

# Group-based block linear successive interference cancellation for DS-CDMA

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**Abstract**— Mobile communication systems based on DS-CDMA suffer from multiple access interference (MAI), which limits the system capacity. Several techniques, such as beamforming with multiple antennas and multiuser detection (MUD), have proved to be effective to mitigate MAI. To reduce the complexity of MUD, it has been proposed to process users in smaller groups. The interference between group is then reduced using beamforming with smart antennas. In this paper, a new group-based block linear successive interference cancellation structure is developed. The structure is shown to provide BER performance close to the full space-time MUD at a lower computational cost.

## I. INTRODUCTION

The limited availability of the bandwidth for mobile communication systems constrains the service providers to make more efficient use of the resource. Most current and future cellular systems are based on direct-spread code-division multiple-access (DS-CDMA), a technology known to be limited by multiple access interference (MAI). To make better use of the bandwidth with DS-CDMA it is essential to reduce MAI, for example by using multiuser detection (MUD) or beamforming with antenna arrays; two approaches that have been widely studied [1, 2].

In its optimal form, multiuser detection requires trellis decoding, an inherently complex task. Fortunately there exists several sub-optimal approaches also effective against MAI such as linear MUD with minimum mean square error (MMSE) or zero forcing (decorrelator) filters [3]. Iterative variants such as parallel interference cancellation (PIC) and successive interference cancellation (SIC) also provide efficient MAI reduction [4, 5].

To further reduce the complexity of the MUD on the uplink, it has been proposed recently to exploit the spatial dimension with beamforming to cluster users in mutually exclusive groups of spatial equivalence [6, 7]. The data symbols from each group are jointly detected using reduced dimension MUD while the inter-group interference (IGI), i.e. the interference created by the users outside of the group of interest, is reduced by using spatial filtering or *beamforming*. In the current literature, grouping is generally accomplished using thresholding on a spatio-temporal cross-correlation metric.

In this work, we propose to use successive interference cancellation (SIC) among groups for improved detection. In this approach, the symbols from each group are detected successively, and the estimated symbols are used to reduce the IGI for the subsequent groups. The proposed linear MMSE

weights design takes into account the interference from the following groups by considering it as a random contribution. The development is carried out under the signal model of [3], which includes the effects of inter-symbol interference (ISI) and MAI in a convenient block matrix form.

We then extend the model to provide multistage SIC. Because IGI is reduced at every stage, this multistage approach allows to reduce the weight design complexity by neglecting the IGI. This multistage structure converges asymptotically to the decorrelator as the number of stages increases.

Finally, in contrast to the current literature on group-based space-time MUD, the proposed approaches avoid the use of a dedicated beamforming unit. Since beamforming in this context reduces the dimension of the observation space and thus the performance of the ensuing linear filters [8], we make use instead of a space-time match filter prior to the linear filters. This avoids the additional complexity of a separate beamforming unit while keeping the sufficient statistics for detection.

The remainder of the paper is organized as follows. Background information and system model are presented in Section II. In Section III the new group-based successive interference cancellation structure is developed. Numerical simulation results are shown in Section IV, and finally some conclusions are drawn in Section V.

## II. BACKGROUND

### A. Signal model

Consider the received signal at the antenna array of a DS-CDMA multiuser synchronous system. The received signal consists of the contribution of  $K$  mobile terminals, transmitting blocks of  $N$  symbols with a spreading factor  $Q$  through a multipath channel of finite duration  $WT_c$ , where  $T_c$  is the duration of a DS-CDMA “chip” ( $Q = T_s/T_c$ , where  $T_s$  is the symbol duration). The received antenna array is assumed to be a standard linear array of  $M$  elements.

