheduling policy. Similar approaches have been investigated in [13]–[20], where sensor scheduling problems with cost considerations are formulated as instances of the partially observable Markov decision problem (POMDP) of the standard stochastic control literature (see, e.g., [21] and [22]). Modeling the sensor scheduling problem as a POMDP enables one to write down a dynamic programming decomposition and use a standard set of numerical techniques to solve the problem. However, these numerical techniques, which are discussed in detail in Section III-A, do not always scale well with the size of the problem. For the application under consideration, a wireless sensor network typically requires a large number of sensors, on the order of 10s. Exactly solving the POMDP for a network of this size is computationally challenging.

We circumvent these computational difficulties by utilizing special features of the system to make the scheduling problem tractable for a larger number of sensors. These features are: 1) the control actions do not affect the underlying state dynamics; 2) the *in situ* sensors we use to measure soil moisture levels are effectively noiseless (see Section II-A2 for further justification); and 3) the sensors have only two modes of operation, namely, active and sleeping. These features are a result of the facts that the *in situ* sensors are passive measuring devices with low observation noise and calibration errors, and the sensors can be turned either on or off by an actuator.

We leverage these three properties by introducing two justifiable assumptions in Section III-B that allow us to reduce the POMDP formulation to a computationally simpler finite-state Markov decision problem (MDP). The solution to the resulting MDP gives us an easily implementable sensor scheduling policy that achieves the objective of monitoring soil moisture evolution in an energy-efficient manner. Moreover, due to the computational savings, this approach scales with the size of the problem better than standard POMDP techniques. Finally, this approach is not restricted to sensing soil moisture; it is also potentially applicable to a number of other sensing applications with these same features, such as temperature, air quality, and water quality monitoring.

The remainder of this paper is organized as follows. In the next section, we describe the control architecture, physics-based model of soil moisture evolution, and sensor measurement model. We also formulate the scheduling problem as a POMDP. In Section III, we focus on the special case of sensors at multiple depths at a single lateral location. We introduce our modeling assumptions, and formulate an equivalent MDP to solve the scheduling problem. We then discuss the scheduling and estimation problem with sensors at multiple lateral locations. We present a numerical example in Section IV, and discuss how to incorporate meteorological observations in Section V. Section VI concludes the paper.

II. PROBLEM DESCRIPTION

The overall system objective is to use *in situ* sensors to track the random evolution of soil moisture over time, while limiting the energy consumed in taking measurements. The physical setup consists of a sensor network deployed over the field of interest. The sensors are placed at multiple lateral locations. At each lateral location, multiple sensors at different depths are wired to a local actuator. The

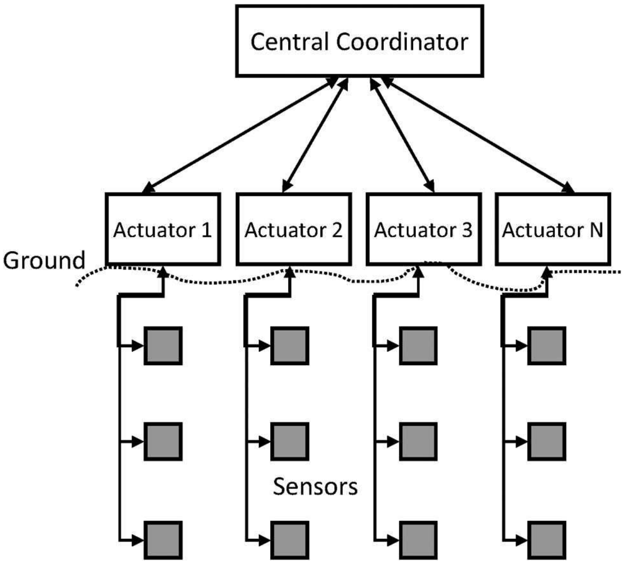


Fig. 1. *The control architecture.*

actuator is capable of wirelessly communicating with a central coordinator, and actuating the colocated *in situ* sensors. The central coordinator dynamically schedules the measurements at each location, transmits the scheduling commands to each local actuator, and subsequently receives the sensor measurement readings back from the actuators. The coordinator uses all of the measurement readings to estimate the soil moisture at all locations and depths, and to schedule future measurements. The coordinator's task is to leverage the spatial and temporal correlations of soil moisture, in order to make the best estimates of its evolution with as few measurements as possible. A diagram of this control architecture is shown in Fig. 1. The physical implementation of this wireless communication and actuation system is discussed further in [23].

A. Modeling Components

In order to determine a scheduling and estimation strategy to be used by the coordinator, we need statistical models of soil moisture evolution and of the sensors. We describe those here.

1) Physical Model of Soil Moisture Evolution: Soil moisture varies as a function of time and 3-D space in response to variable exogenous forcings such as rainfall, temperature, cloud cover, and solar radiation. It is also influenced by landscape parameters such as vegetation cover, soil type, and topography. The soil moisture variations in time and depth, or infiltration, can be modeled as a pair of partial differential equations (the so-called Richards equations [24]) in the case of a flat horizontal surface, subject to constraints such as soil heat flow (represented by another space-time partial differential equation involving soil moisture and temperature), vegetation growth, and solute flow [25]. For a homogeneous and flat landscape, the spatial variations can be assumed to be limited to one dimension (depth only). In the presence of topography, the 1-D infiltration in the direction of the surface normal is redistributed, dominated by gravity, by the lateral fluxes in the vadose (unsaturated) zone as well as the boundary values imposed by the phreatic (saturated) zone at depth. Triangulated irregular network (TIN) surface models, which discretize the surface topography into triangle-shaped mesh elements (also called Voronoi cells) for subsequent numerical analyses, efficiently model local topography for the purpose of this lateral distribution process.

Proper modeling of the soil moisture evolution process has to take into account the water flow mechanisms and the energy balance of the entire landscape, including the surface–atmosphere interactions. It therefore has to include mechanisms such as rainfall, groundwater flow, evapotranspiration demand, and runoff. Among the most sophisticated numerical models capable of predicting the time-space soil moisture evolutions is the TIN-based real-time integrated basin simulator (tRIBS) [26]–[29]. The mesh-generation algorithm within tRIBS is an adaptive discretization scheme that resembles the spatial pattern of the landscape with variable resolution to ensure the impact of the basin response is properly represented.

The tRIBS model can be initialized and further driven with distributed and spatially explicit terrain data such as digital elevation models (DEMs) for topography, satellite- or field-derived vegetation cover information, soil type, precipitation estimates from satellite or ground-based radar sources, temperature measurements, and various other sources of meteorological and landscape data. The sources of uncertainty in the predictions of tRIBS include modeling and discretization errors, as well as the errors in the landscape parameters and time-varying forcing factors such as precipitation and temperature.

We use this model to simulate long time-series realizations of the soil moisture over a basin. We then quantize these realizations and use them to generate a joint spatial–temporal statistical model of soil moisture. Fig. 2 shows a sample basin and the simulated soil moisture evolution for two locations in this basin.

2) Sensor Model: The *in situ* sensors are localized probes, whose measurements are related to soil moisture values via simple empirical models. Several standard methods of *in situ* sensing exist, such as time-domain reflectometer (TDR) probes, neutron probes, capacitance probes, and ring resonators. We use capacitance probes from Decagon, model ECH$_2$O EC-5, which make highly localized measurements. Specifically, these probes have $\pm$1%–2% accuracy if calibrated according to soil type [30]. Such a probe is shown in Fig. 3(a). We calibrate the probes through a standard procedure: we use a local soil sample, dry it, and add water in known proportions. With each addition of

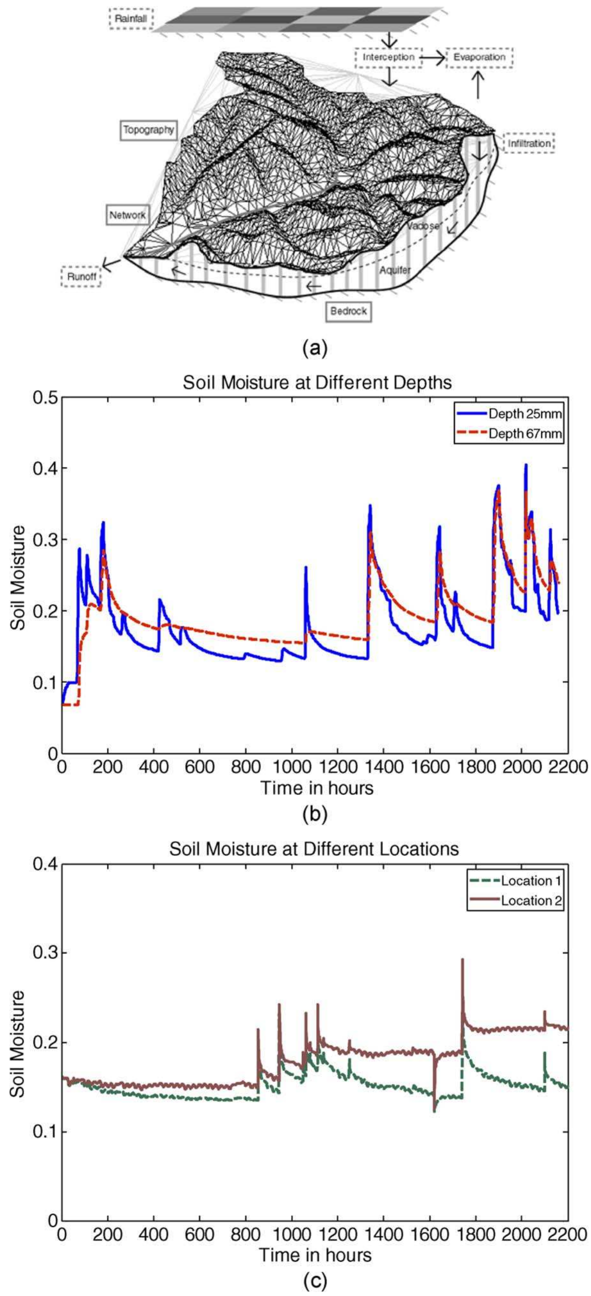


Fig. 2. ***(a) Nominal 2 km $\times$ 2 km basin used for tRIBS simulations. The location in this example has climatology consistent with Oklahoma. (b) Example of temporal evolution of soil moisture at two different depths (25 and 67 mm) at the same lateral position. (c) Example of temporal evolution of soil moisture at the same depth (67 mm), but at two different lateral locations.***

water (which results in a known value of moisture content), probe measurements are taken. This produces a graph of probe voltage versus water content, which is subsequently used to fit a third-order polynomial. This polynomial is shown in Fig. 3(b), along with experimental data points. This simple soil moisture retrieval model (the polynomial) represents a calibration accuracy of about 1%. Because the desired quantization intervals are typically 3%–4%, we believe modeling these *in situ* sensors as noiseless is a reasonable starting assumption. While incorporating the calibration noise into the sensor model increases the accuracy of the model, it also significantly increases the complexity of the problem. Note that, unlike the observation noise typically modeled in POMDPs, the calibration noise is not an independently and identically distributed (i.i.d.) process, and it cannot be directly incorporated into the POMDP framework. Given the small magnitude of the noise, we believe a noiseless assumption provides a computationally tractable approximation to a difficult problem.

