

# SEMI-BLIND ADAPTIVE BEAMFORMING FOR CYCLOSTATIONARY SIGNALS: A KALMAN FILTERING APPROACH

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## ABSTRACT

In this paper, we develop a new adaptive beamforming algorithm for cyclostationary signals. Our algorithm is derived by maximizing the cyclic moment of the beamformer's output subject to a constraint that preserves all the signals within a prescribed uncertainty set. This constraint allows the beamformer to capture the desired signal and suppress any cyclostationary interferers using the (possibly erroneous) prior information about the array manifold. We develop a state-space model for the underlying optimization problem and derive an iterative cyclic beamforming algorithm using the second-order extended Kalman filter (EKF). Numerical simulations are presented showing the superior performance of our beamformer compared to earlier cyclic beamforming techniques.

## 1. INTRODUCTION

Blind cyclic beamforming is capable of extracting a cyclostationary signal-of-interest (SOI) without any knowledge of the array manifold. It was first proposed in [1] where a class of objective functions termed Spectral self-COherence REstoral (SCORE) was introduced. Several cyclic beamforming algorithms were also derived using different cost functions, e.g., [2], [3], and [4]. In all these algorithms the beamformer was capable of extracting the SOI and suppressing noncyclic interference signals. However, it was shown in [3] that if any of the interference signals has nonzero cyclic moment at the same cycle frequency of the SOI, the beamformer might capture the interference signal instead of the SOI. This problem arises because the beamformer does not have any information to discriminate between the SOI and the cyclic interference.

In many applications, some a priori spatial information is available about the SOI and the array aperture. This information can be used to distinguish between the SOI and the cyclic interference signals. In [5], Castedo *et al.* added linear constraints to the cyclic beamforming algorithm in order to exploit the spatial information available about the SOI. However, no attempt has been made in [5] to account for any er-

rors in the available spatial information, and hence, the performance of this beamformer may degrade severely if the prior information about the SOI is inaccurate.

Several approaches have been recently proposed to provide robustness to the minimum variance distortionless response (MVDR) beamformer against mismatches in the steering vector of the SOI, e.g., [6] and [7]. In this paper, we present a new adaptive beamforming algorithm for cyclostationary signals that is capable of suppressing cyclic interference signals using the available possibly erroneous spatial information about the SOI. Our algorithm is derived by maximizing the cyclic moment of the beamformer's output subject to a constraint that provides high gain to all the signals within a prescribed spatial uncertainty set. This constraint was originally employed in the robust MVDR beamformer of [6]. It was shown to be effective in preserving the SOI in spite of mismatches between the presumed and actual steering vector of the SOI. Hence, this constraint provides robustness against any errors in the prior spatial information of the SOI. We present a state-space model describing the constrained cyclic beamforming problem similar to the model developed in [7] for the robust MVDR beamformer. A second-order EKF can be used to estimate the beamformer weight vector iteratively with reduced computational complexity. Numerical simulations are presented showing the superior performance of our beamformer compared to earlier cyclic beamforming techniques in nonstationary signal environments and in the presence of cyclic interference signals.

## 2. SEMI-BLIND CYCLIC BEAMFORMING

A zero-mean complex signal  $s(t)$  generates a spectral line at a frequency  $\alpha$  after passing through the nonlinearity  $(\cdot)^p$  if it has a nonzero  $p$ th-order cyclic moment defined as [1]

$$m_{p,s}^{\alpha} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} s^p(t) e^{-j2\pi\alpha t} dt. \quad (1)$$

Many communication signals have nonzero cyclic moments. The most typical examples are linear digital modulated signals that generate spectral lines at multiples of the symbol rate. For carrier modulated signals, the spectral lines will be centered around  $\alpha = p f_c$  where  $f_c$  is the carrier frequency [8].

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In general, the square nonlinearity is used for one dimensional constellations such as pulse amplitude modulated signals. In this case, the spectral lines are obtained at  $\alpha = 2f_c + \frac{i}{T_B}$  where  $i$  is an integer and  $T_B$  is the symbol duration.