The observation signal consists of the received signal converted to baseband, matched filtered to the transmission pulse, sampled and digitized at the chip rate  $1/T_c$ . Following the model in [3], let the observation signal at antenna  $m$  be given by the complex vector  $\mathbf{x}^{(m)}$  of length  $NQ + W - 1$ . The total observation vector can then be expressed as  $\mathbf{x} = \text{vec}([\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(M)}]^T) \in \mathbb{C}^{M(NQ+W-1)}$ , where  $\text{vec}(\cdot)$  is a

function that concatenates the columns of its argument matrix to form a vector of appropriate dimension.

Similarly, the vector of  $NK$  transmission symbols can be expressed as  $\mathbf{d} = \text{vec}([\mathbf{d}^{(1)}, \dots, \mathbf{d}^{(K)}]^T)$ , where  $\mathbf{d}^{(k)}$  is the vector of  $N$  transmitted symbols of user  $k$ . The symbols are assumed to be independent and identically distributed (iid) and normalized such that  $E[\mathbf{d}\mathbf{d}^H] = \mathbf{I}_{NK}$ , where  $E[\cdot]$  is the statistical expectation operator, the  $H$  exponent denotes Hermitian transposition, and  $\mathbf{I}_{NK}$  is the identity matrix of dimension  $NK \times NK$ . For the rest of the paper it is assumed, without loss of generality, that BPSK is used such that  $\mathbf{d} \in \{\pm 1\}^{NK}$ .

Finally, let the *effective* space-time signature vector, i.e. the space-time channel response to a unit pulse excitation sequence, be given by  $\mathbf{v}_k$  for user  $k$ , and let  $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_K]$ . Then the observation signal can be conveniently expressed in a linear block matrix form as

$$\mathbf{x} = \mathbf{T}\mathbf{d} + \mathbf{n}, \quad (1)$$

where  $\mathbf{T}$  is a  $M(NQ + W - 1) \times NK$  block Toeplitz matrix built from  $N$  blocks  $\mathbf{V}$  as in [3], and  $\mathbf{n}$  is the noise vector. The elements of  $\mathbf{n}$  are iid circular complex white Gaussian noise samples with covariance matrix  $E[\mathbf{n}\mathbf{n}^H] = \sigma^2 \mathbf{I}_{M(NQ+W-1)}$ , where  $\sigma^2$  is the noise power.

### B. Space-time MUD

In multiuser detection, the symbols transmitted from all  $K$  users are jointly estimated, based on the space-time observation vector  $\mathbf{x}$ . In a linear receiver, the soft symbols estimates are obtained from the output of the estimator  $\mathbf{M} \in \mathbb{C}^{NK \times NK}$ . For BPSK, the actual symbols estimates are taken as the sign of the real part of the soft estimates, i.e.:

$$\hat{\mathbf{d}} = \text{sgn}\{\Re(\mathbf{M}^H \mathbf{y})\}, \quad \mathbf{y} = \mathbf{T}^H \mathbf{x}, \quad (2)$$

where  $\mathbf{y}$  is the match filter (MF) output,  $\text{sgn}(\cdot)$  is a function that returns the sign of its argument and  $\Re(\cdot)$  is its real part. In a space-time receiver, the number of observation samples can be quite large and it becomes advantageous to use the output of the MF, which provides a complete set of observations, to estimate the transmitted symbols.

The linear filter minimizing the mean square error  $J^o(\mathbf{M}) = E\|\mathbf{d} - \mathbf{M}^H \mathbf{y}\|^2$  can be shown to take the form

$$\mathbf{M}_0 = (\mathbf{T}^H \mathbf{T} + \sigma^2 \mathbf{I})^{-1}. \quad (3)$$

The complete operation thus consists of a match filter ( $\mathbf{T}^H$ ) followed by a minimum mean square error (MMSE) filter of dimension  $NK \times NK$ . If inverted using traditional techniques, the operation has complexity order  $\mathcal{O}(K^3)$ ; a considerable difficulty for real-time operations. We shall refer to the MUD filter in (3) as the *full space-time MUD (STMUD)*.

