A SUBSPACE METHOD FOR THE BLIND IDENTIFICATION OF MULTIPLE TIME-VARYING FIR CHANNELS

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ABSTRACT

A new method is proposed for the blind subspace-based identification of the coefficients of time-varying (TV) single-input multiple-output (SIMO) FIR channels. The TV channel coefficients are represented via a finite basis expansion model, i.e. linear combination of known basis functions. In contrast to earlier related works, the basis functions need not be limited to complex exponentials, and therefore do not necessitate the a priori estimation of frequency parameters. This considerably simplifies the implementation of the proposed method and provides added flexibility in applications. The merits of the proposed technique, including asymptotic consistency, are demonstrated by numerical simulations.

1. INTRODUCTION

In the application of subspace methods to channel identification, it is necessary to estimate the underlying data correlation matrix by collecting the observed data vectors over an adequate time interval, also called *integration time*. The latter must be large enough to minimize the effects of noise and statistical fluctuations on the correlation estimate, while small enough to ensure that the channel model remains time-invariant (TI) over this interval. Often, as in e.g. mobile wireless communications, these requirements are conflicting and it is not possible to select an adequate integration time.

One possible approach to the problem of blind estimation of a time-varying (TV) channel is to employ an adaptive algorithm. In the case of subspace methods, this can be realized by using a subspace tracker to efficiently update the eigendecomposition of an exponentially weighted sample correlation matrix (see e.g. [1, 2] and references therein). While such an adaptive approach makes it possible to track the channel parameters over time, it does not overcome the fundamental limitation resulting from the above trade-off in selecting an adequate integration time.

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Despite the limitations imposed by TV channel conditions on the performance of blind subspace methods, the literature on this problem is scarce. In [3], a subspace method is proposed for estimating the expansion coefficients of a TV SISO FIR channel along a known set of basis functions, by establishing an equivalence between the TV SISO channel and an unobservable TI SIMO channel. However, the method necessitates restrictive independence assumptions on the products of basis functions to ensure identifiability. In [4], a different approach is proposed for transforming the TV SIMO problem into a TI MIMO problem by making specific use of complex exponential basis functions. Blind subspace-based techniques originally developed for TI MIMO systems can then be applied to estimate the TV SIMO channel expansion coefficients. A similar representation of the TV channels in terms of complex exponential basis functions is used in [5] to derive blind identification algorithms based on the second-order linear prediction error method.

A fundamental problem with this type of approach is that the number and frequencies of the exponential basis functions need to be estimated a priori, which poses major implementation problems for high data-rate broadband systems. Also, the use of complex exponentials is not well-suited to certain simple yet very important models of TV channels: e.g., a low order polynomial variation may require a large number of exponential basis functions for its accurate representation. In this paper, a new method is proposed for the blind subspace-based identification of the coefficients of TV SIMO FIR channels represented in terms of a basis expansion model. Contrary to the above-cited techniques, the proposed method does not make explicit use of complex exponential basis functions and thus offers much flexibility in its applications.

2. PROBLEM FORMULATION

2.1. System model

We consider a discrete-time SIMO TV channel model between a source node and L sink nodes. Let s_n denote the digital symbols emitted by the source at time nT_s, where T_s is the symbol duration. These symbols are modulated,

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transmitted through L communication channels, and demodulated at the receiver side. Modeling these operations as linear, the complex-valued, baseband discrete-time output of the lth channel, $l = 1, ..., L$, is expressed as

$$x_n^{(l)} = \sum_m h_{n,m}^{(l)} s_{n-m} + w_n^{(l)} \tag{1}$$

where $w_n^{(l)}$ is an additive noise perturbation and $h_{n,m}^{(l)}$ is the impulse response of the lth channel. In this work, the $h_{n,m}^{(l)}$ are allowed to be time-varying; hence the use of two discrete-time indices. Specifically, $h_{n,m}^{(l)}$ represents the response of the lth channel at discrete-time n (in units of T_s) to a unit pulse applied at time $n - m$ [6]. At any given time n, it is assumed that $h_{n,m}^{(l)}$ has finite support over its coefficient index m, i.e. $h_{n,m}^{(l)} = 0$ for $m \notin \{0, 1, ..., M\}$, where $M+1$ represents the maximum channel duration in samples.