B. Problem Formulation

We consider a sensor network consisting of sensors at L lateral locations, with sensors at D different depths at each lateral location. We model the quantized soil moisture evolution as a discrete-time, discrete-valued stochastic process $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$, where

$$\mathbf{X}_t = \begin{bmatrix} X_t^{1,1} & X_t^{1,2} & \cdots & X_t^{1,D} \\ X_t^{2,1} & X_t^{2,2} & \cdots & X_t^{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ X_t^{L,1} & X_t^{L,2} & \cdots & X_t^{L,D} \end{bmatrix}$$

and $X_t^{l,d}$ denotes the soil moisture quantile at time t at location l and depth d. We denote the finite sample space of $\mathbf{X}_t$ by $\mathcal{X}$, which represents all possible quantized moisture levels. The statistics of the process $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$ are inferred from the physical models, as described in Section II-A1. We assume that the coordinator observes perfectly the initial soil moisture levels. This observation is given by $\mathbf{Y}_0 = \mathbf{X}_0$. At each time $t = 1, 2, \ldots$, the central coordinator chooses which sensors should take a measurement. Its decision is denoted by the matrix

$$\mathbf{U}_t = \begin{bmatrix} U_t^{1,1} & U_t^{1,2} & \cdots & U_t^{1,D} \\ U_t^{2,1} & U_t^{2,2} & \cdots & U_t^{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ U_t^{L,1} & U_t^{L,2} & \cdots & U_t^{L,D} \end{bmatrix}$$

where $U_t^{l,d} \in \{0,1\}$ indicates whether the sensor at location l and depth d should take a measurement $(U_t^{l,d} = 1)$ or not take a measurement $(U_t^{l,d} = 0)$ at time t. The coordinator then communicates the vector $\mathbf{U}_t^l := (U_t^{l,1}, U_t^{l,2}, \ldots, U_t^{l,D})$ to the actuator at lateral location l. The actuator switches on the colocated sensors corresponding to $\mathbf{U}_t^l$, collects the

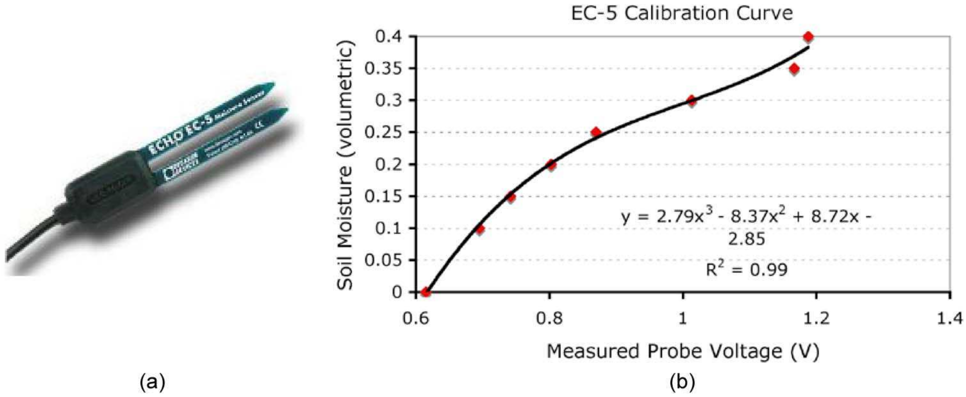


Fig. 3. ***(a) The Decagon ECH$_2$O EC-5 soil moisture probe. (b) The calibration curve derived from experimental data.***

observations $\mathbf{Y}_t^l := (Y_t^{l,1}, Y_t^{l,2} \ldots, Y_t^{l,D})$ of the colocated sensors, and transmits this vector back to the central coordinator. These observations are given by

$$Y_t^{l,d} = \begin{cases} X_t^{l,d}, & \text{if } U_t^{l,d} = 1 \\ b, & \text{if } U_t^{l,d} = 0 \end{cases} \tag{1}$$

where b represents a blank measurement. The central coordinator receives the observations from all L actuators, which collectively form the matrix of observations

$$\mathbf{Y}_t = \begin{bmatrix} Y_t^{1,1} & Y_t^{1,2} & \cdots & Y_t^{1,D} \\ Y_t^{2,1} & Y_t^{2,2} & \cdots & Y_t^{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ Y_t^{L,1} & Y_t^{L,2} & \cdots & Y_t^{L,D} \end{bmatrix}.$$

After receiving these observations, the coordinator forms an estimate of the soil moisture field, and decides which sensors to activate next. The matrix of estimates

$$\hat{\mathbf{X}}_t = \begin{bmatrix} \hat{X}_t^{1,1} & \hat{X}_t^{1,2} & \cdots & \hat{X}_t^{1,D} \\ \hat{X}_t^{2,1} & \hat{X}_t^{2,2} & \cdots & \hat{X}_t^{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{X}_t^{L,1} & \hat{X}_t^{L,2} & \cdots & \hat{X}_t^{L,D} \end{bmatrix}$$

of the soil moisture levels at all locations and all depths is selected as a function of all past observations and scheduling decisions as follows:

$$\hat{\mathbf{X}}_t = h_t(\mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_t, \mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_t). \tag{2}$$

The sequence $\mathbf{h} := (h_1, h_2, \ldots)$ constitutes an *estimation policy*.

The coordinator then selects the next scheduling decision matrix $\mathbf{U}_{t+1}$ as a function of all prior observations and scheduling decisions, as follows:

$$\mathbf{U}_{t+1} = g_t(\mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_t, \mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_t). \tag{3}$$

The sequence $\mathbf{g} := (g_1, g_2, \ldots)$ constitutes a *scheduling policy*.

There are two objectives in determining good scheduling and estimation policies. The first is to conserve energy by limiting the number of sensor measurements. The second is to accurately estimate the soil moisture evolution. Accordingly, we impose energy costs $c(\mathbf{U}_t)$, which are proportional to the number of sensors scheduled to take a measurement at each time t. Additionally, we assess estimation costs $\rho(\mathbf{X}_t, \hat{\mathbf{X}}_t)$, which quantify the accuracy of the soil moisture estimates at each time t. We wish to find scheduling and estimation policies $\mathbf{g}$ and $\mathbf{h}$, respectively, that minimize the infinite-horizon expected discounted cost criterion

$$J^{\mathbf{g},\mathbf{h}} := \mathbb{E}^{\mathbf{g},\mathbf{h}}\left\{\sum_{t=1}^{\infty} \alpha^{t-1} \cdot \left[c(\mathbf{U}_t) + \rho(\mathbf{X}_t, \hat{\mathbf{X}}_t)\right]\right\} \tag{4}$$

where $\alpha \in (0,1)$ is the discount rate.

C. Markovian Assumption

The optimization problem formulated above is, in general, nontractable. One reason for this difficulty is that up to this point, we have not assumed any structure on the statistics of the underlying soil moisture process $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$. In order to identify solvable approximations of the original problem, we need to impose additional statistical structure on the soil moisture process. Motivated by the heuristic reasoning that the current soil moisture is more correlated with the recent past values of the moisture than with moisture values from the distant past, we assume that the soil moisture process is a kth-order Markov process. That is, for all $t = 1, 2, \ldots$

$$\Pr(\mathbf{X}_t | \mathbf{X}_{t-1}, \ldots, \mathbf{X}_0) = \Pr(\mathbf{X}_t | \mathbf{X}_{t-1}, \ldots, \mathbf{X}_{t-k}). \quad (5)$$

As k increases, the statistical behavior described by (5) becomes a better approximation of the statistical behavior of the soil moisture process $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$. Under the kth-order Markovian assumption, one can think of $\mathbf{V}_t := (\mathbf{X}_t, \mathbf{X}_{t-1}, \ldots, \mathbf{X}_{t-k+1})$ as the state that completely describes the future statistical behavior of the soil moisture process. A consequence of (5) is that the process $\{\mathbf{V}_t\}_{t=0,1,2,\ldots}$ is first-order Markovian. The Markovian nature of the process $\{\mathbf{V}_t\}_{t=0,1,2,\ldots}$ allows the optimization problem to be viewed as a POMDP. Note that despite the fact that the sensors are noiseless [i.e., when measurements are taken, they are always correct, as shown in (1)], the process $\{\mathbf{V}_t\}_{t=0,1,2,\ldots}$ is still not perfectly observed. This is because the coordinator receives no observations of the soil moisture when a measurement is not scheduled.

As a first approximation, we assume the soil moisture process $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$ to be first-order Markovian, resulting in the following POMDP.

