# **GROUP-OPTIMAL LINEAR SPACE-TIME MULTIUSER DETECTION**

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#### Abstract

Multiuser detection (MUD) techniques are known to improve the performance of CDMA cellular communication systems. This performance improvement usually comes at a large computational cost. To reduce the complexity, it has been proposed recently to exploit the spatial dimension by grouping users in clusters and apply MUD individually to each group. This approach leads to a potentially significant complexity reduction, at a marginal cost in performance. In this work we propose a new space-time receiver structure based on the group-optimal MMSE linear detector. The numerical results show that the proposed technique performs close to the optimal full space-time linear MMSE MUD receiver but with a significantly lower complexity.

**Keywords**—Multiuser detection, space-time signal processing, DS-CDMA.

## **1** Introduction

Most of the current and future cellular wireless systems based on direct-spread code-division multiple access (DS-CDMA) are interference-limited. For DS-CDMA systems, where multiple access interference (MAI) is known to limit the system capacity, multiuser detection (MUD) and beamforming (BF) with antenna arrays have been widely studied for interference reduction [1–3].

Optimal MUD takes the form of trellis decoding and is very complex due to the size of the search space which increases exponentially with the number of users and sequence length [4]. Several reduced complexity suboptimal techniques for MUD have been proposed, including linear filtering approaches and iterative techniques (see e.g.: [4–6]).

To further reduce the complexity of the MUD receiver and at the same time reduce the co-channel interference on the uplink, it has been proposed in [7,8] to group users within a sector in mutually exclusive spatial equivalence classes or cluster. The data symbols from each group are jointly detected using reduced dimension MUD, while inter-group interference (IGI) is reduced by using spatial filtering or *beamforming* with smart antennas, a concept illustrated in Fig. 1.

When implemented as a matrix inversion, the complexity associated to MUD for each group is proportional to  $K_j^3$ , where  $K_j$  is the number of users in group  $j \in \{1, \ldots, G\}$ , G being the total number of groups. Thus the grouping approach in [7] has the potential to considerably reduce the total complexity, i.e.  $\sum_j K_j^3$ , compared to the full MUD complexity of  $K^3$ , where  $K = \sum_j K_j$ .

We point out in [9] that the beamforming step for spatial filtering is redundant. In fact, beamforming reduces the dimension



Fig. 1: Group-based space-time multiuser detection conceptual diagram.

of the observation space by using a non-invertible linear transformation, resulting in a potential loss of performance. In this work, we develop a new group-based space-time MUD (GRP-STMUD) MMSE linear receiver, which, in contrast to [7–10], does not need a separate and independent beamforming "unit". The linear estimator is applied directly on the complete observation vector. The weights are designed using a MMSE criterion and are optimal with respect to a fixed grouping. Furthermore, we develop a new, practical user grouping algorithm based on the hardware physical limitations. The proposed system is shown to provide BER performance significantly better than the conventional match filter; indeed the numerical BER results obtained for the proposed system are close to the full space-time MUD (STMUD), at a lower computation cost.

The paper is organized as follows. Section 2 introduces the system model and the full STMUD receiver. The proposed group-based structure is developed in section 3. The new group-ing algorithm is discussed in section 4, and numerical results supporting the work are shown in section 5. Finally, a brief conclusion is presented in section 6

# 2 Background

#### 2.1 System model

Consider the uplink of a synchronous DS-CDMA communication system with K users transmitting blocks of N information symbols simultaneously through a dispersive channel to a common multi-antenna receiver. At each antenna, the received signal is converted to baseband, matched filtered to the transmission pulse and sampled at the "chip" rate of  $1/T_c$ , where  $T_c$  denotes the chip duration. The observed signal at the receiver therefore consists of a complex-valued vector of length NQ + W - 1, where  $Q = T_s/T_c$  is the symbol expansion factor (or spreading factor),  $T_s$  is the symbol duration, and W is the length of the finite impulse response channel.