Let  $\mathbf{x}(k)$  be the  $M \times 1$  complex vector representing the sampled output signal of an  $M$ -sensor array. We assume that the signal environment consists of  $D$  statistically independent signals with nonzero  $p$ th-order cyclic moment at the frequency  $\alpha_s$  and  $I$  signals with zero  $p$ th-order cyclic moment at  $\alpha_s$ . We can model  $\mathbf{x}(k)$  as

$$\mathbf{x}(k) = \sum_{i=1}^D s_i(k) \mathbf{a}(\theta_{s,i}) + \sum_{i=1}^I n_i(k) \mathbf{a}(\theta_{n,i}) + \mathbf{v}(k) \quad (2)$$

where  $s_i(k)$  is the  $i$ th cyclostationary signal arriving from the direction  $\theta_{s,i}$ ,  $n_i(k)$  is the  $i$ th noncyclic interference signal arriving from  $\theta_{n,i}$ ,  $\mathbf{a}(\theta)$  is the  $M \times 1$  array manifold vector,  $\mathbf{v}(k)$  is an  $M \times 1$  vector of white Gaussian noise with zero mean and covariance  $\sigma_v^2 \mathbf{I}$ , and  $\mathbf{I}$  denotes the identity matrix.

If there is only one signal with a nonzero  $p$ th-order cyclic moment at the frequency  $\alpha_s$ , i.e.,  $D = 1$ , we can recover this signal and suppress the interference by choosing the weight vector  $\mathbf{w}$  of the narrowband beamformer such that it minimizes the cost function [3]

$$J = \sum_k |e^{j2\pi\alpha_s k} - y^p(k)|^2 \quad (3)$$

where  $y(k) = \mathbf{x}^H(k) \mathbf{w}$  is the complex-valued beamformer output. Several techniques have been proposed to minimize (3), e.g., using the least mean squares algorithm and the recursive least squares (RLS) algorithm [3], [4]. On the other hand, when the environment consists of several signals with nonzero cyclic moment at the frequency  $\alpha_s$ , the beamformer does not have enough information to discriminate between these signals. Nevertheless, in many applications some prior information about the spatial characteristics of the SOI can be obtained. This information can be used to capture the SOI and suppress cyclic interference signals. Without any loss of generality, let  $s_1(k)$  be the SOI and  $\mathbf{a}_0$  be its presumed steering vector. We assume that actual steering vector of the SOI belongs to the uncertainty set [6]

$$\mathcal{A}_\varepsilon = \{\tilde{\mathbf{a}} = \mathbf{a}_0 + \mathbf{e} \mid \|\mathbf{e}\| \leq \varepsilon\} \quad (4)$$

We constrain the beamformer such that it provides a high gain to all the steering vectors in the uncertainty set. The modified cyclic beamforming problem can be written as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \sum_k |e^{j2\pi\alpha_s k} - (\mathbf{x}^H(k) \mathbf{w})^p|^2 \\ \text{s.t.} \quad & |\tilde{\mathbf{a}}^H \mathbf{w}| \geq 1 \quad \forall \tilde{\mathbf{a}} \in \mathcal{A}_\varepsilon. \end{aligned} \quad (5)$$

This constraint has been originally proposed in [6] to prevent performance degradation of the MVDR beamformer due to mismatches in the steering vector of the SOI. We can write the constraint in (5) as

$$\min_{\tilde{\mathbf{a}} \in \mathcal{A}_\varepsilon} |\tilde{\mathbf{a}}^H \mathbf{w}| = 1. \quad (6)$$