Problem (P1): Given the statistics of the Markov process $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$ and the observation model of (1) find a scheduling policy $\mathbf{g} := (g_1, g_2, \ldots)$ of the form in (3) and an estimation policy $\mathbf{h} := (h_1, h_2, \ldots)$ of the form in (2) to minimize the objective

$$J^{\mathbf{g},\mathbf{h}} := \mathbb{E}^{\mathbf{g},\mathbf{h}} \left\{ \sum_{t=1}^{\infty} \alpha^{t-1} \cdot \left[c(\mathbf{U}_t) + \rho(\mathbf{X}_t, \hat{\mathbf{X}}_t) \right] \right\}$$

where $\alpha \in (0, 1)$ is the discount rate.

The statistics of a Markov process $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$ consist of a probability distribution on the initial state $\mathbf{X}_0$ and a transition matrix $\mathbf{P}$ such that

$$\Pr(\mathbf{X}_t = \mathbf{x}' | \mathbf{X}_{t-1} = \mathbf{x}) = \mathbf{P}(\mathbf{x}, \mathbf{x}') \quad (6)$$

for $\mathbf{x}, \mathbf{x}' \in \mathcal{X}$.

Problem (P1) is an approximation to the original problem. Better approximations can be obtained by modeling the soil moisture process as a higher order Markov process. In that case, $\{\mathbf{V}_t\}_{t=0,1,2,\ldots}$ becomes the underlying Markov process in the POMDP. While the underlying theory is the same, the additional gain in approximation from using a higher order Markov process comes at the cost of increased computational complexity.

III. ANALYSIS

A. Exact Solution of the POMDP

POMDPs have been well studied in the literature. Such problems can be transformed into fully observed MDPs by taking the MDP state to be the conditional probability distribution of the partially observed POMDP state, given all the past observations and actions. This conditional probability distribution is commonly referred to as a *belief state*. For this resulting fully observed MDP, it is known (see, e.g., [21] and [22]) that the optimal policies are stationary policies of the form

$$\begin{aligned} \hat{\mathbf{X}}_t &= h(\pi_t) \\ \mathbf{U}_{t+1} &= g(\pi_t) \end{aligned} \quad (7)$$

where π_t is the belief state, a vector with components

$$\pi_t(\mathbf{x}) := \Pr(\mathbf{X}_t = \mathbf{x} | \mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_t, \mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_t), \quad \mathbf{x} \in \mathcal{X}, \quad t = 1, 2, \ldots$$

At time $t+1$, the belief state is computed from π_t, the belief state at time t; $\mathbf{U}_{t+1}$, the scheduling decision at time $t+1$; and $\mathbf{Y}_{t+1}$, the observation at time $t+1$.

While fully observed MDPs can in principle be solved via dynamic programming, the fact that the belief state belongs to a continuous space (the space of all probability distributions on $\mathcal{X}$) makes standard dynamic programming intractable. In [31]–[33], Sondik and Smallwood make the key observation that the value functions involved in each step of the dynamic program are piecewise linear and concave, and develop algorithms that utilize this property to determine optimal policies for finite- and infinite-horizon POMDPs. Subsequent algorithms for solving POMDPs include Monahan's enumeration algorithm [34], the linear support algorithm [35], the witness algorithm [36]–[38], and the incremental pruning algorithm [39]. Software implementations of all of these algorithms have been developed by Anthony Cassandra in the pomdp-solve package, which is available at www.pomdp.org. For more on exact solution methods of POMDPs, see the surveys in [34] and [40]–[43].

The complexity class of infinite-horizon POMDPs is not known; however, results on complexity of finite-horizon POMDPs [44] and infinite-horizon MDPs with uncountable state space (which are a superclass of POMDPs) [45], [46] are known. These results suggest that the complexity of the POMDP algorithms increases rapidly as the number of states of the underlying Markov process increases.

In our experiments with Cassandra's pomdp-solve package, the convergence time of the standard algorithms becomes prohibitively large when the number of states exceeds 100. Note that the number of states is equal to the number of quantiles raised to the power of the number of sensors $(L \cdot D)$, so even with three sensors at different depths at a single lateral location and a ten-level quantization of soil moisture at each depth, the cardinality of the state space $\mathcal{X}$ is 1000, which is too large for the exact POMDP algorithms. In light of these computational difficulties, it becomes important to come up with alternative formulations of our problem that can be solved more efficiently than the POMDPs. We present such a formulation in the next section for the case of multiple sensors at a single location.

B. The Special Case of Sensors at Multiple Depths at a Single Lateral Location

In this section, we consider a special instance of Problem (P1) where there is a single lateral location (i.e., $L = 1$). When the number of depths D is small enough, the pomdp-solve package numerically finds the optimal policies; however, the computational burden increases quickly with the number of depths considered.

To simplify the POMDP, we make the following assumption.

Assumption 1: The action space is restricted to either not taking any measurements or taking measurements at all depths; i.e.,

$$\mathbf{U}_t = \left(U_t^{1,1}, U_t^{1,2}, \ldots, U_t^{1,D}\right) \in \{(0,0,\ldots,0),(1,1,\ldots,1)\}.$$

The energy cost associated with action $\mathbf{U}_t = (0, 0, \ldots, 0)$ is 0, and the energy cost associated with action $\mathbf{U}_t = (1,1,\ldots,1)$ is denoted by $\kappa > 0$. The justification for this restricted action space is that the actuators (which include the radios) consume significantly more energy than the sensors. Thus, the marginal energy cost of scheduling one additional sensor measurement at a lateral location where the actuator is already powered on is negligible.

We now describe the nature of optimal estimation and scheduling policies for Problem (P1) under Assumption 1. We then show that Problem (P1) is equivalent to an MDP with countable state space.

1) Nature of the Estimation Policy: With Assumption 1 in place, the nature of the evolution of the belief state under any policy of the form in (7) is as follows. When a measurement is taken at all depths at time t [i.e., $\mathbf{U}_t = (1, 1, \ldots, 1)$], the controller observes perfectly the current soil moisture levels at all depths, and the belief state π_t is an element of the set

$$\mathcal{C} := \{\pi \in \Pi | \pi(\mathbf{x}) = 1, \text{ for some } \mathbf{x} \in \mathcal{X}\}.$$

The set $\mathcal{C}$ is the set of belief states at which the controller is certain about the soil moisture at all depths. We refer to such states as *corner states*, and for each $\mathbf{x} \in \mathcal{X}$, we denote the belief state at which $\pi(\mathbf{x}) = 1$ by $e_{\mathbf{x}}$. Clearly, if the controller's belief state is $e_{\mathbf{x}}$, the optimal estimate is $\mathbf{x}$. Thus

$$h(e_{\mathbf{x}}) = \mathbf{x}.$$

After taking measurements, the controller does not schedule more measurements for some number of time steps (possibly zero). The length of this period of no measurements depends on the outcome of the previous measurement. During the period of no measurements, the belief state is updated based solely on the soil moisture transition matrix $\mathbf{P}$, according to the update equation

$$\pi_{t+1} = \pi_t \mathbf{P}.$$

At each time step during this period of no measurements, it is optimal for the controller to make the state estimate according to

$$h(\pi_t) \in \arg\min_{\mathbf{a} \in \mathcal{X}} \left\{ \sum_{\mathbf{x} \in \mathcal{X}} \pi_t(\mathbf{x}) \cdot \rho(\mathbf{x}, \mathbf{a}) \right\}$$

and the resulting expected distortion is given by

$$\begin{aligned}\tilde{\rho}(\pi_t) &:= \mathbb{E}[\rho(\mathbf{X}_t, h(\pi_t)) | \pi_t] \\ &= \min_{\mathbf{a} \in \mathcal{X}} \left\{ \sum_{\mathbf{x} \in \mathcal{X}} \pi_t(\mathbf{x}) \cdot \rho(\mathbf{x}, \mathbf{a}) \right\}.\end{aligned}$$

Eventually, the controller schedules another measurement at all depths, and the belief state returns to some element of $\mathcal{C}$, depending on the outcome of the subsequent measurements. Fig. 4 shows one such evolution of the belief state. Note that each time the belief state returns to an element $\pi \in \mathcal{C}$, it traces exactly the same path of belief states until the next measurements are scheduled.

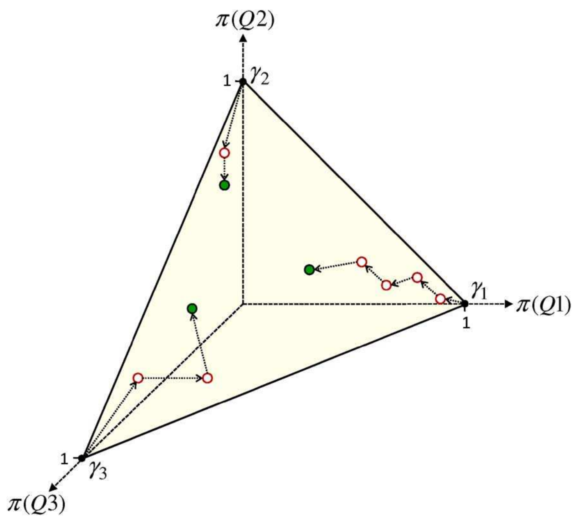


Fig. 4. ***Evolution of the controller's belief state. In this diagram, soil moisture at a single lateral location and single depth is under observation, and there are three possible soil moisture quantiles, so $\mathcal{X} = \{Q1, Q2, Q3\}$. At corner state $\gamma_i \in \mathcal{C}$, $\pi(Q_i) = 1$ and $\pi(Q_j) = 0$ for $j \neq i$. The belief state evolves on the space of probability distributions on $\mathcal{X}$. The unfilled red and filled green circles represent belief states at which it is optimal to not take and to take measurements, respectively.***

2) Nature of the Scheduling Policy: Using Assumption 1 and the nature of the estimation policy described above, the standard dynamic program for the POMDP in Problem (P1) can be written as (see, e.g., [21])

$$V(\pi) = \min\left\{ \begin{array}{l} \tilde{\rho}(\pi) + \alpha\kappa + \alpha \cdot \sum_{\mathbf{x}\in\mathcal{X}} [\pi\mathbf{P}](\mathbf{x}) \cdot V(e_{\mathbf{x}}), \\ \tilde{\rho}(\pi) + \alpha V(\pi\mathbf{P}) \end{array} \right\} \tag{8}$$

where $[\pi\mathbf{P}](\mathbf{x})$ denotes the component of the vector $\pi\mathbf{P}$ corresponding to the state $\mathbf{x}$.

