Dual Transform Domain Echo Canceller for Discrete Multitone Systems

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***Abstract*—In communication systems where full-duplex transmission is required, digital echo cancellers are employed to cancel echo by means of adaptive filtering. In order to reduce the computational complexity of these cancellers, the structure of the Toeplitz matrix containing the transmitted signal is usually exploited to transform the time domain signals and perform the emulation and adaptive update in a more convenient domain (*e.g.* frequency domain). In this paper, we consider a general decomposition of the Toeplitz matrix and examine the effect of different components of the decomposition on the computational complexity and convergence behaviour of the canceller. Based on this general decomposition, a new dual transform domain canceller is proposed which has improved convergence compared to the current echo cancellers and also does not require the transmission of dummy data on the unused tones.**

I. Introduction

Full-duplex transmission on digital subscriber lines (DSL) mainly suffers from echo, the interference caused by the transmitted signal on the collocated received signal. To combat the deteriorating effect of the echo, digital echo cancellers are employed, where an estimate of the echo signal obtained by filtering of the transmitted signal, is subtracted from the received signal. Therefore, the main goal of the echo canceller is to provide the most accurate estimate of the echo signal, as achievable by means of an adaptive filter, with low computation cost and fast convergence.

Current DSL systems widely employ discrete multitone (DMT) modulation, where the channel is divided into orthogonal sub-channels (tones) that can be used either for the downstream or upstream data transmission. The DMT modulation also provides a convenient setting to implement power allocation algorithms, which improve system performance by distributing power efficiently between the tones (*e.g.* water-filling) [1]. Different methods for echo cancellation for DMT-based DSL schemes have been presented in the literature where by exploiting the specific structure in the DMT signals, the complexity of the canceller is reduced.

When the DMT modulation is used, the echo signal can be calculated by multiplying of the estimated echo channel weights with a Toeplitz matrix containing the transmitted signal samples (thereby performing the linear convolution). To avoid the costly matrix multiplication, the Toeplitz data matrix is usually decomposed to allow the calculations to be performed in different domains with reduced complexity. For instance, in [2], Ho *et al.* proposed the circular echo synthesis (CES) canceller, where the Toeplitz data matrix is decomposed into a sum of a circulant and a residual matrix, with the former being diagonalized by means of the Fourier transform. Consequently, the CES complexity is reduced by performing the emulation partially in the time and frequency domains, while a per tone weight update is done entirely in the frequency domain. However, not all the tones are used (excited) in DMT-based systems, because of certain power masks requirements or due to the bit allocation algorithm. As a result, this method suffers from slow convergence since the filter weights corresponding to the unused tones remain unchanged after the weight update step. To improve the convergence, transmission of dummy data with reduced power on the unused tones is proposed in [2]. However, as shown in [3], this approach generates extra interference demanding a more complex equalizer at the receiver; it also requires higher order front end filters to comply with the power mask [4].

To overcome this convergence issue, other methods have been proposed in the literature. In [5], Ysebaert *et al.* suggested the circulant decomposition canceller (CDC), where the Toeplitz data matrix is expressed as a sum of a circulant and a skew-circulant matrix. Similar to the CES canceller, in this method the echo is emulated jointly in the time and frequency domains; however, the tap-input vector used for the weight update, usually has sufficient excitation on all tones. Later, in [6], Pisoni and Bonaventura introduced the symmetric decomposition canceller (SDC), where trigonometric transformations are used to decompose the Toeplitz data matrix. They also derived the relationship between this canceller and CDC, and proposed a more efficient implementation for the latter based on the new decomposition. Eventually, in [7], Pisoni *et al.* introduced a canceller similar to the self-orthogonalizing filter, where the singular value decomposition (SVD) of the correlation matrix is used to boost the slow-modes.

The computational complexity and convergence behavior of different echo cancellers are mainly dictated by the decomposition of the Toeplitz data matrix they employ. In other words, this decomposition determines the transformations that can be used to map the received and transmitted signals in alternate domains where the echo emulation and weight adaptation can be performed more conveniently. In this paper, we consider a general decomposition of the Toeplitz data matrix in terms of

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arbitrary unitary sub-matrices. It is shown that, the decomposition used in the echo cancellers discussed above are special cases of this general decomposition. This decomposition is exploited to derive a novel realization of the echo canceller in dual transform domain. In this proposed canceller, the time domain signals and the filter weights are transformed by a pair of matrices determined by the decomposition of the Toeplitz data matrix, and the emulation and weight update steps are performed in the corresponding transform domains. The symmetric decomposition of the Toeplitz data matrix is then used to realize the proposed dual transform domain echo canceller, which results in improved convergence compared to the previous approach using this decomposition in [6] and the CDC in [5]. It is also notable that the proposed canceller does not require the transmission of the dummy data on the unused tones, which is required for the convergence of the CES canceller in [2].