### III. GROUP-BASED SIC RECEIVER

The full STMUD structure described above is advantageous because it takes into consideration ISI and MAI simultaneously. To reduce the complexity, we propose to process users in smaller group, as in [6, 7]. In this work we assume that

the  $G$  groups have been pre-assigned and are available to the receiver.

To simplify the development, we assume without loss of generality that the symbols in  $\mathbf{d}$  are ordered according to the given grouping. If we let  $\mathbf{d}_j$  be the  $NK_j$  symbols associated to users of group  $j$ , where  $K_j$  is the number of users in group  $j$ , then we have  $\mathbf{d} = [\mathbf{d}_1^T, \dots, \mathbf{d}_G^T]^T$ , and the columns in  $\mathbf{T}$  are re-ordered accordingly. In addition, we define  $\mathbf{P}_j \in \mathbb{R}^{NK \times NK_j}$  such that  $\mathbf{d}_j = \mathbf{P}_j^T \mathbf{d}$ . Thus  $\mathbf{P}_j$  takes the form

$$\mathbf{P}_j = \begin{bmatrix} \mathbf{0}_{NK_j^- \times NK_j} \\ \mathbf{I}_{NK_j} \\ \mathbf{0}_{NK_j^+ \times NK_j} \end{bmatrix}, \quad (4)$$

where  $\mathbf{0}_{A \times B}$  is a zero matrix of dimension  $A \times B$ ,  $K_j^- \triangleq \sum_{l=1}^{j-1} K_l$  and  $K_j^+ \triangleq \sum_{l=j+1}^G K_l$  are the total number of users in the groups before and after group  $j$ , respectively. Also, let  $\mathbf{T}_j \triangleq \mathbf{T}\mathbf{P}_j \in \mathbb{C}^{M(NQ+W-1) \times NK_j}$  be the matrix containing the columns related to the users of group  $j$  only.

We first consider the case where the inter-group interference (IGI) is reduced via the group MUD linear filters by treating the interference as a random contribution to the received signal. Then we simplify the filter design and consider the case where the IGI is iteratively canceled using a multistage approach.

#### A. GRP-STMUD-SIC

The GRP-STMUD-SIC receiver consists of a series of  $G$  linear filters each followed by a decision device  $\mathcal{Q}(\cdot)$ . The symbol estimates for each group are obtained from the interference reduced *error signal* coming from the previous group. The signal contribution from the detected symbols is then subtracted from the error signal and fed to the next group, as illustrated in the block diagram of Fig. 1.

The hard symbol estimate for each group is given by  $\hat{\mathbf{d}}_j = \mathcal{Q}(\mathbf{M}_j^H \mathbf{P}_j^T \mathbf{e}_j)$ , where  $\mathcal{Q}(\cdot)$  is a non-linear device (e.g. for BPSK  $\mathcal{Q}(\cdot) = \text{sgn}(\cdot)$ ), and  $\mathbf{e}_j$  is the input "error" signal vector of dimension  $NK$ . The estimated signal contribution from the detected group is then removed from the error signal:

$$\mathbf{e}_{j+1} = \mathbf{e}_j - \mathbf{T}^H \mathbf{T}_j \hat{\mathbf{d}}_j, \quad j = 1, \dots, G. \quad (5)$$

For the first group,  $\mathbf{e}_1 = \mathbf{y} \triangleq \mathbf{T}^H \mathbf{x}$ , the output of the match filter. The linear detection filter  $\mathbf{M}_j$  acts on the error signal elements of the group of interest, selected by  $\mathbf{P}_j^T$ .