The following is a standard result for such dynamic programs.

Lemma 1 (Smallwood and Sondik, 1973): $V(\pi)$ is concave in π.

Next, let g^* be the optimal scheduling policy for Problem (P1) obtained from the dynamic program (8). We have the following result on the convexity of the optimal measurement region, which is similar in spirit to [47, Lemma 1].

Theorem 1: Let $\mathcal{D} = \{\pi : g^*(\pi) = (1, 1, \ldots)\}$. Then, $\mathcal{D}$ is a convex subset of Π.

Proof: The set $\mathcal{D}$ represents the region of the belief space where the optimal action is to schedule measurements at the next time step. Therefore, the set $\mathcal{D}$ is characterized by the following inequality:

$$\tilde{\rho}(\pi) + \alpha\kappa + \alpha \cdot \sum_{\mathbf{x}\in\mathcal{X}} [\pi\mathbf{P}](\mathbf{x}) \cdot V(e_{\mathbf{x}}) \leq \tilde{\rho}(\pi) + \alpha V(\pi\mathbf{P}).$$

Let $\pi^1, \pi^2 \in \mathcal{D}$ be arbitrary. To show that $\mathcal{D}$ is convex, we need to show that for any $\lambda \in [0, 1]$, $\lambda\pi^1 + (1-\lambda)\pi^2 \in \mathcal{D}$.

Since $\pi^1 \in \mathcal{D}$, we have

$$\tilde{\rho}(\pi^1) + \alpha\kappa + \alpha \cdot \sum_{\mathbf{x}\in\mathcal{X}} [\pi^1\mathbf{P}](\mathbf{x}) \cdot V(e_{\mathbf{x}}) \leq \tilde{\rho}(\pi^1) + \alpha V(\pi^1\mathbf{P})$$

or, equivalently

$$\alpha\kappa + \alpha \cdot \sum_{\mathbf{x}\in\mathcal{X}} [\pi^1\mathbf{P}](\mathbf{x}) \cdot V(e_{\mathbf{x}}) \leq \alpha V(\pi^1\mathbf{P}). \tag{9}$$

By a similar argument, we have

$$\alpha\kappa + \alpha \cdot \sum_{\mathbf{x}\in\mathcal{X}} [\pi^2\mathbf{P}](\mathbf{x}) \cdot V(e_{\mathbf{x}}) \leq \alpha V(\pi^2\mathbf{P}). \tag{10}$$

Now consider

$$\begin{aligned} &\alpha\kappa + \alpha \cdot \sum_{\mathbf{x}\in\mathcal{X}} \left[\left(\lambda\pi^1 + (1-\lambda)\pi^2\right)\mathbf{P}\right](\mathbf{x}) \cdot V(e_{\mathbf{x}}) \\ &\overset{(a)}{\leq} \lambda\alpha V(\pi^1\mathbf{P}) + (1-\lambda)\alpha V(\pi^2\mathbf{P}) \\ &\overset{(b)}{\leq} \alpha V\left(\lambda\pi^1\mathbf{P} + (1-\lambda)\pi^2\mathbf{P}\right) \\ &= \alpha V\left(\left[\lambda\pi^1 + (1-\lambda)\pi^2\right]\mathbf{P}\right) \end{aligned} \tag{11}$$

where (a) follows from (9) and (10), and (b) follows from the concavity of V. Equation (11) implies that

$$\begin{aligned} &\tilde{\rho}\left(\lambda\pi^1 + (1-\lambda)\pi^2\right) + \alpha\kappa \\ &\quad + \alpha \sum_{\mathbf{x}\in\mathcal{X}} \left[\left(\lambda\pi^1 + (1-\lambda)\pi^2\right)\mathbf{P}\right](\mathbf{x}) \cdot V(e_{\mathbf{x}}) \\ &\leq \tilde{\rho}\left(\lambda\pi^1 + (1-\lambda)\pi^2\right) + \alpha V\left(\left[\lambda\pi^1 + (1-\lambda)\pi^2\right]\mathbf{P}\right) \end{aligned}$$

and hence $\lambda\pi^1 + (1-\lambda)\pi^2 \in \mathcal{D}$. Therefore, $\mathcal{D}$ is a convex set. ■

3) Equivalence of Problem (P1) With a Countable State MDP: We now present an MDP that is equivalent to

Problem (P1). For that matter, we define the following for $t \in \{0, 1, \ldots\}$:

$$\begin{aligned} R_t &:= \min\{\tau \geq 0 : Y_{t-\tau} \neq b\} \\ \mathbf{S}_t &:= (\mathbf{X}_{t-R_t}, R_t) \\ \mathbf{A}_t &:= \mathbf{U}_{t+1}. \end{aligned} \tag{12}$$

R_t represents the time since the most recent measurements. Thus, $\mathbf{Y}_{t-R_t} = \mathbf{X}_{t-R_t}$ is the vector of the most recent measurements.

Remark 1: Note that realizations of $(\mathbf{S}_0, \mathbf{S}_1, \ldots, \mathbf{S}_t)$ and $(\mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_{t-1})$ completely specify the realizations of the observation and control processes $(\mathbf{Y}_0, \ldots, \mathbf{Y}_t)$ and $(\mathbf{U}_1, \ldots, \mathbf{U}_t)$, respectively. Hence, any scheduling policy of the form

$$\mathbf{U}_{t+1} = g_{t+1}(\mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_t, \mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_t)$$

can also be written as

$$\mathbf{A}_t = \tilde{g}_{t+1}(\mathbf{S}_0, \mathbf{S}_1, \ldots, \mathbf{S}_t, \mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_{t-1}).$$

We now state the main result of this section.

Theorem 2: Under Assumption 1, Problem (P1) is equivalent to an MDP with state $\mathbf{S}_t \in \mathcal{S} := \mathcal{X} \times \mathbb{Z}_+$, actions $\mathbf{A}_t \in \mathcal{A} = \{(0, 0, \ldots, 0), (1, 1, \ldots, 1)\}$, and cost criterion

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \alpha^t \cdot \kappa \cdot \mathbb{1}_{\{\mathbf{A}_t = (1,1,\ldots,1)\}} + \sum_{t=1}^{\infty} \alpha^{t-1} \cdot \eta(\mathbf{S}_t)\right\} \tag{13}$$

where for $\mathbf{x} \in \mathcal{X}$ and $n = 0, 1, 2, \ldots$

$$\eta(\mathbf{x}, n) := \tilde{\rho}(e_{\mathbf{x}} \mathbf{P}^n). \tag{14}$$

We refer to this equivalent MDP as Problem (MDP-1).

Proof: Due to the known form of the optimal estimation policy, as described in Section III-B1, Problem (P1) amounts to finding a scheduling policy $\mathbf{g} = (g, g, \ldots)$ to minimize the discounted expected cost criterion

$$\mathbb{E}\left\{\sum_{t=1}^{\infty} \alpha^{t-1} \cdot [c(\mathbf{U}_t) + \tilde{\rho}(\pi_t)]\right\}. \tag{15}$$

Because of Remark 1, in order to show this is equivalent to Problem (MDP-1), it suffices to show that (15) can be written as (13), and that $\{\mathbf{S}_t\}_{t=0,1,\ldots}$ is a controlled Markov process with actions $\{\mathbf{A}_t\}_{t=0,1,\ldots}$; i.e.,

$$\Pr(\mathbf{S}_{t+1} | \mathbf{S}_0, \mathbf{S}_1, \ldots, \mathbf{S}_t, \mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_t) = \Pr(\mathbf{S}_{t+1} | \mathbf{S}_t, \mathbf{A}_t). \tag{16}$$

The energy cost of Problem (P1) can be written as

$$\begin{aligned} \mathbb{E}\left\{\sum_{t=1}^{\infty} \alpha^{t-1} \cdot c(\mathbf{U}_t)\right\} &= \mathbb{E}\left\{\sum_{t=1}^{\infty} \alpha^{t-1} \cdot \kappa \cdot \mathbb{1}_{\{\mathbf{U}_t = (1,1,\ldots,1)\}}\right\} \\ &= \mathbb{E}\left\{\sum_{t=0}^{\infty} \alpha^{t} \cdot \kappa \cdot \mathbb{1}_{\{\mathbf{A}_t = (1,1,\ldots,1)\}}\right\}. \end{aligned}$$

The distortion cost of Problem (P1) can be written as

$$\begin{aligned} \mathbb{E}\left\{\sum_{t=1}^{\infty} \alpha^{t-1} \cdot \tilde{\rho}(\pi_t)\right\} &= \mathbb{E}\left\{\sum_{t=1}^{\infty} \alpha^{t-1} \cdot \tilde{\rho}\left(e_{\mathbf{X}_{t-R_t}} \mathbf{P}^{R_t}\right)\right\} \\ &= \mathbb{E}\left\{\sum_{t=1}^{\infty} \alpha^{t-1} \cdot \eta(\mathbf{S}_t)\right\}. \end{aligned}$$

Thus, (15) can be written as (13).

Next, we note that at time t, the controller's belief on the current state is given by

$$\begin{aligned} \pi_t(\mathbf{x}) &:= \Pr(\mathbf{X}_t = \mathbf{x} | \mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_t, \mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_t) \\ &= \Pr\left(\mathbf{X}_t = \mathbf{x} \middle| \begin{array}{l} \mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_{t-R_t}, \\ \mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_{t-R_t}, R_t \end{array}\right) \qquad (17) \\ &= \Pr(\mathbf{X}_t = \mathbf{x} | \mathbf{X}_{t-R_t}, R_t) \\ &= \left[e_{\mathbf{X}_{t-R_t}} \mathbf{P}^{R_t}\right](\mathbf{x}), \quad \mathbf{x} \in \mathcal{X}, \quad t = 1, 2 \ldots \qquad (18) \end{aligned}$$

where (17) follows from the fact that the observations after time $t - R_t$ are blank. Equation (18) follows from the facts that Y_{t-R_t} completely determines X_{t-R_t}, and due to the Markovian nature of $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$, X_t is conditionally independent of observations before time $t - R_t$ given X_{t-R_t}. As defined earlier, $e_{\mathbf{x}}$ is the belief that assigns probability 1 to the state $\mathbf{x}$, and $[e_{\mathbf{X}_{t-R_t}} \mathbf{P}^{R_t}](\mathbf{x})$ denotes the component of the vector $e_{\mathbf{X}_{t-R_t}} \mathbf{P}^{R_t}$ corresponding to state x. To prove (16), we consider two cases.