Let M be the number of antennas and  $\mathbf{x}^{(m)} \in \mathbb{C}^{NQ+W-1}$ for  $m = 1, \ldots, M$ , be the received signal vector for the  $m^{\text{th}}$ antenna element. Following the linear model described in [5], it is convenient to represent the complete set of observations in vector form as

$$\mathbf{x} = \operatorname{vec}([\mathbf{x}^{(1)} \dots \mathbf{x}^{(M)}]^T) \in \mathbb{C}^{M(NQ+W-1)}, \qquad (1)$$

where T denotes matrix transposition and  $vec(\cdot)$  is an operation that sequentially concatenates the columns of a matrix into a column vector of appropriate dimension. Similarly, the vector of NK information symbols transmitted by the K users can be represented in vector form as

$$\mathbf{d} = \operatorname{vec}([\mathbf{d}^{(1)} \dots \mathbf{d}^{(K)}]^T) \in \mathcal{A}^{NK},$$
(2)

where  $\mathbf{d}^{(k)} \in \mathcal{A}^N$  is the vector of information symbols for user k and  $\mathcal{A}$  is the symbol alphabet of  $N_{\mathcal{A}}$  elements (e.g.: for BPSK  $\mathcal{A} = \{\pm 1\}$  and  $N_{\mathcal{A}} = 2$ ). The information symbols are assumed to be independent, identically distributed (iid) and normalized such that  $E[\mathbf{dd}^H] = \mathbf{I}_{NK}$ , where H represents Hermitian transposition,  $\mathbf{I}_{NK}$  is the identity matrix of dimension NK and  $E[\cdot]$  denotes statistical expectation.

Let  $\mathbf{v}_k \in \mathbb{C}^{M(Q+W-1)}$  be the  $k^{\text{th}}$  user space-time *effective* signature vector, i.e. the space-time response to a unit pulse excitation sequence  $\delta = [1, 0, \dots, 0]$  as observed by the multiantenna receiver after demodulation, sampling and vector formatting as described above. Define  $\mathbf{V} = [\mathbf{v}_1 \dots \mathbf{v}_K] \in \mathbb{C}^{M(Q+W-1)\times K}$  to be the effective signature matrix for the set of K users. Then the total received vector may be conveniently expressed as

$$\mathbf{x} = \mathbf{T}\mathbf{d} + \mathbf{n},\tag{3}$$

where  $\mathbf{T} \in \mathbb{C}^{M(NQ+W-1)\times NK}$  is a block-Toeplitz matrix. In particular, assuming a relatively short channel delay-spread so that symbols interfere only with their adjacent neighbors, i.e. W < Q, the matrix  $\mathbf{T}$  takes the special form [5]

$$\mathbf{T} = \begin{bmatrix} \mathbf{v} & MQ \\ MQ \\ \mathbf{v} & MQ \\ \mathbf{v} & \mathbf{v} \end{bmatrix}.$$
(4)

The vector  $\mathbf{n} \in \mathbb{C}^{M(NQ+W-1)}$  in (3) contains white circular complex Gaussian noise samples with covariance matrix  $\mathbf{R_n} \triangleq E[\mathbf{nn}^H] = \sigma^2 \mathbf{I}_{M(NQ+W-1)}$ , where  $\sigma^2$  is the noise power. Notice that the above model and ensuing results can be generalized to account for colored noise and the case W > Q.

# 2.2 Space-time MUD

In multi-user detection, the symbols transmitted from all K users are jointly estimated, based on the space-time observation vector  $\mathbf{x}$ . In a linear receiver, the soft symbols estimates are obtained from the output of the estimator  $\mathbf{M} \in \mathbb{C}^{NK \times NK}$ . For BPSK, the actual symbols estimates are taken as the sign of the real part of the soft estimates, i.e.:  $\hat{\mathbf{d}} = \operatorname{sgn}\{\Re(\mathbf{M}^H \mathbf{y})\}$ , where  $\mathbf{y} = \mathbf{T}^H \mathbf{x}$  is the match filter (MF) output,  $\operatorname{sgn}(\cdot)$  is a function that returns the sign of its argument and  $\Re(\cdot)$  is its real part. It