The minimum of  $|\tilde{\mathbf{a}}^H \mathbf{w}|$  over the uncertainty set can be found by noticing that

$$|\tilde{\mathbf{a}}^H \mathbf{w}| \geq |\mathbf{a}_0^H \mathbf{w}| - |\mathbf{e}^H \mathbf{w}| \geq |\mathbf{a}_0^H \mathbf{w}| - \varepsilon \|\mathbf{w}\| \quad (7)$$

where the two inequalities follow from the triangle and Cauchy Schwartz inequalities, respectively. The worst-case error vector that satisfies (7) with equality is given by  $\mathbf{e} = -\varepsilon e^{j\phi} \mathbf{w} / \|\mathbf{w}\|$  where  $\phi = \arg\{\mathbf{a}_0^H \mathbf{w}\}$  [6]. Hence, we can write (5) as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \sum_k |e^{j2\pi\alpha_s k} - (\mathbf{x}^H(k) \mathbf{w})^p|^2 \\ \text{s.t.} \quad & |\mathbf{a}_0^H \mathbf{w}| - \varepsilon \|\mathbf{w}\| = 1 \end{aligned} \quad (8)$$

### 3. STATE-SPACE MODEL

In order to solve the non-convex optimization problem in (8), we will use a state-space modeling approach similar to that used in [7] to solve the robust MVDR beamforming problem. We note that for slowly varying signal environments we can approximate  $(\mathbf{x}^H(k) \mathbf{w}(k))^p$  by  $\mathbf{q}^H(k) \mathbf{w}(k)$  where  $\mathbf{q}^H(k) = (\mathbf{x}^H(k) \hat{\mathbf{w}}(k-1))^{p-1} \mathbf{x}^H(k)$  and  $\hat{\mathbf{w}}(k)$  is the estimate of the beamforming vector at the  $k$ th instant [4]. Therefore, a state-space model describing the optimization problem in (8) is given by the following process equation

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mathbf{n}_w(k) \quad (9)$$

and the associated measurement equation

$$\begin{bmatrix} e^{j2\pi\alpha_s k} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{q}^H(k) \mathbf{w}(k) \\ |\mathbf{a}_0^H \mathbf{w}(k)| - \varepsilon \|\mathbf{w}(k)\| \end{bmatrix} + \begin{bmatrix} n_{m,1}(k) \\ n_{m,2}(k) \end{bmatrix} \quad (10)$$

where  $\mathbf{w}(k)$  is the state vector,  $\mathbf{n}_w(k)$  is the process noise that allows tracking of the beamforming vector in nonstationary environments and is assumed to be white Gaussian with zero mean and covariance  $\sigma_w^2 \mathbf{I}$ , and  $n_{m,1}(k)$  and  $n_{m,2}(k)$  are the measurement noise sequences assumed to be independent white Gaussian with zero mean and covariances  $\sigma_1^2$  and  $\sigma_2^2$ .

Based on the above state-space model, a state estimator can be used to estimate and track the beamforming vector  $\hat{\mathbf{w}}(k)$ . The estimator will yield a vector that minimizes the mean square values of  $n_{m,1}(k)$  and  $n_{m,2}(k)$ , hence minimizing the cost function in (8) and the mean square error in satisfying the constraint in (8), respectively. The process noise variance  $\sigma_w^2$  should be chosen to reflect the degree of nonstationarity of the environment. Also, the value of  $\sigma_2^2$  should be chosen small enough such that the constraint is satisfied with enough accuracy, e.g.,  $\sigma_2^2 = 10^{-4}$  in our experimental setup. The value of  $\sigma_1^2$  should be selected as the expected value of the cost function. Assuming that the interfering sources are suppressed at convergence of the estimator, i.e.,  $\mathbf{x}^H(k) \mathbf{w} = \mathbf{a}^H(\theta_{s,1}) \mathbf{w} s_1(k) + \mathbf{v}^H(k) \mathbf{w}$ , we can write

$$\begin{aligned} \sigma_1^2 = \mathbb{E} \left\{ \left| e^{j2\pi\alpha_s k} - \mathbf{q}^H(k) \mathbf{w} \right|^2 \right\} &= 1 - 2\text{Re} \{ m_{p,s_1}^{\alpha_s} \mathbf{a}^H(\theta_{s,1}) \mathbf{w} \} \\ &+ (\sigma_s^2 \mathbf{w}^H \mathbf{a}(\theta_{s,1}) \mathbf{a}^H(\theta_{s,1}) \mathbf{w} + \sigma_v^2 \|\mathbf{w}\|^2)^p \end{aligned} \quad (11)$$