Case 1) $\mathbf{A}_t = (0,0,\ldots,0)$

$$\Pr\left(\mathbf{S}_{t+1} = (\mathbf{x}',n') \middle| \begin{array}{l} \mathbf{S}_0 = \mathbf{s}_0, \mathbf{S}_1 = \mathbf{s}_1, \ldots, \mathbf{S}_t = (\mathbf{x},n), \\ \mathbf{A}_0 = \mathbf{a}_0, \mathbf{A}_1 = \mathbf{a}_1, \ldots, \\ \mathbf{A}_t = (0,0,\ldots 0) \end{array}\right)$$
$$= \mathbb{1}_{\{\mathbf{x}'=\mathbf{x}\}} \cdot \mathbb{1}_{\{n'=n+1\}}$$
$$= \Pr(\mathbf{S}_{t+1} = (\mathbf{x}',n') | \mathbf{S}_t = (\mathbf{x},n), \mathbf{A}_t = (0,0,\ldots,0)). \tag{19}$$

In (19), both equalities follow from the fact that if no measurements are scheduled at time $t+1$ [i.e., $\mathbf{A}_t = (0,0,\ldots,0)$], then the time since the most recent measurements increases by 1 (i.e., $R_{t+1} = R_t + 1$), while the most recent observation remains the same.

Case 2) $\mathbf{A}_t = (1,1,\ldots,1)$

$$\Pr\left(\mathbf{S}_{t+1} = (\mathbf{x}',n') \middle| \begin{array}{l} \mathbf{S}_0 = \mathbf{s}_0, \mathbf{S}_1 = \mathbf{s}_1, \ldots, \mathbf{S}_t = (\mathbf{x},n), \\ \mathbf{A}_0 = \mathbf{a}_0, \mathbf{A}_1 = \mathbf{a}_1, \ldots, \\ \mathbf{A}_t = (1,1,\ldots,1) \end{array}\right)$$
$$= \mathbb{1}_{\{n'=0\}} \cdot \Pr\left(\mathbf{X}_{t+1} = \mathbf{x}' \middle| \begin{array}{l} \mathbf{S}_0 = \mathbf{s}_0, \mathbf{S}_1 = \mathbf{s}_1, \ldots, \\ \mathbf{S}_t = (\mathbf{x},n), \\ \mathbf{A}_0 = \mathbf{a}_0, \mathbf{A}_1 = \mathbf{a}_1, \ldots, \\ \mathbf{A}_t = (1,1,\ldots,1) \end{array}\right) \tag{20}$$
$$= \mathbb{1}_{\{n'=0\}} \cdot \sum_{\mathbf{x}''\in\mathcal{X}} \Pr(\mathbf{X}_{t+1} = \mathbf{x}' | \mathbf{X}_t = \mathbf{x}'') \cdot \Pr\left(\mathbf{X}_t = \mathbf{x}'' \middle| \begin{array}{l} \mathbf{S}_0 = \mathbf{s}_0, \mathbf{S}_1 = \mathbf{s}_1, \ldots, \mathbf{S}_t = (\mathbf{x},n), \\ \mathbf{A}_0 = \mathbf{a}_0, \mathbf{A}_1 = \mathbf{a}_1, \ldots, \\ \mathbf{A}_t = (1,1,\ldots,1) \end{array}\right)$$
$$= \mathbb{1}_{\{n'=0\}} \cdot \sum_{\mathbf{x}''\in\mathcal{X}} \Pr(\mathbf{X}_{t+1} = \mathbf{x}' | \mathbf{X}_t = \mathbf{x}'') \cdot \pi_t(x'') \tag{21}$$
$$= \mathbb{1}_{\{n'=0\}} \cdot [\pi_t \mathbf{P}](\mathbf{x}')$$
$$= \mathbb{1}_{\{n'=0\}} \cdot [e_{\mathbf{x}} \mathbf{P}^{n+1}](\mathbf{x}')$$

where (20) follows from the fact that if measurements are scheduled at time $t+1$ (i.e., $\mathbf{A}_t = (1,1,\ldots,1)$), then the time since the most recent measurements becomes 0 (i.e., $R_{t+1} = 0$), while the most recent observation becomes the state at time $t+1$. Equation (21) follows from the fact that realizations of $(\mathbf{S}_0, \mathbf{S}_1, \ldots, \mathbf{S}_t)$ and $(\mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_t)$ completely specify the realizations of the observation and control processes; hence the probability of $\mathbf{X}_t$ conditioned on $(\mathbf{S}_0, \mathbf{S}_1, \ldots, \mathbf{S}_t)$ and $(\mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_t)$ is the same as π_t. Repeating the above arguments for

$$\Pr(\mathbf{S}_{t+1} = (\mathbf{x}',n') | \mathbf{S}_t = (\mathbf{x},n), \mathbf{A}_t = (1,1,\ldots,1))$$

we get

$$\Pr(\mathbf{S}_{t+1} = (\mathbf{x}',n') | \mathbf{S}_t = (\mathbf{x},n), \mathbf{A}_t = (1,1,\ldots,1))$$
$$= \mathbb{1}_{\{n'=0\}} \cdot [e_{\mathbf{x}} \mathbf{P}^{n+1}](\mathbf{x}')$$
$$= \Pr\left(\mathbf{S}_{t+1} = (\mathbf{x}',n') \middle| \begin{array}{l} \mathbf{S}_0 = \mathbf{s}_0, \mathbf{S}_1 = \mathbf{s}_1, \ldots, \\ \mathbf{S}_t = (\mathbf{x},n), \\ \mathbf{A}_0 = \mathbf{a}_0, \mathbf{A}_1 = \mathbf{a}_1, \ldots, \\ \mathbf{A}_t = (1,1,\ldots,1) \end{array}\right). \tag{22}$$

Equations (19) and (22) imply (16). Thus, we have established that the process $\{\mathbf{S}_t\}_{t=0,1,\ldots}$ is a controlled Markov process with actions $\{\mathbf{A}_t\}_{t=0,1,\ldots}$, completing the proof. ■

4) Approximation by Finite MDP: Problem (MDP-1) has the countably infinite state space $\mathcal{X} \times \mathbb{Z}_+$. MDPs with infinite state spaces cannot be easily solved in general [48]. Therefore, we introduce a new MDP by imposing the following assumption on Problem (MDP-1).

Assumption 2: We restrict the allowable scheduling policies of Problem (MDP-1) to those policies that ensure the length of the period of no measurements is no more than a finite bound M.

We refer to this new MDP as Problem (MDP-2). The implication of Assumption 2 is that R_t, defined in (12), is bounded by M. Hence, the *finite* state space of Problem (MDP-2) is $\mathcal{X} \times \{0,1,\ldots,M\}$, and the only allowable scheduling decision at states $(\mathbf{x},M), \mathbf{x} \in \mathcal{X}$ is to schedule measurements (i.e., $\mathbf{A} = (1,1,\ldots,1)$). The solution of the resulting finite-state MDP can be computed through standard dynamic programming (see, e.g., [21]).

5) Dynamic Program: Following standard methodology for MDPs, the dynamic program for Problem (MDP-2) is given by the following equations:

$$V(\mathbf{x},n) = \min\{\tilde{\rho}(e_{\mathbf{x}}\mathbf{P}^n) + \alpha V(\mathbf{x},n+1), W(\mathbf{x},n)\},$$
$$\forall (\mathbf{x},n) \in \mathcal{X} \times \{0,1,\ldots,M-1\}$$
$$V(\mathbf{x},M) = W(\mathbf{x},M), \qquad \forall \mathbf{x} \in \mathcal{X} \tag{23}$$

where

$$W(\mathbf{x},n) := \tilde{\rho}(e_{\mathbf{x}}\mathbf{P}^n) + \alpha\kappa + \alpha \cdot \sum_{\mathbf{x}'\in\mathcal{X}} [e_{\mathbf{x}}\mathbf{P}^{n+1}](\mathbf{x}') \cdot V(\mathbf{x}',0),$$
$$\forall (\mathbf{x},n) \in \mathcal{X} \times \{0,1,\ldots,M\}.$$

This dynamic program can be solved by well-known methods such as the value iteration and policy iteration algorithms [21].

6) Sufficient Condition for Equivalence of Problem (MDP-1) and Problem (MDP-2): One natural question that arises is when an optimal policy for Problem (MDP-2) [computed from the dynamic program (23)] is also optimal for Problem (MDP-1).

Conjecture 1: If i) $\tilde{\rho}(e_{\mathbf{x}}\mathbf{P}^n)$ is nondecreasing in n for every $x \in \mathcal{X}$, and ii) there exists an optimal policy for Problem (MDP-2) and a sequence $\{n^*_{\mathbf{x}}\}_{\mathbf{x}\in\mathcal{X}}$ with $n^*_{\mathbf{x}} < M$ for all $\mathbf{x} \in \mathcal{X}$, such that the optimal control action at state $(\mathbf{x}, n^*_{\mathbf{x}})$ is given by

$$U^*\left(\mathbf{x}, n^*_{\mathbf{x}}\right) = (1, 1, \ldots, 1), \qquad \forall \mathbf{x} \in \mathcal{X}$$

then Assumption 2 is without loss of optimality; i.e., Problem (MDP-1) and Problem (MDP-2) are equivalent, and both are equivalent to Problem (P1) under Assumption 1.