The filter matrix can be designed using several different criteria. To take into consideration the interference from the remaining groups, the minimum mean square error (MMSE) criterion is used, leading to the cost function

$$J_j^o(\mathbf{M}) = E\|\mathbf{d}_j - \mathbf{M}^H \mathbf{P}_j^T \mathbf{e}_j\|^2. \quad (6)$$

The error signal as seen at the filter input for group  $j$  can be expressed as

$$\mathbf{e}_j = \mathbf{y} - \mathbf{T}^H \sum_{l=1}^{j-1} \mathbf{T}_l \hat{\mathbf{d}}_l \quad (7)$$

$$= \mathbf{y} - \mathbf{T}^H \tilde{\mathbf{T}}_j \tilde{\mathbf{d}}_j, \quad (8)$$

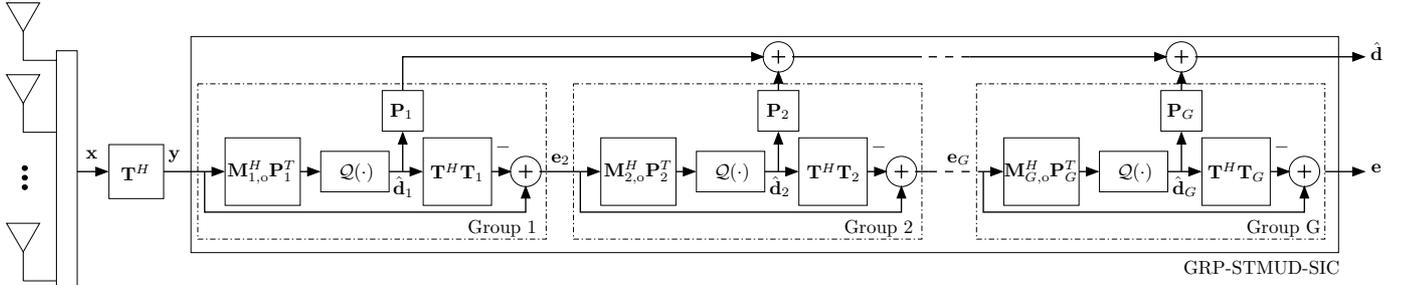


Fig. 1: Block diagram for the proposed GRP-STMUD-SIC receiver.

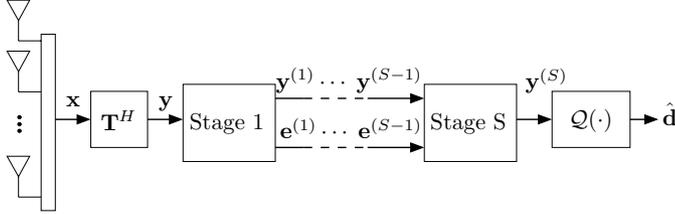


Fig. 2: Multistage GRP-STMUD-SIC receiver block diagram.

where  $\tilde{\mathbf{T}}_j$  are the columns of  $\mathbf{T}$  associated to the users of the groups  $l = 1, \dots, j-1$ , and  $\tilde{\mathbf{d}}_j = [\hat{\mathbf{d}}_1^T, \dots, \hat{\mathbf{d}}_{j-1}^T]^T$ . To optimize (6) with respect to  $\mathbf{M}_j$ , it is assumed that the symbols from the previous groups have been detected without error. This is a common assumption in the design of decision feedback detectors. Using  $\mathbf{d}_j$  instead of its estimate in (8), it can be shown following a development similar to [9] that the optimal linear filter minimizing  $J_j^o(\mathbf{M})$  in (6) is given by

$$\mathbf{M}_{j,o} = (\mathbf{R}_j \mathbf{R}_j^H + \tilde{\mathbf{C}}_j \tilde{\mathbf{C}}_j^H + \sigma^2 \mathbf{R}_j)^{-1} \mathbf{R}_j^H, \quad (9)$$

where  $\mathbf{R}_j \triangleq \mathbf{T}_j^H \mathbf{T}_j$ , and  $\tilde{\mathbf{C}}_j \triangleq \mathbf{T}_j^H \tilde{\mathbf{T}}_j$ . As it can be observed, the dimension of the new filter is now  $NK_j \times NK_j$ , reduced from the full STMUD filter in (3).