can be shown that the linear filter that minimizes the mean square error, defined here as  $J^{o}(\mathbf{M}) = E \|\mathbf{d} - \mathbf{M}^{H}\mathbf{y}\|^{2}$ , takes the form

$$\mathbf{M}_{\mathbf{o}} = (\mathbf{T}^H \mathbf{T} + \sigma^2 \mathbf{I})^{-1}.$$
 (5)

Notice that the complete operation consists of a match filter  $(\mathbf{T}^H)$  followed by a minimum mean square error (MMSE) filter of dimension  $NK \times NK$ . If inverted using traditional techniques, the operation has complexity of order  $\mathcal{O}(K^3)$ ; a considerable difficulty for real-time operations. We shall refer to the MUD filter in (5) as the *full space-time MUD (STMUD)*.

#### **3** Linear ST-MUD with grouping

The proposed receiver structure for linear ST-MUD with grouping is shown in Fig. 2. As illustrated, each group has its own STMUD unit, and the soft estimate output is given by  $\mathbf{z}_j \triangleq \mathbf{M}_j^H \mathbf{T}_j^H \mathbf{x}$  with  $\mathbf{T}_j$  and  $\mathbf{M}_j$  being the matched filter and MMSE linear filter matrices for users of group j, respectively. The hard symbol estimates are obtained through a non-linear decision device  $\mathcal{Q}(\cdot)$  (e.g. for BPSK  $\mathcal{Q}(\cdot) = \operatorname{sgn}(\cdot)$ ).



Fig. 2: Block diagram for the proposed GRP-STMUD receiver.

# 3.1 Optimal linear ST-MUD with grouping

The linear estimators in the GRP-STMUD structure in Fig. 2 in general depend on the grouping, which in turns may depend on the choice of linear weights. The optimal weights and grouping must therefore be chosen jointly.

Since in Fig. 2 symbol detection is performed independently among groups and the choice of weights for a given group does not affect the other groups, it is reasonable to define the cost function associated to group j for a given set of user grouping as  $J(\mathbf{M}_j, \mathcal{G}_j)$ , and define the total cost as the sum of the individual costs from each group. The set of optimal filters and grouping may be expressed as

$$[\mathbf{M}_{1,o},\ldots,\mathbf{M}_{G,o},\mathcal{G}^{o}] = \arg\min_{[\mathbf{M}_{1},\ldots,\mathbf{M}_{G},\mathcal{G}]} \sum_{j=1}^{G} J(\mathbf{M}_{j},\mathcal{G}_{j}),$$
(6)

where the system grouping is given by  $\mathcal{G} = \{\mathcal{G}_1, \ldots, \mathcal{G}_G\}$  with  $\mathcal{G}_j \subseteq \mathcal{K}$  being the set of user indices belonging to group  $j \in \{1, \ldots, G\}$  and  $\mathcal{K} = \{1, \ldots, K\}$  is the set of all users indices. To complete this notation, let  $\overline{\mathcal{G}}_j$  be the complement of  $\mathcal{G}_j$  such that  $\mathcal{G}_j \cup \overline{\mathcal{G}}_j = \mathcal{S}$  and  $\mathcal{G}_j \cap \overline{\mathcal{G}}_j = \emptyset, \forall j$ .

Due to the discrete nature of G and the extremely large number of possibilities, finding a solution for (6) may be a very difficult

task for real-time operations. For this reason, the optimization problem is relaxed and carried out in two separate steps; filter design, discussed below, and grouping algorithm, discussed in section 4.