Defining the $L \times 1$ vectors $x_n = (x_n^{(1)}, \cdots, x_n^{(L)})^T$, $h_{n,m} = (h_{n,m}^{(1)}, \cdots, h_{n,m}^{(L)})^T$, and $w_n = (w_n^{(1)}, \cdots, w_n^{(L)})^T$, we can express the set of L equations in (1) compactly as

$$x_n = \sum_{m=0}^{M} h_{n,m} s_{n-m} + w_n. \tag{2}$$

Next, considering a block of N successive samples of x_n, we form the stacked $LN \times 1$ data and noise vectors

$$X_n = \begin{pmatrix} x_n \\ \vdots \\ x_{n-N+1} \end{pmatrix}, \quad W_n = \begin{pmatrix} w_n \\ \vdots \\ w_{n-N+1} \end{pmatrix}, \tag{3}$$

respectively, and we also define the source symbol vector

$$S_n = (s_n, \cdots, s_{n-(N+M)+1})^T \tag{4}$$

of size $(N + M) \times 1$. It follows from (2) that

$$X_n = \mathcal{H}_n S_n + W_n \tag{5}$$

where the $LN \times (N + M)$ filtering matrix

$$\mathcal{H}_n = \begin{pmatrix} h_{n,0} & \cdots & h_{n,M} & 0 & \cdots & 0 \\ 0 & h_{n-1,0} & \cdots & h_{n-1,M} & \cdots & 0 \\ \vdots & & & & & \vdots \\ 0 & \cdots & \cdots & h_{n-N+1,0} & \cdots & h_{n-N+1,M} \end{pmatrix} \tag{6}$$

The sequences of digital symbol vectors $\{S_n\}$ and additive noise vectors $\{W_n\}$ in (5) are modeled as fourth-order stationary, correlation ergodic complex vector random processes. These two sequences are assumed to be statistically independent, each with zero-mean and correlation matrices $R_S = E[S_n S_n^H]$ and $R_W = E[W_n W_n^H]$. We assume that R_S is full rank and that the noise is white (over both time and channel dimensions), so that $R_W = \sigma_w^2 I_{LN}$.

2.2. Basis expansion model

In this work, the integration time is assumed to span the set of indices from $n = 0$ to $n = N_a - 1$, and is thus of duration $N_a T_s$. In practice, the integration time must be large enough to minimize the effects of noise and statistical fluctuations on the correlation estimate, but small enough to ensure that the channel model remains time-invariant. These requirements are conflicting and it is often not possible to select an adequate integration time. This motivates the use of the general time-varying channel model (1).

In the sequel, we shall further assume that the time-varying channel impulse responses $h_{n,m}^{(l)}$ in (1), or equivalently the vectors $h_{n,m}$ in (2), can be linearly expanded in terms of a known set of scalar basis functions, that is:

$$h_{n,m} = \sum_{b=1}^{B} k_m^{(b)} \phi_n^{(b)}, \tag{7}$$

where B is the number of basis functions, $\phi_n^{(b)}$ is the bth basis function and $k_m^{(b)}$ is the corresponding expansion coefficient vector of size $L \times 1$. In the scalar case $L = 1$, this model reduces to that used in [3]. Furthermore, the TV SIMO channel model in [4] is a special case of (7) with $\phi_n^{(b)} = e^{j\omega_b n}$, i.e. complex exponentials with known angular frequencies ω_b. In practice, the choice of basis functions is application specific and reflects a priori knowledge about the time evolution of the channel coefficients over the integration time.

It is convenient to represent the collection of basis functions by a $B \times 1$ vector function, defined as

$$\Phi_n = (\phi_n^{(1)}, \cdots, \phi_n^{(B)})^T. \tag{8}$$

In terms of Φ_n, we make the following assumptions about the variations of the basis functions over the integration time $0 \leq n \leq N_a - 1$:

A1) The basis functions are absolutely bounded, that is: $\|\Phi_n\| \leq \phi_{max}$ for all n, where ϕ_{max} is a positive bound;

A2) The basis functions are linearly independent, which can be equivalently expressed as:

$$\Psi(N_a) \triangleq \frac{1}{N_a} \sum_{n=0}^{N_a-1} \Phi_n \Phi_n^H > 0 \tag{9}$$

A3) The basis functions do not vary appreciably over the duration of the stacked data vector X_n in (3), i.e. $\Phi_{n_1} \approx \Phi_{n_2}$ whenever $|n_1 - n_2| \leq N$.