C. Independent Scheduling and Joint Estimation

In the case of sensors at a single location, Assumptions 1 and 2 allowed us to formulate the scheduling and estimation problem as a finite-state MDP. A key element in obtaining this reduction was that the belief state always returns to a corner state after a bounded number of time steps. In the case of sensors at multiple lateral locations ($L > 1$), we can still assume that at each location, the length of a no measurement phase cannot exceed a bound M. However, since measurements at different locations may not be synchronized, the belief state (on the moisture levels at all locations and all depths) may not necessarily return to a corner state within M steps. Thus, we cannot in general model this problem as an MDP with state space $\mathcal{X} \times \{0, 1, \ldots, M\}$ [like Problem (MDP-2)].

One simple approximation for the multiple location scheduling problem is to obtain a scheduling policy for each location independently, by solving an instance of Problem (MDP-2) for each location via the dynamic program (23). The central coordinator then employs these polices to schedule measurements at the respective locations. Since the soil moisture values at different locations are correlated, the coordinator can then use the measurements received from all locations to form a *joint* belief π_t on the current soil moisture levels at all locations and depths. It is then optimal for it to produce estimates according to

$$h(\pi_t) \in \arg\min_{\mathbf{a}\in\mathcal{X}} \left\{ \sum_{\mathbf{x}\in\mathcal{X}} \pi_t(\mathbf{x}) \cdot \rho(\mathbf{x}, \mathbf{a}) \right\}.$$

Note that this approach represents a compromise between *jointly* scheduling measurements and *jointly* estimating the soil moisture levels at all locations and depths, and *independently* scheduling measurements and *independently* estimating the soil moisture levels at each lateral location. Namely, independently scheduling measurements allows us to avoid the additional computational complexity that would be required to jointly schedule the sensor measurements at all locations. At the same time, jointly estimating the soil moisture levels enables the coordinator to leverage the soil moisture correlations across space to make better estimates.

D. Discussion

For the scheduling problem with sensors at multiple depths at *the same* lateral location (and hence sharing the same actuator), the energy cost of communication between the actuator and the coordinator far exceeds the energy cost of taking a measurement. If the actuator is powered on and instructed to activate the sensor at any one depth, activating the rest of the sensors to collect additional data does not add significant additional cost to the system. This reasoning forms the basis of our Assumption 1. Moreover, under any realistic statistical model, one does not expect a scheduling policy with arbitrarily long periods of no measurement to perform well. In other words, it is reasonable to expect that all good scheduling policies take at least one measurement in any consecutive M time periods (for some choice of M). This motivates our Assumption 2. Under these two assumptions, the dynamic program (23) provides the *optimal* scheduling policy for the problem with multiple sensors at different depths at the same lateral location.

For the scheduling problem with sensors at *multiple* lateral locations, an additional benefit of the independent scheduling and joint estimation approach is that the central coordinator communicates to each node the duration of its next sleep cycle immediately after it receives the measurements from that node. Thus, the local nodes do not need to keep their radios powered on to listen for further instructions from the coordinator while they are asleep.

IV. NUMERICAL EXAMPLE

In this section, we present a numerical example to illustrate our methodology. We use the tRIBS simulations to generate the matrix of transition probabilities describing the soil moisture evolution, and use these dynamics as a basis to evaluate the three policies. In practice, to evaluate the method and compare it to other scheduling heuristics, we do the following.

1) Either collect field data or use the tRIBS model to generate realizations of soil moisture evolution across time and space.

Table 1 Performance Comparison of Different Control and Estimation Policies

	Expected Total Cost	Expected Measurement Cost	Expected Estimation Cost	% of Time Measurements Taken
Take measurements with every sensor at every time	57.0	57.0	0.0	100%
Independent measurement scheduling and independent estimation	16.8	8.0	8.8	12%
Independent measurement scheduling and joint estimation	15.0	8.0	7.0	12%

2) Use the data from 1) to generate the matrix of transition probabilities.
3) Generate scheduling and estimation policies based on the system dynamics from 2) using the methodology discussed in Section III.
4) Evaluate the policies by testing them on new field data.

One example of this process using field data collected at the University of Michigan Matthaei Botanical Gardens is discussed in [23].

We consider an arrangement of sensors at two different locations, and three depths (25, 67, and 123 mm) at each location. We use the tRIBS physical model described in Section II-A1 to generate soil moisture realizations at all sensor locations for a time horizon of 2209 time steps. We then quantize the soil moisture realizations into eight quantization levels (0%–12%, 12%–14%, 14%–16%, 16%–18%, 18%–20%, 20%–22%, 22%–24%, and 24%–30%). Next, we count the frequency of transitions between all possible pairs of soil moisture matrices. For example, one such frequency is the number of transitions from the joint quantile matrix

$$q_i = \begin{bmatrix} 16\% - 18\% & 18\% - 20\% \\ 14\% - 16\% & 16\% - 18\% \\ 12\% - 14\% & 16\% - 18\% \end{bmatrix}$$

to the joint quantile matrix

$$q_j = \begin{bmatrix} 18\% - 20\% & 20\% - 22\% \\ 16\% - 18\% & 16\% - 18\% \\ 12\% - 14\% & 16\% - 18\% \end{bmatrix}.$$

We then normalize these frequencies to compute a Markovian transition matrix for the soil moisture process.

The objectives are to conserve energy and estimate the soil moisture at all three depths at both locations. At each location and depth, we penalize estimation errors by the absolute difference between the quantile index of the true moisture and the estimated quantile index. Relative to one unit of estimation error, the energy cost of taking measurements at all depths at a given location is 1.5. We assume that these measurements are noiseless. We use a discount factor of 0.95, and a time horizon of 200 steps to approximate an infinite horizon.

We use both Assumptions 1 and 2, with $M = 30$ for Assumption 2. To find a scheduling policy for each location, we solve the dynamic program (23). If measurements at all depths are scheduled at the current time, the scheduling policy table tells the coordinator the number of time steps after which the next measurements must be taken. This number of time steps depends on the outcomes of the current measurements. The system operation can therefore be described as follows: at time $t = 0$, the actuator takes measurements and transmits them to the central coordinator. Given these measurements, the coordinator commands the actuator to sleep for the number of time steps specified by the scheduling policy table. After the specified duration of the sleep mode, the actuator wakes up and takes new measurements, which it communicates back to the central coordinator. The process is then repeated.

For the purpose of comparison, we consider three different scheduling and estimation strategies. The first strategy is to take measurements with all sensors at every time step. The second is to independently schedule the sensor measurements at each location according to the solution of Problem (MDP-2) for that location, and independently estimate the soil moisture levels at a given location based only on the measurement readings at that specific location. The third strategy is to independently schedule the sensor measurements at each location according to the solution of Problem (MDP-2) for that location, but to have the coordinator jointly estimate all soil moisture quantiles using the measurements from all locations. The resulting expected costs are shown in Table 1.

Table 1 demonstrates that using the scheduling policy to take fewer measurements results in significant savings in measurement costs, at the expense of some estimation cost. The second strategy of independent scheduling and independent estimation uses the correlations in the soil moisture process across time and depth to produce good estimates without taking measurements all the time. The third strategy of independent scheduling and joint estimation effectively exploits spatial correlations as well, in order to reduce the expected estimation cost.

V. INCORPORATING METEOROLOGICAL OBSERVATIONS

So far we have assumed that the coordinator does not observe any meteorological data such as rainfall, ambient temperature, or solar radiation. In this section, we show how to adapt the scheduling and estimation policies when the coordinator observes rainfall. Other types of meteorological observations can be handled in a similar manner.

Let $Z_t = (Z_t^1, Z_t^2, \cdots, Z_t^L)$ denote the rainfall at the L lateral sensor locations at time t, where $Z_t^l \in \{0,1\}$. Here, $Z_t^l = 0$ indicates that rainfall at location l is below a fixed threshold, and $Z_t^l = 1$ indicates that rainfall at location l is above the threshold. The coordinator forms an estimate of the soil moisture field as

$$\hat{\mathbf{X}}_\mathbf{t} = \mathbf{h}_\mathbf{t}(\mathbf{Y}_\mathbf{0}, \ldots, \mathbf{Y}_\mathbf{t}, \mathbf{U}_\mathbf{1}, \ldots, \mathbf{U}_\mathbf{t}, \mathbf{Z}_\mathbf{0}, \ldots, \mathbf{Z}_\mathbf{t})$$

and selects the scheduling decision as

$$\mathbf{U}_{t+1} = g_t(\mathbf{Y}_0, \ldots, \mathbf{Y}_t, \mathbf{U}_1, \ldots, \mathbf{U}_t, \mathbf{Z}_0, \ldots, \mathbf{Z}_{t+1}).$$

As before, we wish to find scheduling and estimation policies $\mathbf{g}$ and $\mathbf{h}$, respectively, that minimize the infinite-horizon discounted cost criterion in (4).

In general, finding optimal scheduling and estimation policies is intractable without any structure on the statistics of the rainfall and soil moisture processes. Similar to Section II-C, we approximate rainfall evolution with a kth-order Markov process, i.e.,

$$\Pr(\mathbf{Z}_t | \mathbf{Z}_{t-1}, \ldots, \mathbf{Z}_0) = \Pr(\mathbf{Z}_t | \mathbf{Z}_{t-1}, \ldots, \mathbf{Z}_{t-k})$$

and soil moisture evolution with a kth-order Markov process, i.e.,

$$\begin{aligned}\Pr(&\mathbf{X}_t | \mathbf{X}_{t-1}, \ldots, \mathbf{X}_0, \mathbf{Z}_{t-1}, \ldots, \mathbf{Z}_0) \\ &= \Pr(\mathbf{X}_t | \mathbf{X}_{t-1}, \ldots, \mathbf{X}_{t-k}, \mathbf{Z}_{t-1}, \ldots, \mathbf{Z}_{t-k}).\end{aligned}$$

As a first approximation, we assume that these processes are first-order Markovian, resulting in a POMDP similar to Problem (P1). We refer to this POMDP as Problem (R1). The statistics of $\{\mathbf{Z}_t\}_{t=0,1,2,\ldots}$ consist of a transition matrix $\mathbf{Q}$ such that

$$\Pr(\mathbf{Z}_t = \mathbf{z}' | \mathbf{Z}_{t-1} = \mathbf{z}) = \mathbf{Q}(\mathbf{z}, \mathbf{z}')$$

for $\mathbf{z}, \mathbf{z}' \in \{0,1\}^L$, and the statistics of $\{\mathbf{X}_t\}_{t=0,1,2,\ldots}$ consist of a transition matrix P such that

$$\Pr(\mathbf{X}_t = \mathbf{x}' | \mathbf{X}_{t-1} = \mathbf{x}, \mathbf{Z}_{t-1} = \mathbf{z}) = \mathbf{P}(\mathbf{z}, \mathbf{x}, \mathbf{x}')$$

for $\mathbf{x}, \mathbf{x}' \in \mathcal{X}$ and $\mathbf{z} \in \{0,1\}^L$.