## 3.2 MMSE ST-MUD with grouping

The GRP-STMUD weights are derived under the assumption that the set of groups  $\mathcal{G}$  is known and fixed. The MMSE GRP-STMUD weights are therefore optimal with respect to the fixed grouping  $\mathcal{G}$ . To derive the weights, the received vector is first expressed as a sum of three signal contributions: the signal from the users within the group of interest  $\mathcal{G}_j$ , the so-called intergroup interference which comes from the users outside of the group of interest  $(\overline{\mathcal{G}}_j)$ , and the additive Gaussian noise.

Let  $\mathbf{P}^{(j)} \in \mathbb{R}^{K \times K_j}$  be the matrix whose columns consist of the set of  $K_j$  elementary vectors  $\{\mathbf{e}_k\}, k \in \mathcal{G}_j$ , where  $\mathbf{e}_k \in \mathbb{R}^K$ is a column vector containing zeros except at position k, where it contains the value 1. Also let  $\mathbf{P}_j \triangleq (\mathbf{I}_N \otimes \mathbf{P}^{(j)}) \in \mathbb{R}^{NK \times NK_j}$ such that  $\mathbf{d}_j = \mathbf{P}_j^T \mathbf{d}$  represents the  $NK_j$  data symbols transmitted by users in group j only. Similarly, define  $\bar{\mathbf{P}}_j$  as a complement of  $\mathbf{P}_j$ , so that  $\bar{\mathbf{d}}_j = \bar{\mathbf{P}}_j^T \mathbf{d} \in \mathbb{R}^{N(K-K_j)}$  is the vector of all symbols transmitted from the users outside the group j. Observe that  $\mathbf{P}_j^T \mathbf{P}_j = \mathbf{I}, \bar{\mathbf{P}}_j^T \bar{\mathbf{P}}_j = \mathbf{I}$  and  $\bar{\mathbf{P}}_j^T \mathbf{P}_j = \mathbf{0}$ , so that the matrices  $\mathcal{P}_j \triangleq \mathbf{P}_j \mathbf{P}_j^T$  and  $\bar{\mathcal{P}}_j \triangleq \bar{\mathbf{P}}_j \bar{\mathbf{P}}_j^T$  provide a pair of complementary orthogonal projection matrices that can be used to express the observation vector as the sum of "in-group" and "out-of-group" components (i.e.  $\mathcal{P} + \bar{\mathcal{P}} = \mathbf{I}$ ). The received signal in (3) can then be expressed as

$$\mathbf{x} = \mathbf{T}(\mathcal{P}_j + \bar{\mathcal{P}}_j)\mathbf{d} + \mathbf{n} = \mathbf{T}_j\mathbf{d}_j + \bar{\mathbf{T}}_j\bar{\mathbf{d}}_j + \mathbf{n},$$
(7)

where  $\mathbf{T}_j \triangleq \mathbf{TP}_j \in \mathbb{C}^{M(NQ+W-1) \times NK_j}$  and  $\bar{\mathbf{T}}_j \triangleq \mathbf{TP}_j \in \mathbb{C}^{M(NQ+W-1) \times N(K-K_j)}$  are the matrices containing the columns related to the users of group j and its complement, respectively.

The MMSE filter is applied at the output of the group matched filter; using this approach the optimal MMSE filter output for group *j* can be expressed as  $\mathbf{z}_j = \mathbf{M}_{j,o}^H \mathbf{y}_j$ , where  $\mathbf{M}_{j,o}$  is the group-optimal MMSE filter and  $\mathbf{y}_j \triangleq \mathbf{T}_j^H \mathbf{x}$  is the matched filter output for group *j*. The proposed cost function for the MMSE linear estimator of group *j* is given by

$$J_j(\mathbf{M}) \triangleq J(\mathbf{M}, \mathcal{G}_j) = E \| \mathbf{d}_j - \mathbf{M}^H \mathbf{y}_j \|^2,$$
(8)

where the dimension of the matrix **M** is now  $NK_j \times NK_j$ , and the MMSE linear weights for the GRP-STMUD receiver are obtained by solving

$$\mathbf{M}_{j,o}(\mathcal{G}_j) \equiv \mathbf{M}_{j,o} = \arg\min_{\mathbf{M}} J_j(\mathbf{M}).$$
(9)