### B. Multistage GRP-STMUD-SIC

A logical extension of the GRP-STMUD-SIC approach is to use multiple stages for the detection. Several GRP-STMUD-SIC blocks are used successively and the decisions on the symbols are delayed until the last stage, as illustrated in Fig. 2. The detection block for each stage are similar to the GRP-STMUD-SIC block of Fig. 1, except that the non-linear device is removed to allow soft output to be passed on to subsequent stages. This technique is advantageous because it allows to reduce the impact of decision errors propagation on interference cancellation [10].

In the context of multistage GRP-STMUD-SIC, it is expected that the interference decreases at each stage. Under this assumption, the linear filter design can then be simplified to reduce the complexity of (9). To this end, we assume that the IGI can be neglected for all stages, leading to the new sub-optimal cost function

$$J_j(\mathbf{M}) = E \|\mathbf{d}_j - \mathbf{M}^H \mathbf{P}_j^T (\mathbf{y} - \mathbf{T}^H \tilde{\mathbf{T}}_j \tilde{\mathbf{d}}_j)\|^2, \quad (10)$$

where  $\tilde{\mathbf{T}}_j$  and  $\tilde{\mathbf{d}}_j$  are the columns of  $\mathbf{T}$  and the elements of  $\mathbf{d}$  related to the users *outside* of group  $j$ . It follows that the matrix filter minimizing (10) can be expressed as

$$\mathbf{M}_j = (\mathbf{R}_j + \sigma^2 \mathbf{I})^{-1}. \quad (11)$$

Notice that (11) takes the same form as (3), and has the same dimension as (9). This filter structure replaces  $\mathbf{M}_{j,o}$  in the GRP-STMUD-SIC of Fig. 1 for all the stages.

Let  $\mathbf{y}_j^{(s)}$  denote the  $j$  group linear filter soft output vector of dimension  $NK_j$  and  $\mathbf{e}_j^{(s)}$  denote the error signal vector of dimension  $NK$  at the input of group  $j$  at stage  $s$ . Following the development in [5], it can be shown that the soft linear output is given by

$$\mathbf{y}_j^{(s)} = \mathbf{M}_j^H \mathbf{P}_j^T \prod_{k=1}^{j-1} (\mathbf{I} - \mathbf{T}^H \mathbf{T}_k \mathbf{M}_k^H \mathbf{P}_k^T) \left( \sum_{p=0}^{s-1} \Phi^p \right) \mathbf{y}, \quad (12)$$

and the corresponding error signal is expressed by

$$\mathbf{e}_j^{(s)} = \prod_{k=1}^{j-1} (\mathbf{I} - \mathbf{T}^H \mathbf{T}_k \mathbf{M}_k^H \mathbf{P}_k^T) \Phi^{s-1} \mathbf{y}, \quad (13)$$

where  $\Phi \triangleq \prod_{k=1}^G (\mathbf{I} - \mathbf{T}^H \mathbf{T}_k \mathbf{M}_k^H \mathbf{P}_k^T)$ . The hard symbol estimates are obtained at the output of the non-linear device after the last stage  $S$ , as illustrated in Fig. 2, where  $\mathbf{y}^{(s)} = [\mathbf{y}_1^{(s)T}, \dots, \mathbf{y}_G^{(s)T}]^T$  is the soft symbol estimate vector at stage  $s$  and  $\mathbf{e}^{(s)} \triangleq \mathbf{e}_{G+1}^{(s)} \equiv \mathbf{e}_1^{(s+1)}$  is the error signal at the output of stage  $s$ . The hard symbol estimate is obtained at the output of the decision device, located after the last stage  $S$ , i.e.  $\hat{\mathbf{d}} = \mathcal{Q}(\mathbf{y}^{(S)})$ .

### C. Convergence

It is important to know the convergence conditions of the multistage receiver described by (12) and (13) above as the number of stages  $S$  increases, and in particular for  $S \rightarrow \infty$ . In [11], it is shown that the group-wise successive interference canceller converges to the decorrelator, provided that the group-linear filter is invertible, which is usually the case.