where  $\sigma_s^2 = E\{|s_1(k)|^2\}$  is the power of the SOI. Note that the above expression for  $\sigma_1^2$  is a function of  $\mathbf{w}^H \mathbf{a}(\theta_{s,1})$  and  $\|\mathbf{w}\|$  which are unknown a priori. Nevertheless, at high signal-to-noise ratio (SNR), we can approximate  $\sigma_1^2$  as

$$\sigma_1^2 \approx \sigma_s^{2p} |\mathbf{w}^H \mathbf{a}(\theta_{s,1})|^{2p} = (M \sigma_s)^{2p} \quad (12)$$

where we have assumed that the SOI is added coherently by the beamformer. We will show through numerical simulations that our state estimator performs well for a wide range of  $\sigma_1^2$ .

Due to the nonlinearity of the measurement equation, we will use a real-valued second-order EKF to estimate the beamformer weight vector. We define the  $2M \times 1$  real-valued weight vector  $\tilde{\mathbf{w}}(k) = [\text{Re}\{\mathbf{w}^T(k)\}, \text{Im}\{\mathbf{w}^T(k)\}]^T$ . The equivalent state-space model is given by

$$\tilde{\mathbf{w}}(k+1) = \tilde{\mathbf{w}}(k) + \tilde{\mathbf{n}}_w(k) \quad (13)$$

$$\tilde{\mathbf{z}}(k) = \mathbf{h}(k, \tilde{\mathbf{w}}(k)) + \tilde{\mathbf{n}}_m(k) \quad (14)$$

where  $\tilde{\mathbf{n}}_w(k) = [\text{Re}\{\mathbf{n}_w^T(k)\}, \text{Im}\{\mathbf{n}_w^T(k)\}]^T$  is the  $2M \times 1$  process noise vector with covariance matrix  $\mathbf{Q} = (\sigma_w^2/2)\mathbf{I}$ ,  $\tilde{\mathbf{n}}_m(k) = [\text{Re}\{n_{m,1}(k)\}, \text{Im}\{n_{m,1}(k)\}, n_{m,2}(k)]^T$  is the measurement noise with the diagonal covariance matrix  $\mathbf{R} = \text{diag}\{\sigma_1^2/2, \sigma_1^2/2, \sigma_2^2\}$ ,  $\tilde{\mathbf{z}}(k) = [\cos(2\pi\alpha_s k), \sin(2\pi\alpha_s k), 1]^T$ ,  $\mathbf{h}(k, \tilde{\mathbf{w}}) = [\mathbf{q}_r^T(k)\tilde{\mathbf{w}}, \mathbf{q}_i^T(k)\tilde{\mathbf{w}}, \sqrt{\tilde{\mathbf{w}}^T \tilde{\mathbf{A}}_0 \tilde{\mathbf{w}} - \varepsilon} \|\tilde{\mathbf{w}}\|]^T$ , the  $2M \times 1$  vectors  $\mathbf{q}_i(k) = [-\text{Im}\{\mathbf{q}^T(k)\}, \text{Re}\{\mathbf{q}^T(k)\}]^T$  and  $\mathbf{q}_r(k) = [\text{Re}\{\mathbf{q}^T(k)\}, \text{Im}\{\mathbf{q}^T(k)\}]^T$ , and the  $2M \times 2M$  matrix

$$\tilde{\mathbf{A}}_0 = \begin{bmatrix} \text{Re}\{\mathbf{a}_0 \mathbf{a}_0^H\} & -\text{Im}\{\mathbf{a}_0 \mathbf{a}_0^H\} \\ \text{Im}\{\mathbf{a}_0 \mathbf{a}_0^H\} & \text{Re}\{\mathbf{a}_0 \mathbf{a}_0^H\} \end{bmatrix}. \quad (15)$$