Beyond these general assumptions, and in contrast to other works previously cited, no specific assumptions are made regarding a particular choice of basis functions.

2.3. Problem statement

The problem of interest in this work is the estimation of the set of channel coefficient vectors $\{k_m^{(b)}\}$, where $b = 1, \cdots, B$

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and $m = 0, \cdots, M$, from the observed data over the integration time, represented by the set of vectors $\{X_n\}$, where $n = 0, \cdots, N_a - 1$. Once the coefficient vectors $\{k_m^{(b)}\}$ are known, they can be used in turn to estimate the time-varying channel impulse responses by means of (7).

3. AN EQUIVALENT TI MIMO SYSTEM

In [4], the multiplicative property of the complex exponential basis functions (i.e. $e^{j\omega(n-m)} = e^{j\omega n}e^{-j\omega m}$) is used to transform the TV SIMO system into an equivalent TI MIMO system. We first show how assumption A3 can be used as a substitute of the complex exponential model to derive an alternative, yet equivalent TI MIMO system for the problem under consideration.

Making use of the basis expansion model in (7), the TV filtering matrix in (6) can be decomposed as

$$\mathcal{H}_n = \sum_{b=1}^{B} \mathcal{H}_n^{(b)} \tag{10}$$

where we have introduced the $LN \times (N+M)$ matrices

$$\mathcal{H}_n^{(b)} = \begin{pmatrix} k_0^{(b)}\phi_n^{(b)} & \cdots & k_M^{(b)}\phi_n^{(b)} & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & k_0^{(b)}\phi_{n-N+1}^{(b)} & \cdots & k_M^{(b)}\phi_{n-N+1}^{(b)} \end{pmatrix} \tag{11}$$

We shall take this model for $\mathcal{H}_n$ one step further by making use of assumption A3, i.e. variations in the basis functions $\phi_n^{(b)}$ over one data stack of size N can be neglected; that is, our concern is about channel variations over the integration time. In application of subspace methods, the integration time is much larger than the block size, i.e. $N_a \gg N$, so that this assumption is well justified

Under this condition, matrix $\mathcal{H}_n^{(b)}$ in (11) can be approximated as $\mathcal{H}_n^{(b)} = \phi_n^{(b)}\mathcal{K}^{(b)}$, where we define

$$\mathcal{K}^{(b)} = \begin{pmatrix} k_0^{(b)} & \cdots & k_M^{(b)} & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & k_0^{(b)} & \cdots & k_M^{(b)} \end{pmatrix} \tag{12}$$

and the system model (5) reduces to

$$X_n = \sum_{b=1}^{B} \mathcal{K}^{(b)}\phi_n^{(b)} S_n + W_n \tag{13}$$

Equation (13) provides the desired link between the TV SIMO channel model under consideration and an equivalent TI MIMO channel model with B unobservable input signals $\phi_n^{(b)} S_n$. We note that this equivalent TI MIMO system differs from the one in [4] in two aspects: the basis functions $\phi_n^{(b)}$ need not be exponentials and the multiple inputs are obtained by modulation of the original input *vector* (as opposed to modulation at the sample level).

It will be convenient to express (13) in a more compact matrix form. To this end, define the $L \times B$ matrices

$$K_m = [k_m^{(1)}, \cdots, k_m^{(B)}], \tag{14}$$

which contain the channel expansion coefficient vectors corresponding to the mth channel lag, and also define

$$K = (K_0^T, K_1^T, \cdots, K_M^T)^T \tag{15}$$

Then, it follows from (12)-(14) that

$$X_n = \mathcal{K} T_n + W_n \tag{16}$$

where $\mathcal{K}$ is a TI MIMO filtering matrix of size $LN \times B(N+M)$ obtained from K via the general construction

$$\begin{aligned} \mathcal{K} &= \mathcal{S}_N(K_0, K_1, \cdots, K_M) \qquad (17) \\ &\triangleq \begin{pmatrix} K_0 & K_1 & \cdots & K_M & 0 & \cdots & 0 \\ 0 & K_0 & \cdots & K_{M-1} & K_M & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & \cdots & K_0 & K_1 & \cdots & K_M \end{pmatrix} \end{aligned}$$

and T_n is an *extended* source symbol vector of size $B(N+M)$ obtained by rearrangement of the B modulated input vectors $\phi_n^{(b)} S_n$, i.e.

$$T_n = S_n \otimes \Phi_n \tag{18}$$

with Φ_n as defined in (8).