Below, we derive the expression for the equivalent linear filter at each stage, based on the work in [5]. For the purpose of the derivation, we assume that  $\mathbf{M}_j = (\mathbf{T}_j^H \mathbf{T}_j)^{-1}$ , that is,  $\mathbf{M}_j$  is the group decorrelator. Based on this assumption, the

soft symbol estimate can be expressed using the following recursive relation

$$\mathbf{y}_j^{(s)} = \mathbf{M}_j^H \mathbf{P}_j^T \left( \mathbf{y} - \sum_{k=1}^{j-1} \mathbf{T}^H \mathbf{T}_k \mathbf{y}_k^{(s)} - \sum_{m=j+1}^G \mathbf{T}^H \mathbf{T}_m \mathbf{y}_m^{(s-1)} \right), \quad (14)$$

where the second and third term in the parenthesis correspond to the reconstructed signal based on the symbol estimates of the current and previous stage, respectively. This recursive relation can be conveniently expressed in matrix form as

$$\mathbf{y}^{(s)} = \mathbf{M}^H (\mathbf{y} - \mathbf{L} \mathbf{y}^{(s)} - \mathbf{L}^H \mathbf{y}^{(s-1)}), \quad (15)$$

where  $\mathbf{M} \triangleq \text{diag}(\mathbf{M}_1, \dots, \mathbf{M}_G)$  and  $\mathbf{L}$  is a strict lower block triangular matrix with  $(i, j)$  block element  $\mathbf{T}_i^H \mathbf{T}_j$  for  $j < i$  with  $i = 2, \dots, G$ . Solving the recursive equation (15) for  $\mathbf{y}^{(s)}$  at the last stage, we obtain

$$\begin{aligned} \mathbf{y}^{(S)} &= \sum_{l=0}^{S-1} (-\mathbf{W})^l (\mathbf{I} + \mathbf{M}^H \mathbf{L})^{-1} \mathbf{M}^H \mathbf{y} \\ &= (\mathbf{I} - (-\mathbf{W})^S) (\mathbf{I} + \mathbf{W})^{-1} (\mathbf{I} + \mathbf{M}^H \mathbf{L})^{-1} \mathbf{M}^H \mathbf{y}, \end{aligned} \quad (16)$$

where  $\mathbf{W} \triangleq (\mathbf{I} + \mathbf{M}^H \mathbf{L})^{-1} \mathbf{M}^H \mathbf{L}^H$ . Clearly, the convergence of the multistage receiver depends entirely on  $\rho(\mathbf{W})$ , the spectral radius of  $\mathbf{W}$ . Indeed, if  $\rho(\mathbf{W}) < 1$  then  $\mathbf{W}^S \rightarrow \mathbf{0}$  as  $S \rightarrow \infty$  and the converged solution becomes

$$\mathbf{y}^{(\infty)} = (\mathbf{I} + \mathbf{W})^{-1} (\mathbf{I} + \mathbf{M}^H \mathbf{L})^{-1} \mathbf{M}^H \mathbf{y}, \quad (17)$$

provided that the inverses exist. Using the fact that  $\mathbf{M}$  is full rank, it can be shown that  $\mathbf{y}^{(\infty)} = \mathbf{S}^{-1} \mathbf{y}$ , where  $\mathbf{S} \triangleq (\mathbf{M}^{-H} + \mathbf{L} + \mathbf{L}^H) = (\mathbf{T}^H \mathbf{T})$ , and thus the Multistage GRP-STMUD-SIC with  $\mathbf{M}_j = \mathbf{T}_j^H \mathbf{T}_j$  converges as expected to the full decorrelator.