The following notations are used throughout the paper. For a general matrix $\mathcal{A}$, the element on the ith row and the kth column is denoted as $\mathcal{A}(i,k)$, the kth column as $\mathcal{A}(:,k)$, and the ith row as $\mathcal{A}(i,:)$. Similarly, for a vector $\mathbf{v}$, the ith element is denoted by $v(i)$. The square identity matrix of size N is denoted by $\mathcal{I}_N$. The all zero matrix of size $N\times M$ is denoted by $\mathbf{0}_{N\times M}$. The discrete Fourier transformation and inverse discrete Fourier transformation matrices of size $N\times N$ are denoted by $\mathcal{F}_N$ and $\mathcal{F}_N^{-1}$, respectively, where $[\mathcal{F}_N]_{k,l}=\frac{1}{\sqrt{N}}e^{-j2\pi kl/N}$ and $\mathcal{F}_N^{-1}=\mathcal{F}_N^H$. The superscripts of $(.)^H$ and $(.)^T$ indicate the conjugate transpose and transpose operations, respectively. Finally, $\mathrm{diag}\{\mathbf{v}\}$ indicates a diagonal matrix whose diagonal elements are given by vector $\mathbf{v}$.

II. Background

In this section, we briefly examine some of the existing methods for echo cancellation in DMT-based systems. In this paper, for convenience we assume symmetric data rate at the transmitter and the receiver where the modulation and demodulation are performed by the use of IDFT/DFT of equal length N; however, generalization to multirate systems are possible. The transmitted time domain symbol at symbol period k, is represented by $\mathbf{u}^k=[u_0^k,\cdots,u_{N-1}^k]^T$, where u_i^k $(i=0,\cdots,N-1)$ represent the samples obtained as the N-point IDFT of the vector containing the QAM modulated data in the frequency domain. To ensure that the channel is now divided into parallel independent subchannels, a cyclic prefix with the length υ larger than the assumed length of the far end channel, is added to each time domain symbol [1].

The emulated echo $\mathbf{y}_e^k$ at symbol period k is generated by the linear convolution of the transmitted symbols and the weight vector of the adaptive filter modeling the echo channel, expressed by

$$\mathbf{y}_e^k=\mathcal{U}^k\,\mathbf{w}^k. \tag{1}$$

In the general asynchronous case, there is a misalignment or delay of Δ samples between the echo frames and received far-end frames. Therefore, matrix $\mathcal{U}^k$ in (1), is an $N\times N$ Toeplitz matrix consisting of elements from symbols, $\mathbf{u}^{k-1}$, $\mathbf{u}^k$ and $\mathbf{u}^{k+1}$: its first row is given by $[u_\Delta^k,\cdots,u_0^k,u_{N-1}^k,\cdots,u_{N-\upsilon}^k,u_{N-1}^{k-1},\cdots,u_{\Delta+\upsilon+1}^{k-1}]$ and its first column is $[u_\Delta^k,\cdots,u_{N-1}^k,u_{N-\upsilon}^{k+1},\cdots,u_{N-1}^{k+1},u_0^{k+1},\cdots,u_{\Delta-\upsilon-1}^{k+1}]^T$. [1] $\mathbf{w}^k$ in (1) is a $N\times 1$ vector representing the estimate of the true echo channel, which models the effects of the hybrid circuit, the digital and analog front end filters and the time domain equalizer. The emulated echo is then subtracted from the received symbol $\mathbf{y}^k$, resulting in the error signal $\mathbf{e}^k=\mathbf{y}^k-\mathbf{y}_e^k$. Consequently, the echo weights can be obtained adaptively using various methods, *e.g.* the least mean square (LMS) algorithm, in which the error signal is used to update the weights iteratively [8].

A. Circular Echo synthesis Echo Canceller

In order to avoid the costly matrix multiplication in (1), Ho *et al.* introduced the circular echo synthesis (CES) canceller, in [2], where the matrix $\mathcal{U}^k$ is decomposed as a sum:

$$\mathcal{U}^k=\mathcal{X}^k+\mathcal{L}^k \tag{2}$$

where $\mathcal{L}^k$ is a circulant matrix with first column given by $[u_\Delta^k,\cdots,u_{N-1}^k,u_0^k,\cdots,u_{\Delta-1}^k]^T$ and $\mathcal{X}^k=\mathcal{U}^k-\mathcal{L}^k$ is a residual component. [2] The circulant matrix $\mathcal{L}^k$ can now be diagonalized using the DFT and IDFT matrices, as given by

$$\mathcal{L}^k=\mathcal{F}_N^{-1}\,\mathrm{diag}\{\mathbf{V}^k\}\,\mathcal{F}_N \tag{3}$$

where the elements of $\mathbf{V}^k$ are obtained as the Fourier transform of the first column of $\mathcal{L}^k$. Furthermore, the error signal transformed into the frequency domain (*i.e.* $\mathbf{E}^k=\mathcal{F}_N\,\mathbf{e}^k$) can be written as

$$\mathbf{E}^k=\mathcal{F}_N\left(\mathbf{y}^k-\mathcal{X}^k\,\mathbf{w}^k\right)-\mathrm{diag}\{\mathbf{V}^k\}\,\mathbf{W}^k \tag{4}$$

where $\mathbf{W}^k=\mathcal{F}_N\,\mathbf{w}^k$ is the Fourier transform of the echo channel weights. The error signal is then used to update the echo weights, where the diagonalization in the frequency domain allows a per tone approximate LMS update, given by

$$\mathbf{W}^{k+1}=\mathbf{W}^k\,+\,\mu\,\mathrm{diag}\{\mathbf{V}^{k^*}\}\,\mathbf{E}^k \tag{5}$$

with μ denoting the step size. Since not all the tones are excited in the frequency domain due to the power mask requirements and specific bit power allocation, this method suffers from poor convergence, unless dummy data with reduced power are transmitted on the unused tones.