As with Problem (P1), Problem (R1) is an approximation of the original problem, and better approximations can be obtained by modeling the rainfall and soil moisture processes by higher order Markov processes.

An exact solution of Problem (R1) can be found by solving a dynamic program, but the solution has a high computational complexity. We therefore consider an approximate solution for the case of multiple sensors at a single location. Under Assumption 1, Problem (R1) is equivalent to a countable state MDP similar to the one defined in Theorem 2, with R_t and $\mathbf{A}_t$ defined as before, and

$$\mathbf{S}_t := (\mathbf{Z}_t, \ldots, \mathbf{Z}_{t-R_t}, \mathbf{X}_{t-R_t}, R_t). \tag{24}$$

We call this Problem (MDP-R1).

Under Assumption 2, Problem (MDP-R1) reduces to a finite-state MDP, Problem (MDP-R2), with state space $\{0,1\}^M \times \mathcal{X} \times \{0,1,\ldots,M\}$. This state space, although finite, increases exponentially with M, making a computational solution intractable. Therefore, we introduce a third MDP by imposing the following assumption.

Assumption 3: We restrict the allowable scheduling policies of Problem (MDP-R2) to those that always take a measurement if rainfall is high ($Z_{t+1}^1 = 1$).

We refer to this new MDP as problem (MDP-R3). Assumption 3 restricts the possible values that $\mathbf{S}_t$, defined in (24), can take, thereby restricting the state space to $\mathcal{X} \times \{0,1,\ldots,M\}$. The solution of the resulting MDP can be computed through a dynamic program similar to the one given by (23). Therefore, meteorological observations such as rainfall can be incorporated without significantly increasing the complexity of the computational solution procedure.

VI. CONCLUSION

In this paper, we considered the problem of monitoring soil moisture evolution using a wireless network of *in situ* sensors. The key idea was that, at the cost of some inaccuracy in estimating the soil moisture evolution, we can significantly reduce energy consumption by taking a sparser set of measurements. The physical model of soil moisture evolution allows us to leverage soil moisture correlations across time, depth, and space to both schedule measurements when they are expected to yield the most information, and generate estimates based on the sparser set of measurements. After formulating the problem as a POMDP, we took advantage of the problem structure to approximate the original problem by a computationally simpler MDP. The resulting measurement scheduling and estimation policies represent a scalable and implementable technology that we have tested and validated numerically and in the field. ■

Acknowledgment

The authors would like to thank A. Flores of Boise State University for providing tRIBS simulations, and the anonymous reviewers for helpful suggestions.

REFERENCES

[1] *NASA Strategic Plan*, 2006. [Online]. Available: www.nasa.gov/pdf/142302main_2006_NASA_Strategic_Plan.pdf

[2] M. H. Cosh, T. J. Jackson, R. Bindlish, and J. H. Prueger, "Watershed scale temporal and spatial stability of soil moisture and its role in validating satellite estimates," *Remote Sens. Environ.*, vol. 92, no. 4, pp. 427–435, Sep. 2004.

[3] M. H. Cosh, T. J. Jackson, S. Moran, and R. Bindlish, "Temporal persistence and stability of surface soil moisture in a semi-arid watershed," *Remote Sens. Environ.*, vol. 112, no. 2, pp. 304–313, Feb. 2008.

[4] Digi International. [Online]. Available: http://www.digi.com

[5] R. Willett, A. Martin, and R. Nowak, "Backcasting: Adaptive sampling for sensor networks," in *Proc. 3rd Int. Symp. Inf. Process. Sensor Netw.*, Berkeley, CA, Apr. 2004, pp. 124–133.

[6] M. Rahimi, M. Hansen, W. Kaiser, G. Sukhatme, and D. Estrin, "Adaptive sampling for environmental field estimation using robotic sensors," in *Proc. Int. Conf. Intell. Robots Syst.*, Aug. 2005, pp. 3692–3698.

[7] L. Meier, J. Peschon, and R. M. Dressler, "Optimal control of measurement subsystems," *IEEE Trans. Autom. Control*, vol. 12, no. 5, pp. 528–536, Oct. 1967.

[8] M. Athans, "On the determination of optimal costly measurement strategies for linear stochastic systems," *Automatica*, vol. 8, pp. 397–412, 1972.

[9] M. Athans and C. Schweppe, "Optimal waveform design via control theoretic concepts," *Inf. Control*, vol. 10, pp. 335–377, 1967.

[10] H. Kushner, "On the optimum timing of observations for linear control systems with unknown initial state," *IEEE Trans. Autom. Control*, vol. 9, no. 2, pp. 144–150, Apr. 1964.

[11] M. S. Andersland and D. Teneketzis, "Measurement scheduling for recursive team estimation," *J. Optim. Theory Appl.*, vol. 89, pp. 615–636, 1996.

[12] J. C. Van Dam, "Field-scale water flow and solute transport: SWAP model concepts, parameter estimation, and case studies," Ph.D. dissertation, Dept. Environ. Sci., Wageningen Univ., Wageningen, The Netherlands, 2000.

[13] J. Evans and V. Krishnamurthy, "Optimal sensor scheduling for hidden Markov model state estimation," *Int. J. Control*, vol. 74, no. 18, pp. 1737–1742, Dec. 2001.

[14] V. Krishnamurthy, "Algorithms for optimal scheduling and management of hidden Markov model sensors," *IEEE Trans. Signal Process.*, vol. 50, no. 6, pp. 1382–1397, Jun. 2002.

[15] J. S. Baras and A. Bensoussan, "Sensor scheduling problems," in *Proc. IEEE Conf. Decision Control*, Austin, TX, Dec. 1988, vol. 3, pp. 2342–2345.

[16] J. S. Baras and A. Bensoussan, "Optimal sensor scheduling in nonlinear filtering of diffusion processes," *SIAM J. Control Optim.*, vol. 27, no. 4, pp. 786–813, Jul. 1989.

[17] D. A. Castañón, "Approximate dynamic programming for sensor management," in *Proc. IEEE Conf. Decision Control*, San Diego, CA, Dec. 1997, vol. 2, pp. 1202–1207.

[18] D. A. Castañón and L. Carin, "Stochastic control theory for sensor management," in *Foundations and Applications of Sensor Management*. New York: Springer-Verlag, 2008, pp. 7–32.

[19] Y. He and E. K. P. Chong, "Sensor scheduling for target tracking: A Monte Carlo sampling approach," *Digital Signal Process.*, vol. 16, no. 5, pp. 533–545, Sep. 2006.

[20] Y. Li, L. W. Krakow, E. K. P. Chong, and K. N. Groom, "Approximate stochastic dynamic programming for sensor scheduling to track multiple targets," *Digital Signal Process.*, vol. 19, no. 6, pp. 978–989, Dec. 2009.

[21] P. R. Kumar and P. Varaiya, *Stochastic Systems: Estimation, Identification, and Adaptive Control*. Englewood Cliffs, NJ: Prentice-Hall, 1986.

[22] O. Hernández-Lerma, *Adaptive Markov Control Processes*. New York: Springer-Verlag, 1989.

[23] M. Moghaddam, D. Entekhabi, Y. Goykhman, K. Li, M. Liu, A. Mahajan, A. Nayyar, D. Shuman, and D. Teneketzis, "A wireless soil moisture smart sensor web using physics-based optimal control: Concept and initial demonstrations," *IEEE J. Special Top. Appl. Earth Observations Remote Sens.*, to be published.

[24] L. A. Richards, "Capillary conduction of liquids through porous mediums," *Physics*, vol. 1, no. 5, pp. 318–333, 1931.

[25] J. C. Van Dam, P. Groenendijk, R. F. A. Hendriks, and J. G. Kroes, "Advances of modeling water flow in variably saturated soils with SWAP," *Vadose Zone J.*, vol. 7, no. 2, May 2008.

[26] V. Y. Ivanov, E. R. Vivoni, R. L. Bras, and D. Entekhabi, "Catchment hydrologic response with a fully-distributed triangulated irregular network model," *Water Resources Res.*, vol. 40, no. W11102, 2004, DOI: 10.1029/2004WR003218.

[27] V. Y. Ivanov, R. L. Bras, and E. R. Vivoni, "Vegetation-hydrology dynamics in complex terrain of semiarid areas: I. A mechanistic approach to modeling dynamic feedbacks," *Water Resources Res.*, vol. 44, 2008, DOI: 10.1029/2006WR005588W03429.

[28] E. Vivoni, V. Teles, V. Ivanov, R. Bras, and D. Entekhabi, "Embedding landscape processes into triangulated terrain models," *Int. J. Geograph. Inf. Sci.*, vol. 19, no. 4, pp. 429–457, Apr. 2005.

[29] A. Flores, V. Ivanov, D. Entekhabi, and R. Bras, "Impact of Hillslope-Scale organization of topography, soil moisture, soil temperature, and vegetation on modeling surface microwave radiation emission," *IEEE Trans. Geosci. Remote Sens.*, vol. 47, no. 8, pp. 2557–2569, Aug. 2009.

[30] *Decagon*. [Online]. Available: http://www.decagon.com/soil_moisture/ec5/index.php?pg=specs

[31] E. J. Sondik, "The optimal control of partially observable Markov processes," Ph.D. dissertation, Dept. Electr. Eng., Stanford Univ., Stanford, CA, 1971.