To derive the convergence conditions, we follow the development in [5]. Let  $\mathbf{W}_1 \triangleq \mathbf{M}^{-H/2} \mathbf{W} \mathbf{M}^{H/2} = (\mathbf{I} + \mathbf{L}_1)^{-1} \mathbf{L}_1^H$  where  $\mathbf{L}_1 \triangleq \mathbf{M}^{1/2} \mathbf{L} \mathbf{M}^{H/2}$  and  $\mathbf{M} = \mathbf{M}^{H/2} \mathbf{M}^{1/2}$ . It can then be observed that the eigenvalues of  $\mathbf{W}_1$  are the same as those of  $\mathbf{W}$ . Since we are only interested in the spectral radius of  $\mathbf{W}$ , it can be shown that

$$|\lambda|^2 = \left| \frac{\mathbf{z}^H \mathbf{L}_1 \mathbf{z}}{1 + \mathbf{z}^H \mathbf{L}_1 \mathbf{z}} \right|^2, \quad (18)$$

where  $\lambda$  is an eigenvalue of  $\mathbf{W}$  and  $\mathbf{z}$  is the corresponding normalized eigenvector. For a spectral radius less than one, (18) requires  $1 + 2\text{Re}(\mathbf{z}^H \mathbf{L}_1 \mathbf{z}) > 0$ . Using the transformation identities, it can be shown that this condition is equivalent to  $S > 0$ , which is always the case by assumption.

#### IV. COMPUTER EXPERIMENTS

We consider the received signal model of (1) for the uplink of a DS-CDMA system. The users have orthogonal spreading codes of length  $Q = 16$  and transmit BPSK data symbols in blocks of  $N = 50$ . The signals are received by  $M = 6$  antennas in a standard linear array configuration. The hardware

resources are limited to support up to  $G_{\max} = 4$  groups of a maximum of  $K_{\max} = 4$  users each.

The channel consists of six multipaths with power-delay profile following the vehicular channel type A. For a transmission rate of  $1/T_c = 3.84$  MHz, the total channel spread in terms of  $T_c$  is  $W = 11$ . Table I shows the channel taps and their relative power. The main path has DOA  $\theta_0$  uniformly

Tap #	Rel. power (dB)
1	0.0
2	-1.0
4	-9.0
5	-10.0
8	-15.0
11	-20.0

TABLE I: Channel power-delay profile.

distributed within the sector width of  $120^\circ$ , and all other paths are uniformly distributed within  $[\theta_0 + \Delta\theta, \theta_0 - \Delta\theta]$ , with  $\Delta\theta = 30^\circ$ .

#### A. Performance results

Figure 3 shows the BER obtained via Monte-Carlo simulations for the scenario described above with a 80% load. Over  $10^6$  symbols are processed to obtain sufficient precision in the results. Ideal power control is assumed and the SNR is thus given by  $P/\sigma^2$ , where  $P \equiv P_j = \|\mathbf{v}_j\|^2$  is the received power of each user. As expected, the full STMUD outperforms the other algorithms and the MF or conventional receiver performs poorly; at a BER of  $10^{-3}$ , there is a difference of approximately 2.4dB between the two approaches.

The GRP-STMUD approach corresponds to the  $G$  groups being detected independently using (11) and with no interference cancellation. We observe in this scenario an improvement over the MF of approximately 1.8dB at BER of  $10^{-3}$  when using the GRP-STMUD approach. The approaches that reduce IGI perform better than GRP-STMUD alone. The GRP-STMUD-SIC and MS-GRP-STMUD-SIC with a single stage (SIC 1) perform similarly better than GRP-STMUD. When using 2 stages (SIC 2), the difference at BER of  $10^{-3}$  between MS-GRP-STMUD-SIC and the full STMUD reduces to a negligible 0.2dB, making this approach perform almost as good as the full STMUD.

#### B. Convergence

Figure 4 shows the BER for the multistage algorithm at SNR=10dB. As it can be observed, only a few SIC iterations are necessary for the algorithm to converge, confirming what has already been observed in Fig. 3.

As observed in [11], the BER converges slowly to the decorrelator but performs slightly better in the first few iterations after the first stage. The convergence rate depends on several parameters and in general, the grouping and order of cancellation can have a significant effect [11].