B. Circulant Decomposition Echo Canceller

To ameliorate the convergence of the CES echo canceller, Ysebaert *et al.* propose the circulant decomposition canceller (CDC) in [5], where the Toeplitz matrix $\mathcal{U}^k$ is decomposed into a sum of circulant and skew-circulant matrices, *i.e.*:

$$\mathcal{U}^k=\frac{1}{2}\overbrace{(\mathcal{U}^k+\mathcal{S}^k)}^{\text{circulant part}}+\frac{1}{2}\overbrace{(\mathcal{U}^k-\mathcal{S}^k)}^{\text{skew-circulant part}} \tag{6}$$

[1] In previous literature, matrix $\mathcal{U}^k$ is denoted as $\mathcal{U}^{k-1,k,k+1}$, for brevity we preferred the former notation.

[2] Our formulation for asynchronous case is slightly different from the one given in [2], for more details see [4].

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where the $N \times N$ matrix $\mathcal{S}^k$ is defined such that the $2N \times 2N$ matrix $\tilde{\mathcal{L}}^k$, defined as

$$\tilde{\mathcal{L}}^k \triangleq \begin{bmatrix} \mathcal{U}^k & \mathcal{S}^k \\ \mathcal{S}^k & \mathcal{U}^k \end{bmatrix} \quad (7)$$

is circulant. This matrix can also be diagonalized by the DFT and IDFT matrices, resulting into

$$\tilde{\mathcal{L}}^k = \mathcal{F}_{2N}^{-1} \operatorname{diag}\{\tilde{\mathbf{V}}^k\} \mathcal{F}_{2N} \quad (8)$$

where elements of $\tilde{\mathbf{V}}^k$ are obtained from the Fourier transform of the first column of $\tilde{\mathcal{L}}^k$. Using the diagonalization of $\tilde{\mathcal{L}}^k$ in (8), the marix $\mathcal{U}^k$ in (6) can be written as (for more details see [4])

$$\mathcal{U}^k = \frac{1}{2}\Big(\mathcal{F}_N^{-1}\operatorname{diag}\{\tilde{\mathbf{V}}^k_{\text{even}}\}\mathcal{F}_N + \mathcal{Q}^H \mathcal{F}_N^{-1}\operatorname{diag}\{\tilde{\mathbf{V}}^k_{\text{odd}}\}\mathcal{F}_N \mathcal{Q}\Big) \quad (9)$$

where $\tilde{\mathbf{V}}^k_{\text{even}}$ and $\tilde{\mathbf{V}}^k_{\text{odd}}$ are obtained from the even and odd numbered elements of the $\tilde{\mathbf{V}}^k$, respectively and $\mathcal{Q} = \operatorname{diag}\{[1, \cdots, e^{-j\pi(i-1)/N}, \cdots, e^{-j\pi(N-1)/N}]^T\}$.

Consequently, the transformed error signal is now given by

$$\mathbf{E}^k = \mathcal{F}_N\left(\mathbf{y}^k - \frac{1}{2}\mathcal{Q}^H \mathcal{F}_N^{-1}\operatorname{diag}\{\tilde{\mathbf{V}}^k_{\text{odd}}\}\mathcal{F}_N \mathcal{Q}\,\mathbf{w}^k\right) - \frac{1}{2}\operatorname{diag}\{\tilde{\mathbf{V}}^k_{\text{even}}\}\mathbf{W}^k \quad (10)$$

where $\mathbf{W}^k$ is the echo weights in the frequency domain. The per tone approximate weight update is performed similarly to (5) in the frequency domain as

$$\mathbf{W}^{k+1} = \mathbf{W}^k + \hat{\mu}\operatorname{diag}\{\tilde{\mathbf{V}}^{k^*}_{\text{even}}\}\,\mathbf{E}^k. \quad (11)$$

As shown in [5], if $\Delta \neq -\upsilon$, then the CDC algorithm provides an acceptable convergence since the elements of $\tilde{\mathbf{V}}^k_{\text{even}}$ are mostly nonzero (without requiring the transmission of dummy data on the unused tones).

C. Symmetric Decomposition Echo Canceller

In [6], Pisoni and Bonaventura propose a symmetric decomposition canceller (SDC) where the Toeplitz matrix $\mathcal{U}^k$ is decomposed using Discrete Cosine and Sine Transforms (DCT and DST). They also derive the relationship between the SDC and CDC and use the symmetric decomposition to achieve a more efficient implementation for the latter. The symmetric decomposition is based on [9], where it is shown that trigonometric transformers can be utilized to diagonalize a Toeplitz matrix written as a combination of specific Toeplitz and Hankle matrices.