The Jacobian of the measurement vector  $\mathbf{h}(k, \tilde{\mathbf{w}})$  and the Hessian of its third component are given respectively by

$$\mathbf{H}(k, \tilde{\mathbf{w}}) = \left[ \mathbf{q}_r, \mathbf{q}_i, \frac{\tilde{\mathbf{A}}_0 \tilde{\mathbf{w}}}{\sqrt{\tilde{\mathbf{w}}^T \tilde{\mathbf{A}}_0 \tilde{\mathbf{w}}} - \varepsilon} - \frac{\tilde{\mathbf{w}}}{\|\tilde{\mathbf{w}}\|} \right]^T \quad (16)$$

$$\mathbf{H}_{ww}^{(3)}(k, \tilde{\mathbf{w}}) = \frac{\tilde{\mathbf{A}}_0}{\sqrt{\tilde{\mathbf{w}}^T \tilde{\mathbf{A}}_0 \tilde{\mathbf{w}}} - \varepsilon} - \frac{\tilde{\mathbf{A}}_0 \tilde{\mathbf{w}} \tilde{\mathbf{w}}^T \tilde{\mathbf{A}}_0^T}{(\tilde{\mathbf{w}}^T \tilde{\mathbf{A}}_0 \tilde{\mathbf{w}})^{\frac{3}{2}} - \varepsilon} - \frac{\varepsilon \mathbf{I}}{\|\tilde{\mathbf{w}}\|} + \varepsilon \frac{\tilde{\mathbf{w}} \tilde{\mathbf{w}}^T}{\|\tilde{\mathbf{w}}\|^3}$$

The recursion for the estimated weight vector starts with an initial weight vector estimate  $\hat{\mathbf{w}}(0)$  with the associated covariance  $\mathbf{P}(0|0)$ , and updates the weight vector estimate through

$$\hat{\mathbf{w}}(k) = \hat{\mathbf{w}}(k-1) + \mathbf{G}(k) (\tilde{\mathbf{z}} - \hat{\mathbf{z}}(k|k-1)), \quad (17)$$

where the filter gain  $\mathbf{G}(k)$  and the predicted measurement  $\hat{\mathbf{z}}(k|k-1)$  are given by

$$\mathbf{G}(k) = \mathbf{P}(k|k-1) \mathbf{H}^T(k, \hat{\mathbf{w}}(k-1)) \mathbf{S}^{-1}(k) \quad (18)$$

$$\hat{\mathbf{z}}(k|k-1) = \frac{1}{2} \text{tr} \left\{ \mathbf{H}_{ww}^{(3)}(k, \hat{\mathbf{w}}(k-1)) \mathbf{P}(k|k-1) \right\} \mathbf{e}_3 + \mathbf{h}(k, \hat{\mathbf{w}}(k-1)) \quad (19)$$

where  $\mathbf{e}_3 = [0, 0, 1]^T$ . The innovation covariance matrix and the covariance matrix of the predicted weight vector are given respectively by

$$\begin{aligned} \mathbf{S}(k) &= \mathbf{H}(k, \hat{\mathbf{w}}(k-1)) \mathbf{P}(k|k-1) \mathbf{H}^T(k, \hat{\mathbf{w}}(k-1)) + \mathbf{R} \\ &+ \frac{\varepsilon_3 \mathbf{e}_3^T}{2} \text{tr} \left\{ \mathbf{H}_{ww}^{(3)}(k, \hat{\mathbf{w}}(k-1)) \mathbf{P}(k|k-1) \right. \\ &\quad \left. \mathbf{H}_{ww}^{(3)}(k, \hat{\mathbf{w}}(k-1)) \mathbf{P}(k|k-1) \right\} \end{aligned} \quad (20)$$