**Proposition 1** Let  $\mathbf{R}_j \triangleq \mathbf{T}_j^H \mathbf{T}_j$  and  $\mathbf{C}_j \triangleq \mathbf{T}_j^H \bar{\mathbf{T}}_j$ , then the solution to the group MMSE linear weights optimality criterion of (9) is given by

$$\mathbf{M}_{j,o} = (\mathbf{R}_j \mathbf{R}_j^H + \mathbf{C}_j \mathbf{C}_j^H + \sigma^2 \mathbf{R}_j)^{-1} \mathbf{R}_j^H.$$
(10)

*Proof:* Using the technique in [4], the cost in (8) is expanded to give

$$J_j(\mathbf{M}) = \operatorname{tr} \left[ E(\mathbf{d}_j - \mathbf{M}^H \mathbf{y}_j) (\mathbf{d}_j - \mathbf{M}^H \mathbf{y}_j)^H \right]$$
(11)

$$= \operatorname{tr} \left[ \mathbf{I} - \mathbf{R}_{j}^{H} \mathbf{M} - \mathbf{M}^{H} \mathbf{R}_{j} + \mathbf{M}^{H} \mathbf{Q}_{j} \mathbf{M} \right], \quad (12)$$

where  $\mathbf{Q}_j \triangleq (\mathbf{R}_j \mathbf{R}_j^H + \mathbf{C}_j \mathbf{C}_j^H + \sigma^2 \mathbf{R}_j)$  and (12) follows from the statistical independence between the information symbols and the noise. Notice that the signal contribution from the users outside of the group of interest, which appear in (12) through  $\mathbf{C}_j$  in  $\mathbf{Q}_j$ , is considered as an unknown random signal contribution by the optimization procedure. Through algebraic manipulations, (12) becomes

$$J_j(\mathbf{M}) = \operatorname{tr} \left[ \mathbf{I} - \mathbf{R}_j \mathbf{Q}^{-1} \mathbf{R}_j^H + (\mathbf{M} - \bar{\mathbf{M}})^H \mathbf{Q}_j (\mathbf{M} - \bar{\mathbf{M}}) \right], (13)$$

where  $\bar{\mathbf{M}}^{H} \triangleq \mathbf{R}_{j} \mathbf{Q}_{j}^{-1}$ . It is clear that the cost function is minimized when  $\mathbf{M} = \bar{\mathbf{M}}$ . Therefore  $\mathbf{M}_{j,o} = \mathbf{Q}_{j}^{-1} \mathbf{R}_{j}$  as in (10), and the minimum cost is given by

$$J_j(\mathbf{M}_{j,o}) = \operatorname{tr} \left[ \mathbf{I} - \mathbf{R}_j \mathbf{Q}_j^{-1} \mathbf{R}_j^H \right].$$
(14)

#### 4 Practical grouping approach

In a practical receiver, the number of available resources is limited. Based on the diagram in Fig. 2, we assume in this work that the proposed receiver structure can accommodate a maximum of  $G_{\text{max}}$  groups and  $K_{g,\text{max}}$  users per group, where  $G_{\text{max}}$  and  $K_{g,\text{max}}$  depend on the hardware implementation.

The basic principle of the algorithm is to combine pairs of users or groups with short "distance" (not in the strict mathematical sense) first. As in [7,8], we choose to define the pairwise distance between user k and l by  $\delta_{k,l} \triangleq |\mathbf{v}_k^H \mathbf{v}_l|^{-2}$ . For the more general case, we define the distance measure as the shortest pairwise distance between all possible pairs of users:

$$\delta_{\mathcal{S}_p, \mathcal{S}_q} = \arg\min_{\substack{\forall k \in \mathcal{S}_p \\ \forall l \in \mathcal{S}_q}} |\mathbf{v}_k^H \mathbf{v}_l|^{-2}, \tag{15}$$

where  $S_p$  is a non-empty subset of user indices and  $S_p \cap S_q = \emptyset$ , for  $p \neq q$ . This choice of measure is intuitively justified since users with strong effective signature cross-correlation interfere the most with each other.