Consider column vector $\mathbf{a}^k$, $a^k(i) = \mathcal{U}^k(0,i) + \mathcal{U}^k(i,0)$ for $i = 0, \cdots, N-1$, and $\mathbf{b}^k$, with $b^k(i) = \mathcal{U}^k(0,i) - \mathcal{U}^k(i,0)$ for $i = 0, \cdots, N-1$. Define a symmetric Toeplitz matrix $\mathcal{T}_S^k$ with its first row given by $[a^k(0), a^k(1), \cdots, a^k(N-1)]$, and an anti-symmetric Toeplitz matrix $\mathcal{T}_A^k$ with its first row as $[0, b^k(1), \cdots, b^k(N-1)]$. Similarly, define a persymmetric Hankel matrix $\mathcal{H}_S^k$ with its first row given as $[a^k(1), \cdots, a^k(N-1), 0]$ and an anti-persymmetric Hankle matrix $\mathcal{H}_A^k$ with first row $[b^k(1), \cdots, b^k(N-1), 0]$. The original Toeplitz matrix $\mathcal{U}^k$ is rewritten as a combination of the above matrices, *i.e.*

$$2\,\mathcal{U}^k = \frac{1}{2}(\mathcal{T}_S^k + \mathcal{H}_S^k) + \frac{1}{2}(\mathcal{T}_S^k - \mathcal{H}_S^k) - \frac{1}{2}(\mathcal{T}_A^k + \mathcal{H}_A^k) - \frac{1}{2}(\mathcal{T}_A^k - \mathcal{H}_A^k). \quad (12)$$

Using the DCT and DST, each term in (12) can be individually diagonalized, yielding

$$\mathcal{U}^k = \mathcal{C}^{\text{II}^T}\tilde{\mathcal{Z}}^T\mathcal{D}^k\tilde{\mathcal{Z}}\mathcal{C}^{\text{II}} + \mathcal{S}^{\text{II}^T}\mathcal{Z}^T\mathcal{D}^k\mathcal{Z}\mathcal{S}^{\text{II}} + \mathcal{C}^{\text{II}^T}\tilde{\mathcal{Z}}^T\tilde{\mathcal{D}}^k\mathcal{Z}\mathcal{S}^{\text{II}} - \mathcal{S}^{\text{II}^T}\mathcal{Z}^T\tilde{\mathcal{D}}^k\tilde{\mathcal{Z}}\mathcal{C}^{\text{II}} \quad (13)$$

where $\mathcal{C}^{\text{II}}$ and $\mathcal{S}^{\text{II}}$ are $N \times N$ Type-II DCT and DST matrices [10], with elements:

$$\mathcal{C}^{\text{II}}(i,k) = \sqrt{2/N}\,\epsilon_i \cos\frac{i(2k+1)\pi}{2N}$$
$$\mathcal{S}^{\text{II}}(i,k) = \sqrt{2/N}\,\epsilon_{i+1} \sin\frac{(i+1)(2k+1)\pi}{2N}$$

with $i, k = 0, \cdots, N-1$ and $\epsilon_i = 1/\sqrt{2}$ for $i = 0, N$ and $\epsilon_i = 1$ otherwise. In (13), the $(N+1) \times N$ matrices $\mathcal{Z} = [\mathbf{0}_{N\times 1}|\mathcal{I}_N]^T$ and $\tilde{\mathcal{Z}} = [\mathcal{I}_N|\mathbf{0}_{N\times 1}]^T$. The $(N+1) \times (N+1)$ matrices $\mathcal{D}^k$ and $\tilde{\mathcal{D}}^k$ are as follows

$$\mathcal{D}^k = \frac{1}{2}\operatorname{diag}\{[\tilde{\mathcal{C}}^I[a(0), \cdots, a(N-1), 0]^T\}$$
$$\tilde{\mathcal{D}}^k = \frac{1}{2}\operatorname{diag}\{[0,\ \tilde{\mathcal{S}}^I[b(1), \cdots, b(N-1)]^T, 0]\}$$

where the non-normalized $(N+1) \times (N+1)$ DCT-I matrix $\tilde{\mathcal{C}}^I$ and $(N-1) \times (N-1)$ DST-I matrix $\tilde{\mathcal{S}}^I$ are defined as

$$\tilde{\mathcal{C}}^I(i,k) = \epsilon_i^2 \cos\frac{ik\pi}{N} \qquad i, k = 0, \cdots, N$$
$$\tilde{\mathcal{S}}^I(i,k) = \sin\frac{ik\pi}{N} \qquad i, k = 1, \cdots, N-1$$

In the SDC, the echo emulation is performed using the decomposition in (13). For updating the weights, the connection between CDC and SDC is derived, showing that the elements of the vector $\tilde{\mathbf{V}}^k_{\text{even}}$ used in the CDC update (11) can be written in terms of diagonal elements of the matrices $\mathcal{D}^k$ and $\tilde{\mathcal{D}}^k$, *i.e.*

$$\tilde{V}^k_{even}(i) = 2\,\mathcal{D}^k(2i,2i) + \tilde{\mathcal{L}}^k(i,0) - 2\,j\,\tilde{\mathcal{D}}^k(2i,2i) \quad (14)$$

with $i = 0, \cdots, \frac{N}{2}-1$ and $\tilde{\mathcal{L}}^k$ is defined in (8). Thus, the vector $\tilde{\mathbf{V}}^k_{\text{even}}$ is calculated directly from SDC elements and (11) is employed to update the weights in the frequency domain.