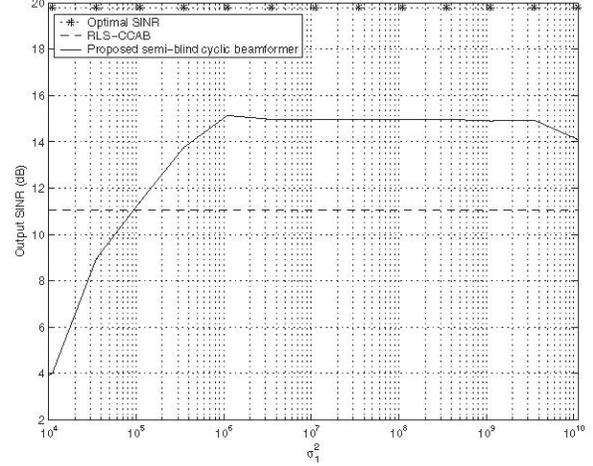


Fig. 1. Average output SINR versus  $\sigma_1^2$ .

$$\mathbf{P}(k|k-1) = \mathbf{P}(k-1|k-1) + \mathbf{Q}, \quad (21)$$

and the updated state covariance matrix is given by

$$\mathbf{P}(k|k) = \mathbf{P}(k|k-1) - \mathbf{G}(k) \mathbf{S}(k) \mathbf{G}^H(k). \quad (22)$$

The consistency of the beamformer can be checked using the normalized innovation square (NIS) test [9]. Under the Gaussian assumption for the measurement noise, the NIS

$$\epsilon_\nu(k) = (\tilde{\mathbf{z}} - \hat{\mathbf{z}}(k|k-1))^H \mathbf{S}^{-1}(k) (\tilde{\mathbf{z}} - \hat{\mathbf{z}}(k|k-1)) \quad (23)$$

is Chi-square distributed with three degrees of freedom and should lie within the confidence region of the Chi-square distribution if the beamformer is consistent [9].

For initialization of the iterative algorithm, a random vector estimate  $\hat{\mathbf{w}}(0)$  can be used together with an initial covariance matrix estimate  $\mathbf{P}(1|0) = \beta \mathbf{I}$ , where  $\beta$  is selected such that the NIS of the first iteration is acceptable. Therefore, by ignoring the second-order terms and the measurement noise covariance matrix in (20), we can approximate  $\beta$  as

$$\beta \approx \frac{1}{3} (\tilde{\mathbf{z}}(1) - \hat{\mathbf{z}}(1|0))^H (\mathbf{H}(1, \hat{\mathbf{w}}(0)) \mathbf{H}^T(1, \hat{\mathbf{w}}(0)))^{-1} (\tilde{\mathbf{z}}(1) - \hat{\mathbf{z}}(1|0)). \quad (24)$$

#### 4. NUMERICAL SIMULATIONS

We consider a linear array of  $M = 10$  elements. The desired signal is a BPSK signal with normalized carrier frequency  $f_c = 0.15$  and bit rate 0.02 impinging on the array from  $\theta = -10^\circ$  with SNR 10 dB. A second BPSK signal with normalized carrier frequency of 0.17 and bit rate 0.01 arrives from  $\theta = 50^\circ$  with INR = 30 dB. Symbols are transmitted using Nyquist-shaped modulation pulses with 100% excess bandwidth. The presumed angle of arrival of the SOI is  $\theta_0 = -8^\circ$ . The sensors are assumed to be uniformly spaced with spacing equal to  $\lambda/2$  where  $\lambda$  is the wavelength of the

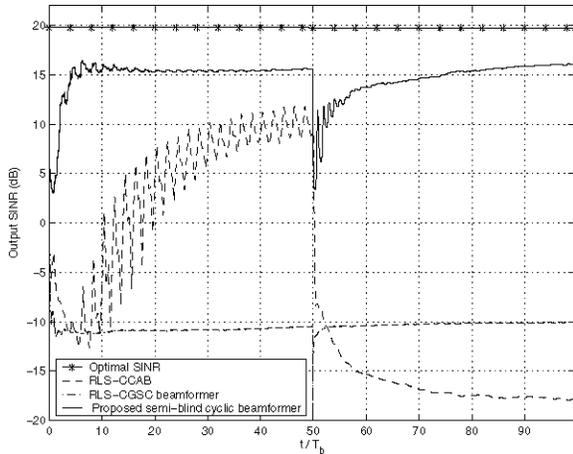


Fig. 2. Average output SINR versus transmitted bits.