The flow diagram for the proposed grouping algorithm is illustrated in Fig. 3. Upon initialization of the algorithm, there are no groups  $\mathcal{G} = \emptyset$  and the pool of unallocated users contains all the users in the system. For each iteration, the first step is to compute and sort in increasing order the distances between each pair of the *G* groups and *U* unallocated users, i.e.:  $(S_p, S_q) \in S^2$ , where  $S = \{\mathcal{G}_1, \ldots, \mathcal{G}_G, \{u_1\}, \ldots, \{u_U\}\}, S_p \neq S_q$  for  $p \neq q, G$  and *U* are the current number of groups and unallocated users at the current iteration, and  $u_l$  is the index for the  $l^{th}$  unallocated user. Then for each pair, starting with the one with shortest distance, grouping is attempted. If the elements of a particular pair can be grouped together, the list of unallocated users and groups are

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updated to reflect the new arrangement and the iteration is completed. Otherwise, the next pair is selected and another attempt is made. The algorithm completes when attempts for grouping have failed for all of the L pairs, as illustrated in Fig. 3. Note that the total number of resources is  $K_{\text{max}} \triangleq G_{\text{max}}K_{g,\text{max}}$  and if  $K \leq K_{\text{max}}$ , the algorithm will allocate all the users successfully.



Fig. 3: Flow diagram for grouping algorithm w/o sharing.

# 5 Results

We consider the received signal model of (3) for the uplink of a DS-CDMA system. The users have orthogonal spreading codes of length Q = 16 and transmit BPSK data symbols in blocks of N = 50. The signals are received by M = 6 antennas in a standard linear array configuration. The channel consists of W = 6 equal power multi-paths, with the main path having DOA  $\theta_0$  uniformly distributed within the sector width of 120°, and all other paths uniformly distributed within  $[\theta_0 + \Delta\theta, \theta_0 - \Delta\theta]$ , with  $\Delta\theta = 30^\circ$ . The received signals are under ideal power control with normalized power  $P \equiv P_g = 1$ , and the SNR is thus defined as SNR  $= P/\sigma^2 = \sigma^{-2}$ . The GRP-STMUD structure has  $G_{\text{max}} = 4$  groups of a maximum of  $K_{g,\text{max}} = 4$  users each. The active users share the  $K_{\text{max}} = 16$  detection units.

# 5.1 Complexity reduction

The expressions for the optimal MMSE linear estimators in (5) and (10) include a matrix inversion and several matrix multiplications. Fortunately, the structure of the data matrix T in (4) can be exploited extensively, leading to significant complexity reduction. The most important reduction results from the struc-

ture in the  $\mathbf{T}^{H}\mathbf{T}$  matrix product. This structure can be exploited in particular to reduce the Cholesky factorization complexity to solve the inverse problem [9].

To compare the complexity between the two approaches, the number of complex floating point operations (CFLOPS) is counted for the different parts of equations (5) and (10) by taking advantage of the symmetries as in [9]. The total complexity is divided in two distinct parts: overhead and linear system solution (lss).

We compare three systems that can support a minimum of K = 16 users. The full STMUD can support K = 16 users simultaneously, the GRP-STMUD A has  $G_{\max}^A = 4$  groups of  $K_{g,\max}^A = 4$  users, and the GRP-STMUD B has  $G_{\max}^B = 3$  groups of  $K_{g,\max}^B = 6$  users. While the full STMUD and the GRP-STMUD A have 16 detection units, GRP-STMUD B has 18; having more detection units brings more flexibility in the grouping but increases the complexity. The results show that the total complexity associated to the full STMUD is more than three times that of the proposed GRP-STMUD A system and approximately twice that of the GRP-STMUD B system.