The use of the trigonometric transformations in [6] provides a more cost efficient implementation of CDC than in [5]. Yet both algorithms have similar convergence, since they use the same weight update formula (11). However, as shown in the next section, the symmetric decomposition from [6] can be employed to derive a realization of a dual transform domain echo canceller which has a better convergence than the above cancellers.

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III. Dual Transform Domain Echo Canceller

So far, we have seen that various echo cancellers utilize different decompositions of the Toeplitz matrix $\mathcal{U}^k$ in (1), which result in varying convergence behavior and computational complexity. In this section, we propose a general decomposition of the matrix $\mathcal{U}^k$ in terms of arbitrary unitary sub-matrices. This general form provides a uniform description of the decompositions used in the echo cancellers discussed in the previous section, and also outlines the effect of the different elements of the decomposition on the implementation and the behaviour of the echo canceller. Based on the general decomposition, a dual transform domain canceller (DTDC) is proposed, and later a realization of this echo canceller utilizing the symmetric decomposition is derived. The proposed canceller provides an improved convergence rate compared to the results in [6] and the CDC algorithm in [5]. Furthermore, its convergence does not require the transmission of dummy data on the unused tones as is needed in [2].

A. General Decomposition of the Toeplitz Data Matrix

In general, $\mathcal{U}^k$ can be decomposed as follows

$$\mathcal{U}^k = [\mathcal{G}_1^H \quad \mathcal{G}_2^H] \begin{bmatrix} \mathcal{S}_k^1 & \mathcal{S}_2^k \\ \mathcal{S}_3^k & \mathcal{S}_4^k \end{bmatrix} \begin{bmatrix} \mathcal{G}_1 \\ \mathcal{G}_2 \end{bmatrix} \tag{15}$$

or

$$\mathcal{U}^k = \mathcal{G}_1^H \mathcal{S}_1^k \mathcal{G}_1 + \mathcal{G}_2^H \mathcal{S}_3^k \mathcal{G}_1 + \mathcal{G}_1^H \mathcal{S}_2^k \mathcal{G}_2 + \mathcal{G}_2^H \mathcal{S}_4^k \mathcal{G}_2. \tag{16}$$

where $\mathcal{G}_i$ $(i = 1, 2)$ are $N \times N$ unitary matrices which are constant for all symbol periods, and $\mathcal{S}_i^k$ $(i = 1, \cdots, 4)$ are $N \times N$ matrices with elements depending on the transmitted symbols. This general form provides a proper structure to incorporate existing echo cancellers discussed in Section II.

Considering the CES echo canceller presented by Ho *et al.* in [2], it can be represented in the general form (15) where the matrix $\mathcal{U}^k$ is decomposed as

$$\mathcal{U}^k = \begin{bmatrix} \mathcal{F}_N^{-1} & \mathcal{I}_N \end{bmatrix} \begin{bmatrix} \text{diag}\{\mathbf{V}^k\} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \mathcal{X}^k \end{bmatrix} \begin{bmatrix} \mathcal{F}_N \\ \mathcal{I}_N \end{bmatrix} \tag{17}$$

with the matrices $\mathbf{V}^k$ and $\mathcal{X}^k$ are defined in Section II-A, and the matrices $\mathcal{G}_1 = \mathcal{F}_N$ and $\mathcal{G}_2 = \mathcal{I}_N$. Similarly, the decomposition used in the CDC algorithm in [5], can be expressed as the general format, given by

$$\mathcal{U}^k = \begin{bmatrix} \mathcal{F}_N^{-1} & \mathcal{Q}^H \mathcal{F}_N^{-1} \end{bmatrix} \begin{bmatrix} \frac{1}{2}\text{diag}\{\tilde{\mathbf{V}}^k_{\text{even}}\} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \frac{1}{2}\text{diag}\{\tilde{\mathbf{V}}^k_{\text{odd}}\} \end{bmatrix} \begin{bmatrix} \mathcal{F}_N \\ \mathcal{F}_N \mathcal{Q} \end{bmatrix} \tag{18}$$

where $\tilde{\mathbf{V}}^k_{\text{even}}$, $\tilde{\mathbf{V}}^k_{\text{odd}}$ and $\mathcal{Q}$ are defined in Section II-B, and $\mathcal{G}_1 = \mathcal{F}_N$ and $\mathcal{G}_2 = \mathcal{F}_N \mathcal{Q}$. Finally, the symmetric decomposition used in the SDC algorithm in [6], can likewise expressed as

$$\mathcal{U}^k = \begin{bmatrix} \mathcal{C}^{\text{II}^T} & \mathcal{S}^{\text{II}^T} \end{bmatrix} \begin{bmatrix} \tilde{\mathcal{Z}}^T \mathcal{D}^k \tilde{\mathcal{Z}} & \tilde{\mathcal{Z}}^T \tilde{\mathcal{D}}^k \mathcal{Z} \\ -\mathcal{Z}^T \tilde{\mathcal{D}}^k \tilde{\mathcal{Z}} & \mathcal{Z}^T \mathcal{D}^k \mathcal{Z} \end{bmatrix} \begin{bmatrix} \mathcal{C}^{\text{II}} \\ \mathcal{S}^{\text{II}} \end{bmatrix} \tag{19}$$