#### C. Numerical complexity

The expressions for the optimal MMSE linear estimators in (3), (9) and (11) include a matrix inversion and several matrix

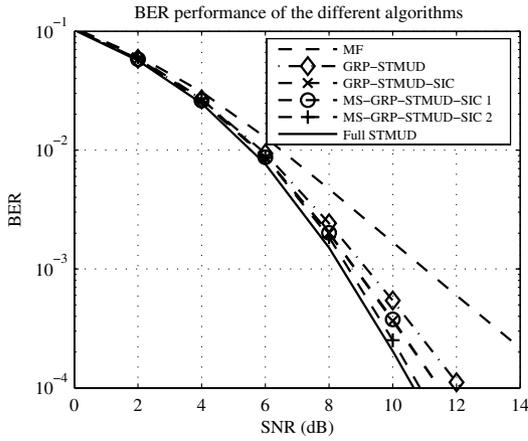


Fig. 3: BER results for a system with  $Q = 16$  at a 80% load with  $G_{\max} = 4$  groups and  $K_{\max} = 4$  users.

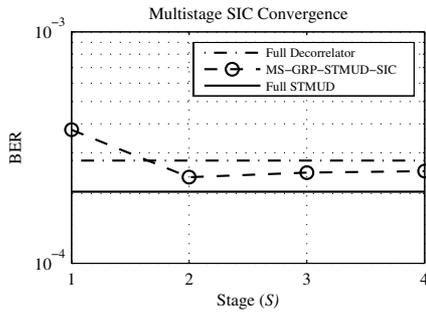


Fig. 4: Convergence of the BER for MS-GRP-STMUD-SIC at SNR=10dB.

multiplications. Fortunately, the structure of the data matrix  $\mathbf{T}$  can be exploited extensively, in particular using the Cholesky decomposition, leading to significant complexity reduction. The most important reduction results from the structure in the  $\mathbf{T}^H \mathbf{T}$  matrix product [8].

To compare the complexity between the approaches, the number of complex floating point operations (CFLOPS) is counted for the different parts of relevant equations by taking advantage of the symmetries. The total complexity is classified in three distinct parts: overhead (the different matrix products and sums to obtain the matrices to invert in (3), (9) and (11)), linear system solution (lss) and the cost for each SIC stage, which requires two back substitutions and signal reconstruction for the interference cancellation. Figure 5 shows the numerical complexity in terms of CFLOPS for solving a system with a typical block size of  $N = 50$  data symbols.

The results show that the total complexity associated to the full STMUD for  $K = 16$  is more than three times the complexity of the GRP-STMUD-SIC approach of Section III-A and approximately 2.2 times the complexity of the multistage approach of Section III-B with  $S = 2$ . Together, Figures 3 and 5 illustrate the possible performance vs complexity tradeoff of the different algorithms and structures proposed. For example the GRP-STMUD algorithm discussed above has very low complexity but its BER is the highest of the proposed group-

based approaches.

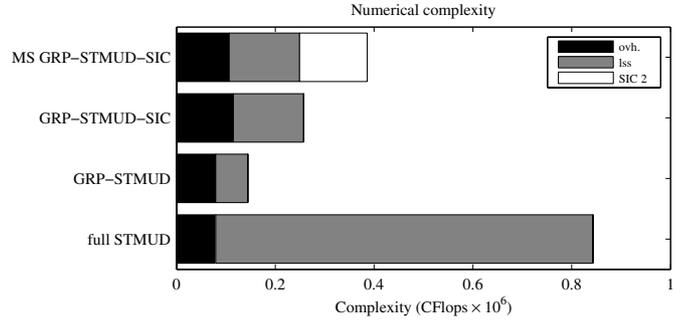


Fig. 5: Numerical complexity in number of complex floating point operations for  $N = 50$ ,  $K = 16$ , and 4 groups of 4 users.

## V. CONCLUSION

We have proposed in this work a new group-based space-time receiver structure for DS-CDMA systems that uses successive interference cancellation to reduce the problematic interference among groups. The new structure is shown to provide BER performance close to the full STMUD at a fraction of the complexity.

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