Fig. 4: Numerical complexity comparison between the full STMUD, GRP-STMUD A with  $G_{\text{max}}^{\text{A}} = 4$ ,  $K_{\text{g,max}}^{\text{A}} = 4$ , and GRP-STMUD B with  $G_{\text{max}}^{\text{B}} = 3$ ,  $K_{\text{g,max}}^{\text{B}} = 6$ .

# 5.2 Grouping algorithm

The purpose of this experiment is to validate the grouping criterion of section 4, and at the same time show that the proposed grouping algorithm performs well. The experiment proceeds in two steps: first, the optimal grouping ( $\mathcal{G}^{\circ}$ ) with respect to the MSE is found through exhaustive search of all possible mutually exclusive grouping. The corresponding MSE is given by

$$J^{\mathrm{o}} = \min_{\mathcal{G}} \sum_{\forall j} J_j(\mathbf{M}_{j,\mathrm{o}}(\mathcal{G}^{\mathrm{o}}), \mathcal{G}^{\mathrm{o}}).$$
(16)

Then the proposed grouping algorithm in section 4 is used to obtain  $\mathcal{G}_{grp}$ , and the corresponding MSE is given by

$$J_{\rm grp} = \sum_{\forall j} J_j(\mathbf{M}_{j,\rm o}(\mathcal{G}_{\rm grp}), \mathcal{G}_{\rm grp}).$$
(17)

This experiment is repeated 250 times for sampling different correlation matrices corresponding to different user positions and channel conditions. We define the normalized MSE difference for the grouping algorithm  $\Delta_{grp} \triangleq (J_{grp} - J^o)/J^o$ . In practice it is highly desirable to have a small  $\Delta_{grp}$ ; this would indicate that the grouping obtained provides a MSE which is close to the minimum MSE achievable.

Table I shows the proportion of grouping scenarios that resulted in a difference in normalized MSE, less than or equal to t = 2.5%, and t = 5%, respectively. The SNR here is fixed to 10dB. Table I indicates that 91% of the grouping obtained us-

Table I: Statistics on the grouping algorithm performance.

ing the sub-optimal algorithm of section 4 results in a MSE no more than 2.5% away from the optimal one. Our experimentations show that such a difference in MSE is equivalent to a loss in SNR of approximately 0.2dB. This is a very good performance considering the computational advantages of the proposed algorithm compared to an exhaustive search.

#### 5.3 BER performance

We consider the BER as the measure of performance, which we obtain using Monte-Carlo software simulations with K = 12users and other parameters as described above. To obtain representative results,  $10^7$  symbols are transmitted and processed using the different algorithms discussed. The channel conditions are determined at the beginning of the simulation and are kept constant throughout. Figure 5 shows the average BER for all users in the system for the different algorithms.

The results show an important performance improvement of both GRP-STMUD A and B over the conventional receiver (MF). Indeed, a gain in SNR of more than 3dB is measurable between the MF and GRP-STMUD A at a BER of  $10^{-3}$ . The full STMUD of section 3 performs only slightly better than the group-based techniques; for this particular experiment, at a BER of  $10^{-3}$ , there is a measurable difference over GRP-STMUD A and B of approximately 1dB and 0.4dB in SNR, respectively. The GRP-STMUD B approach performs better than GRP-STMUD A because it has larger groups and is not as sensitive to IGI. The difference in performance between the algorithms increases with the SNR, showing the sensitivity of the grouping approach to IGI as it becomes the dominant factor.

#### 6 Conclusion

We have presented a new group-based optimal space-time linear multiuser receiver system. The system consists of a set of reduced-dimensions linear MUD units for each of the group, and a new grouping algorithm that takes into consideration practical hardware limitations. The proposed system allows performance and complexity trade-offs, filling the gap between match filtering and full STMUD. The results show that the proposed structure provides a means for reduced complexity MUD, at a small cost in BER performance.



Fig. 5: BER versus SNR average performance.

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