where the matrices involved are introduced in Section II-C, and $\mathcal{G}_1 = \mathcal{C}^{\text{II}}$ and $\mathcal{G}_2 = \mathcal{S}^{\text{II}}$. As it can be seen, in all the cases the matrices $\mathcal{G}_i$ $(i = 1, 2)$ are unitary and the preference is to have diagonal $\mathcal{S}_i^k$ $(i = 1, \cdots, 4)$ sub-matrices, resulting in a more efficient implementation.

B. Dual Transform Domain Echo Canceller

In the previous section, a general decomposition of the Toeplitz matrix $\mathcal{U}^k$ (15) is introduced. In this section, a novel dual transform domain canceller (DTDC) is proposed, based on this general decomposition. In this canceller, the unitary matrices $\mathcal{G}_1$ and $\mathcal{G}_2$, obtained from the decomposition of the matrix $\mathcal{U}^k$, are used to transform the time domain signals and filter weights. This dual transformation provides a more convenient domain to perform the echo emulation and adaptive weight update.

In the proposed DTDC, the transformed emulated echo is given by

$$\mathbf{Y}_e^k = \begin{bmatrix} \mathcal{G}_1 \\ \mathcal{G}_2 \end{bmatrix} \mathbf{y}_e^k. \tag{20}$$

Using the definition for the time domain emulated echo in (1) and substituting $\mathcal{U}^k$ by its general decomposition in (15), the transformed emulated echo can be rewritten as

$$\mathbf{Y}_e^k = \begin{bmatrix} \mathcal{G}_1\mathcal{G}_1^H & \mathcal{G}_1\mathcal{G}_2^H \\ \mathcal{G}_2\mathcal{G}_1^H & \mathcal{G}_2\mathcal{G}_2^H \end{bmatrix} \begin{bmatrix} \mathcal{S}_1^k & \mathcal{S}_2^k \\ \mathcal{S}_3^k & \mathcal{S}_4^k \end{bmatrix} \begin{bmatrix} \mathbf{W}_1^k \\ \mathbf{W}_2^k \end{bmatrix} \tag{21}$$

where $\mathbf{W}_i^k = \mathcal{G}_i \ \mathbf{w}^k$ $(i = 1, 2)$ are the transformed echo weights by matrix $\mathcal{G}_i$. $\mathbf{Y}_e^k$ can be rewritten as

$$\mathbf{Y}_e^k = \Phi^k \boldsymbol{\omega}^k \tag{22}$$

where Φ^k represents the product of the first two matrices in the right side of (21) and the transformed weight is defined as

$$\boldsymbol{\omega}^k = \begin{bmatrix} \mathbf{W}_1^k \\ \mathbf{W}_2^k \end{bmatrix}. \tag{23}$$

Using the above definitions, the transformed error signal for the proposed DTDC is given by

$$\mathbf{E}^k = \mathbf{Y}^k - \Phi^k \boldsymbol{\omega}^k \tag{24}$$

where $\mathbf{Y}^k$ is the transformed received signal, *i.e.*

$$\mathbf{Y}^k = \begin{bmatrix} \mathcal{G}_1 \\ \mathcal{G}_2 \end{bmatrix} \mathbf{y}^k. \tag{25}$$

Using the LMS algorithm, the echo weights are updated by

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \mu \, \Phi^{k^H} \, \mathbf{E}^k. \tag{26}$$

The above equation offers a complete and exact adaptation for the transform domain canceller. In echo cancellers discussed in Section II, the dual transformation is partially used in the emulation part but avoided in the weight update part, in order to reduce the computational cost. However, as shown later, the convergence of the echo canceller can be improved by the use of the dual transformation (as in (26)). Furthermore, with proper approximation it results in minor additional computational cost.

The notion of an extended weight vector such as in (23) was proposed [11], where the extended weight vector contained the weights in the time and frequency domain. In [11], we

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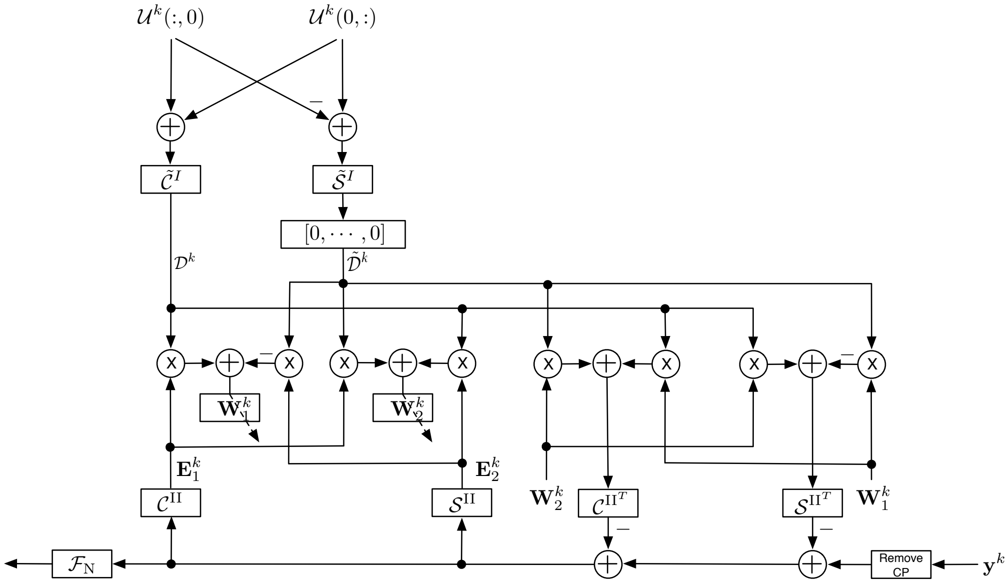