[32] R. D. Smallwood and E. J. Sondik, "The optimal control of partially observable Markov processes over a finite horizon," *Oper. Res.*, vol. 21, no. 5, pp. 1071–1088, Sep.–Oct. 1973.

[33] E. J. Sondik, "The optimal control of partially observable Markov processes over the infinite horizon: Discounted costs," *Oper. Res.*, vol. 26, no. 2, pp. 282–304, Mar.–Apr. 1978.

[34] G. E. Monahan, "A survey of partially observable Markov decision processes: Theory, models, and algorithms," *Manage. Sci.*, vol. 28, no. 1, pp. 1–16, Jan. 1982.

[35] H.-T. Cheng, "Algorithms for partially observable Markov decision processes," Ph.D. dissertation, Univ. British Columbia, Vancouver, BC, Canada, 1988.

[36] M. L. Littman, "The witness algorithm: Solving partially observable Markov decision processes," Dept. Comput. Sci., Brown Univ., Providence, RI, Tech. Rep. CS-94-40, Dec. 1994.

[37] M. L. Littman, A. R. Cassandra, and L. P. Kaelbling, "Efficient dynamic programming updates in partially observable Markov decision processes," Dept. Comput. Sci., Brown Univ., Providence, RI, Tech. Rep. CS-95-19, Nov. 1995.

[38] A. R. Cassandra, L. P. Kaelbling, and M. L. Littman, "Acting optimally in partially observable stochastic domains," in *Proc. Nat. Conf. Artif. Intell.*, Seattle, WA, 1994, pp. 1023–1028.

[39] N. L. Zhang and W. Liu, "Planning in stochastic domains: Problem characteristics and approximations," Dept. Comput. Sci., The Hong Kong Univ. Sci. Technol., Clear

Water Bay, Kowloon, Hong Kong, Tech. Rep. HKUST-CS96-31, 1996.

[40] C. C. White, III, "A survey of solution techniques for the partially observed Markov decision process," *Ann. Oper. Res.*, vol. 32, no. 1, pp. 215–230, Dec. 1991.

[41] W. S. Lovejoy, "A survey of algorithmic methods for partially observed Markov decision processes," *Ann. Oper. Res.*, vol. 28, no. 1, pp. 47–66, Dec. 1991.

[42] D. A. Aberdeen, "Policy-gradient algorithms for partially observable Markov decision processes," Ph.D. dissertation, Dept. Syst. Eng., The Australian Nat. Univ., Acton, A.C.T., Australia, Apr. 2003.

[43] J. Mickova, "Stochastic scheduling with multi-armed bandits," M.S. thesis, Dept. Electr. Electron. Eng., Univ. Melbourne, Melbourne, Vic., Australia, Mar. 2000.

[44] C. H. Papadimitriou and J. N. Tsitsiklis, "The complexity of Markov decision processes," *Math. Oper. Res.*, vol. 12, no. 3, pp. 441–450, Aug. 1987.

[45] C.-S. Chow and J. N. Tsitsiklis, "The complexity of dynamic programming," *J. Complexity*, vol. 5, no. 4, pp. 466–488, Dec. 1989.

[46] J. Rust, "Using randomization to break the curse of dimensionality," *Econometrica*, vol. 65, no. 3, pp. 487–516, May 1997.

[47] W. S. Lovejoy, "On the convexity of policy regions in partially observed systems," *Oper. Res.*, vol. 35, no. 4, pp. 619–621, Jul.–Aug. 1987.

[48] J. Rust, "Numerical dynamic programming in economics," in *Handbook of Computational Economics*, vol. 1. New York: Elsevier Science, 1996, pp. 619–730.

ABOUT THE AUTHORS

David I Shuman (Member, IEEE) received the B.A. degree in economics and the M.S. degree in engineering-economic systems and operations research from Stanford University, Stanford, CA, in 2001 and the M.S. degree in electrical engineering: systems, the M.S. degree in applied mathematics, and the Ph.D. degree in electrical engineering: systems from the University of Michigan, Ann Arbor, in 2006, 2009, and 2010, respectively.

Currently, he is a Postdoctoral Researcher at the Institute of Electrical Engineering, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland. His research interests include stochastic control, stochastic scheduling and resource allocation problems, energy-efficient design of wireless communication networks, and inventory theory.

Ashutosh Nayyar (Student Member, IEEE) received the B. Tech. degree in electrical engineering from the Indian Institute of Technology, Delhi, India, in 2006 and the M.S. degree in electrical engineering and computer science from the University of Michigan, Ann Arbor, in 2008, where he is currently working towards the Ph.D. degree in electrical engineering and computer science.

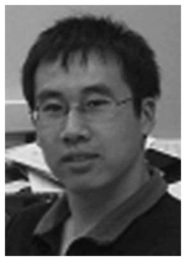
His research interests include decentralized stochastic control, stochastic scheduling and resource allocation, game theory, and mechanism design.

Aditya Mahajan (Member, IEEE) received the B.Tech. degree in electrical engineering from the Indian Institute of Technology, Delhi, India, in 2003 and the M.S. and Ph.D. degrees in electrical engineering and computer science from the University of Michigan, Ann Arbor, in 2006 and 2008, respectively.

Currently, he is an Assistant Professor of Electrical and Computer Engineering, McGill University, Montreal, QC, Canada. From 2008 to 2010, he was a Postdoctoral Researcher at the Department of Electrical Engineering, Yale University, New Haven, CT. His research interests include decentralized stochastic control, team theory, real-time communication, information theory, and discrete event systems.

Yuriy Goykhman (Student Member, IEEE) received the B.S. degree in electrical and computer engineering from Carnegie Mellon University, Pittsburgh, PA, in 2005 and the M.S. degree in electrical engineering from the University of Michigan, Ann Arbor, in 2007, where he is currently working towards the Ph.D. degree.

His research interests include forward and inverse scattering, radar systems, radar data processing, and sensor models.



Ke Li received the B.S. degree in mechanical engineering from Beijing University of Aeronautics and Astronautics, Beijing, China, in 2006. He is currently working towards the Ph.D. degree in instrument science and technology at the Department of Precision Instruments and Mechanology, Tsinghua University, Beijing, China.

He is a Visiting Researcher at the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor. His research interests include architectures, protocols, and performance analysis of wireless networks.

Mingyan Liu (Member, IEEE) received the B.Sc. degree in electrical engineering from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 1995 and the M.Sc. degree in systems engineering and the Ph.D. degree in electrical engineering from the University of Maryland, College Park, in 1997 and 2000, respectively.

She joined the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, in September 2000, where she is currently an Associate Professor. Her research interests are in optimal resource allocation, performance modeling and analysis, and energy efficient design of wireless, mobile *ad hoc*, and sensor networks.

Dr. Liu is the recipient of the 2002 National Science Foundation (NSF) CAREER Award, and the University of Michigan Elizabeth C. Crosby Research Award in 2003. She serves on the editorial board of the IEEE/ACM Transactions on Networking and the IEEE Transactions on Mobile Computing.

Demosthenis Teneketzis (Fellow, IEEE) received the diploma in electrical engineering from the University of Patras, Patras, Greece, in 1974 and the M.S., E.E., and Ph.D. degrees in electrical engineering, from the Massachusetts Institute of Technology, Cambridge, in 1976, 1977, and 1979, respectively.

Currently, he is the Professor of Electrical Engineering and Computer Science at the University of Michigan, Ann Arbor. In winter and spring 1992, he was a Visiting Professor at the Swiss Federal Institute of Technology (ETH), Zurich, Switzerland. Prior to joining the University of Michigan, he worked for Systems Control, Inc., Palo Alto, CA, and Alphatech, Inc., Burlington, MA. His research interests are in stochastic control, decentralized systems, queueing and communication networks, stochastic scheduling and resource allocation problems, mathematical economics, and discrete-event systems.

Mahta Moghaddam (Fellow, IEEE) received the B.S. degree (with highest distinction) from the University of Kansas, Lawrence, in 1986 and the M.S. and Ph.D. degrees from the University of Illinois at Urbana-Champaign, Urbana, in 1989 and 1991, respectively, all in electrical and computer engineering.

Currently, she is the Professor of Electrical Engineering and Computer Science at the University of Michigan, Ann Arbor, where she has been since 2003. From 1991 to 2003, she was with the Radar Science and Engineering Section, Jet Propulsion Laboratory (JPL), California Institute of Technology, Pasadena, CA. She has introduced new approaches for quantitative interpretation of multichannel SAR imagery based on analytical inverse scattering techniques applied to complex and random media. She was a Systems Engineer for the Cassini Radar, the JPL Science group Lead for the LightSAR project, and served as Science Chair of the JPL Team X (Advanced Mission Studies Team). Her most recent research interests include the development of new radar instrument and measurement technologies for subsurface and subcanopy characterization, development of forward and inverse scattering techniques layered random media including those with rough interfaces, and transforming concepts of radar remote sensing to near-field and medical imaging.

Dr. Moghaddam is a member of the NASA Soil Moisture Active and Passive (SMAP) mission Science Definition Team and the Chair of the Algorithms Working Group for SMAP.

Dara Entekhabi (Senior Member, IEEE) received the Ph.D. degree in civil engineering from the Massachusetts Institute of Technology (MIT), Cambridge, in 1990.

Currently, he is a Professor at the Department of Civil and Environmental Engineering, MIT. He serves as the Director of the MIT Ralph M. Parsons Laboratory for Environmental Science and Engineering as well as the MIT Earth System Initiative. His research activities are in terrestrial remote sensing, data assimilation, and coupled land–atmosphere systems behavior.

Dr. Entekhabi is a Fellow of the American Meteorological Society (AMS) and the American Geophysical Union (AGU). He served as the Technical Cochair of the 2008 IEEE International Geoscience and Remote Sensing Symposium. He is the Science Team Leader of the NASA Soil Moisture Active and Passive (SMAP) satellite mission scheduled for launch in 2014.