SOI. The actual sensor locations are displaced along the array line by random displacements uniformly distributed between  $[-0.05\lambda, 0.05\lambda]$ . The parameters of our beamformer are selected as  $\varepsilon = 3$ ,  $\sigma_w^2 = 0$ , and  $\sigma_2^2 = 10^{-4}$ . Simulation results are averaged over 100 Monte Carlo runs.

Fig. 1 shows the average output signal-to-interference-plus-noise ratio (SINR) of our semi-blind cyclic beamformer after 50 transmitted bits versus different values of the parameter  $\sigma_1^2$ . Fig. 1 also shows the output SINR of the RLS implementation of the constrained cyclic adaptive beamformer (RLS-CCAB) of [2] and the optimal output SINR obtained when the beamformer has exact knowledge of the array manifold. Note that the approximate value of  $\sigma_1^2$  in (12) is equal to  $10^8$ . We can clearly see from Fig. 1 that our beamformer performs well and is superior to the RLS-CCAB over a wide range of the parameter  $\sigma_1^2$ .

Next, we evaluate the performance of our beamformer in a nonstationary environment. We consider the same scenario as that in the previous simulation for the first 50 transmitted bits. A second cyclic interference signal having the same carrier frequency and bit rate as those of the SOI impinges on the array from  $\theta = 30^\circ$  with INR = 30 dB after the 50th transmitted bit. The parameters of our semi-blind cyclic beamformer are selected as  $\sigma_w^2 = 10^{-8}$ ,  $\sigma_1^2 = 10^8$ , and  $\sigma_2^2 = 10^{-4}$ . Fig. 2 compares the performance of our semi-blind cyclic beamformer with that of the RLS-CCAB and the cyclic generalized sidelobe canceler (RLS-CGSC) beamformer of [5]. We can clearly see from Fig. 2 that our beamformer has superior performance to both beamformers in terms of the output SINR. The RLS-CCAB beamformer is unable to discriminate the cyclic interference from the desired signal, and hence, the SINR drops after the addition of the cyclic interference. Also, the RLS-CGSC is unable to extract the desired signal due to the error in the prior information about its steering vector. On the other hand, our proposed semi-blind beamformer can extract the desired signal and suppress the cyclic interference in spite of those mismatches. This can be seen from Fig. 3

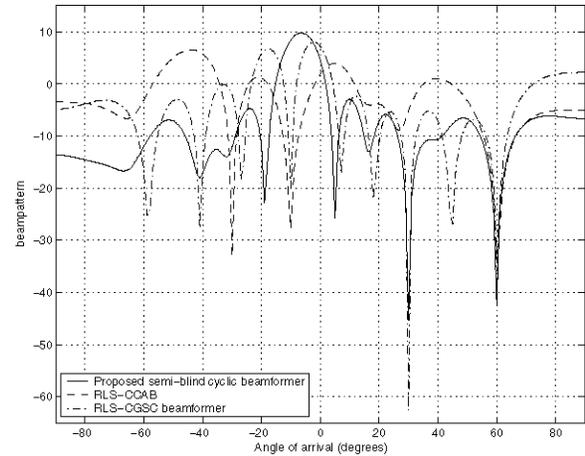


Fig. 3. Beampattern.

which shows the beampattern of different beamformers computed at the frequency of the desired signal at the 100th transmitted bit. We can clearly see the superior performance of our proposed beamformer as it provides a high gain towards the desired signal while suppressing the cyclic interference signal and maintaining a low sidelobe level.

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