Fig. 1. Block diagram of the dual transform domain echo canceller

presented the constrained echo canceller for systems using the decomposition in (2) based on the CES canceller. Therefore, as show in (17), the constrained echo canceller is also a special case of the proposed dual transform domain canceller, and the algorithms derived in [11] are applicable to DTDC.

C. Dual Transform Domain Echo Canceller Realization

As discussed in the previous section, the echo emulation and updating of the weights can be performed in the transform domain. Considering (24) and (26) the dual transform domain echo canceller can be implemented with low complexity if ideally matrix Φ^k contains only diagonal blocks and the matrices $\mathcal{G}_i$ are not dependent on the transmitted symbols; however, such decomposition is not achievable for a Toeplitz matrix $\mathcal{U}^k$. Yet, a sensible realization of the transform domain echo canceller can result in a canceller with improved convergence using the dual transformation without increase in the computational complexity.

Assuming the symmetric decomposition, using (19), the matrix Φ^k is given by

$$\Phi^k = \begin{bmatrix} \mathcal{I}_N & \mathcal{C}^{\mathrm{II}}\mathcal{S}^{\mathrm{II}^T} \\ \mathcal{S}^{\mathrm{II}}\mathcal{C}^{\mathrm{II}^T} & \mathcal{I}_N \end{bmatrix} \begin{bmatrix} \tilde{\mathcal{Z}}^T\mathcal{D}^k\tilde{\mathcal{Z}} & \tilde{\mathcal{Z}}^T\tilde{\mathcal{D}}^k\mathcal{Z} \\ -\mathcal{Z}^T\tilde{\mathcal{D}}^k\tilde{\mathcal{Z}} & \mathcal{Z}^T\mathcal{D}^k\mathcal{Z} \end{bmatrix} \tag{27}$$

where the orthogonality of matrices $\mathcal{C}^{\mathrm{II}}$ and $\mathcal{S}^{\mathrm{II}}$ is used to simplify the equation. However, because the columns of $\mathcal{C}^{\mathrm{II}}$ and $\mathcal{S}^{\mathrm{II}}$ are not orthogonal to each other, the calculation of the emulated echo involves the computation of cross echoes which adds to the computational complexity. To avoid the additional computation, we propose to perform the echo emulation in the time domain, while the transformed echo weight $\mathbf{W}_i^k$ $(i = 1, 2)$ are used. This results into

$$\mathbf{y}_e^k = \begin{bmatrix} \mathcal{C}^{\mathrm{II}^T} & \mathcal{S}^{\mathrm{II}^T} \end{bmatrix} \begin{bmatrix} \tilde{\mathcal{Z}}^T\mathcal{D}^k\tilde{\mathcal{Z}} & \tilde{\mathcal{Z}}^T\tilde{\mathcal{D}}^k\mathcal{Z} \\ -\mathcal{Z}^T\tilde{\mathcal{D}}^k\tilde{\mathcal{Z}} & \mathcal{Z}^T\mathcal{D}^k\mathcal{Z} \end{bmatrix} \begin{bmatrix} \mathbf{W}_1^k \\ \mathbf{W}_2^k \end{bmatrix}. \tag{28}$$

Nevertheless, in the weight update step, (26) can be replaced by an approximate LMS update, as follows

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \mu\hat{\Phi}^{k^H} \begin{bmatrix} \mathbf{E}_1^k \\ \mathbf{E}_2^k \end{bmatrix} \tag{29}$$

where $\mathbf{E}_i^k (i = 1, 2)$ are the error signal transformed by $\mathcal{C}^{\mathrm{II}}$ and $\mathcal{S}^{\mathrm{II}}$ matrices, respectively. The approximate matrix $\hat{\Phi}^k$ is defined as

$$\hat{\Phi}^k = \begin{bmatrix} \tilde{\mathcal{Z}}^T\mathcal{D}^k\tilde{\mathcal{Z}} & \tilde{\mathcal{Z}}^T\tilde{\mathcal{D}}^k\mathcal{Z} \\ -\mathcal{Z}^T\tilde{\mathcal{D}}^k\tilde{\mathcal{Z}} & \mathcal{Z}^T\mathcal{D}^k\mathcal{Z} \end{bmatrix} \tag{30}$$

which is derived from (27) by approximating the first matrix in the right hand side with the identity matrix. Since the approximate matrix $\hat{\Phi}^k$ contains only diagonal blocks, the weights can be updated very efficiently using (29). The block diagram for the proposed DTDC using the symmetric decomposition is given in Fig. 1.

Since the elements of $\mathcal{D}^k$ and $\tilde{\mathcal{D}}^k$ are generally nonzero for the transmitted symbols, this canceller does not require the transmission of dummy data on the unused tones for achieving acceptable convergence rate as required by the CES echo canceller [2]. In addition, as the simulation results given in the next section show, this algorithm has better convergence rate compared to the previous implementations of SDC [6] and CDC [5]. This decomposition also has a cost-effective implementation, since it can be calculated using only DCT and DST, for which various efficient algorithms are available [10].

IV. Simulation Results and Complexity Analysis

In this section, we provide the simulation results comparing the convergence rate of the proposed dual transform domain echo canceller with current echo cancellers. In the simulations, an ADSL system over the carrier serving area (CSA) loop #1

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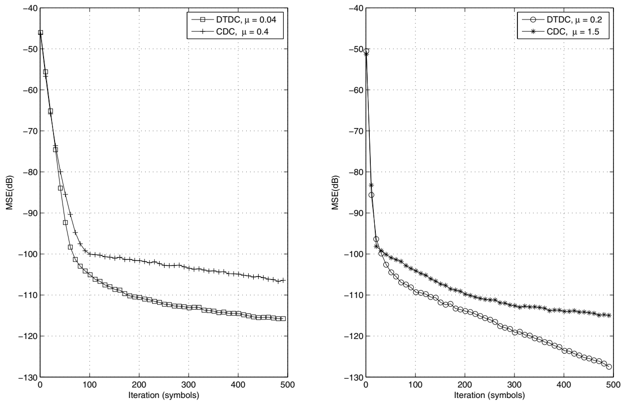

Fig. 2. Comparison of the convergence behaviour of the dual transform domain and circulant decomposition echo cancellers

setup is used [1]. In addition, DMT modulation is employed where, tones 7-31 and 33-255 are allocated for upstream and downstream, respectively, where each tone transmits a 4-QAM signal constellation. The downstream and upstream signal transmit with -40 dbm/Hz, and the external additive noise is white Gaussian noise at -140 dBm/Hz. The transmit block length N at the upstream and downstream is 64 and 512, respectively and the corresponding cyclic prefix length υ is 5 and 40, respectively. The true echo channel contains 512 samples at 2.2 MHz, while the number of echo canceller taps used is $M = 220$. The taps are initialized with all zero, and the weights are updated after each frame is received. The echo channel transfer function includes the effect of the hybrid and the transmitter and receiver filters [1]. This setup is widely used in literature for studying the convergence of echo cancellers such as in [2] and [4].

The mean square error (MSE) for the dual transform domain echo canceller using symmetric decomposition (as described in (28) and (29)) and the circulant decomposition canceller in [5] is depicted in Fig.2 for various step sizes μ. The step sizes are chosen in a way that both algorithms have similar slope in the first part of the curve. As it can be seen, the DTDC converges faster than CDC and does not require the transmission of dummy data on the unused tones as required in [2] for CES canceller.

The computational complexity of the two algorithms is evaluated in terms of the total number of required real multiplications for each iteration. As it can be seen in Table I, where the CDC and DTDC methods are compared, the complexity of the two algorithms is very close. In this comparison, we assumed that both the transmitter and the receiver use the same data rate and are frame synchronous. To evaluate the DFT complexity, the split-radix FFT algorithm is used and for the DCT and DST and their transposes, a generic radix-2 like fast recursive algorithm is considered (same as in [5] and [6]). It should be noted that for the calculation of matrices $\mathcal{D}^k$ and $\tilde{\mathcal{D}}^k$, the FFT of length $2N$ is implemented and the relationship in

TABLE I
COMPLEXITY COMPARISON

Application	**CDC**	**DTDC**
Echo Emulation	$2N\log_2 N + N + 6$	$2N\log_2 N + 2N + 2$
Adaptive Update	$1.5N\log_2 N - \frac{N}{2} + 6$	$N\log_2 N + 2.5N + 2$
Total	$3.5N\log_2 N + \frac{N}{2} + 12$	$3N\log_2 N + 4.5N + 4$

(14) is used. For the dual transformation of the error signal, the computational cost is reduced by using the relationship between the DCT and DST (see *e.g.* [10]), and calculating only one of the transformations.

V. CONCLUSION

In this paper, we have considered a general decomposition of the Toeplitz data matrix used in DMT-based echo cancellers. This general form provides a uniform representation of the previous echo cancellers, and also allows a better understanding of the effect of different components of the decomposition on the computational complexity and the convergence behaviour of the echo canceller. Based on the general decomposition, a novel dual transform domain echo canceller is proposed, and its realization based on the symmetric decomposition is derived. The proposed canceller provides an improved convergence rate compared to the results in [6] and the CDC algorithm in [5], and does not require the transmission of dummy data on the unused tones for its convergence as is required in [2]. Finally, the complexity analysis results show that the proposed algorithm can be implemented very efficiently.

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