4. SUBSPACE APPROACH BASED ON TIME AVERAGING

We first investigate the structure and convergence properties of the sample correlation matrix of the observation vectors X_n in (16). We then discuss necessary conditions which together, ensure that a subspace approach based on time averaging of $X_n X_n^H$ is feasible for the blind identification of the TI-MIMO channel filtering matrix $\mathcal{K}$ (17). Finally, a subspace-based algorithm is formulated.

4.1. The sample correlation matrix

The $LN \times LN$ sample correlation matrix $\hat{R}_X(N_a)$ of the blocked data vectors X_n, over the integration time $0 \leq n < N_a$ is given by

$$\hat{R}_X(N_a) = \frac{1}{N_a}\sum_{n=0}^{N_a-1} X_n X_n^H. \tag{19}$$

For the general model equation (5) with an arbitrary time-varying channel matrix $\mathcal{H}_n$, $\hat{R}_X(N_a)$ does not exhibit the traditional structure which would allow the application of a useful subspace decomposition. That is, it is not possible to

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factor $\mathcal{H}_n$ out of the summation in (19) and so decouple the channel effects from the source symbols in the time averaging operation. However, under assumption A3 of slowly-varying basis functions, which results in the approximate model equation (16), such a factorization is clearly possible.

Let us first consider the noise free case. Substituting (16) into (19) with $W_n = 0$, we immediately obtain

$$\hat{R}_X(N_a) = \mathcal{K}\,\hat{R}_T(N_a)\,\mathcal{K}^H \tag{20}$$

where

$$\hat{R}_T(N_a) = \frac{1}{N_a}\sum_{n=0}^{N_a-1} T_n T_n^H \tag{21}$$

is the sample covariance matrix of the extended source symbol vector T_n. To exploit the structure of (20) in a subspace-based estimation approach, we will need to ensure that $\hat{R}_T(N_a)$ converges to a full rank matrix as $N_a \to \infty$. We note that the random sequence $\{T_n\}$ is not stationary in general due to the presence of the time-varying basis functions $\phi_n^{(b)}$ in (18). Nonetheless, basic results from the theory of ergodic processes can be extended to this case to obtain a set of sufficient conditions which guarantee that $\hat{R}_T(N_a)$ is full rank in the limit of N_a large:

Theorem 1. Assume that the sequence S_n is 4th-order stationary with covariance matrix $R_S = E[S_n S_n^H]$ and let $F_S(m-n) = E[S_n S_n^H S_m S_m^H] - R_S^2$. If $\lim_{n\to\infty} \mathrm{tr}\,[F_S(n)] = 0$, then $\hat{R}_T(N_a)$ converges in the mean-square sense to

$$E[\hat{R}_T(N_a)] = R_S \otimes \Psi(N_a) \tag{22}$$

with $\Psi(N_a)$ given by (9).

The condition on $F_S(m-n)$ is equivalent to demanding that the sequence of source symbol vectors S_n is correlation ergodic for a shift of 0 (see e.g. [7, 8]), which is satisfied in most applications. Under this condition, the theorem states that the mean-square error between $\hat{R}_T(N_a)$ and its mean value in (22), which depends on N_a, can be made arbitrarily small by letting $N_a \to \infty$.

Under the mild assumptions that the source symbol correlation matrix R_S if full rank and the basis expansion functions are linearly independent (i.e. A2), it follows from (22) that $E[\hat{R}_T(N_a)]$ is also of full rank, i.e.

$$\mathrm{rank}\{E[\hat{R}_T(N_a)]\} = (N+M)B. \tag{23}$$

In essence, Theorem 1 thus guarantees that $\hat{R}_T(N_a)$ is full rank if N_a is sufficiently large. In this work, we assume that (23) is satisfied.

More generally, in the presence of additive noise as in (5), and under the assumptions stated at the end of Section 2.1 for the sequences $\{S_n\}$ and $\{W_n\}$, the sample correlation matrix $\hat{R}_X(N_a)$ in (19) converges in the mean-square sense to

$$E[\hat{R}_X(N_a)] = \mathcal{K}\,E[\hat{R}_T(N_a)]\,\mathcal{K}^H + \sigma_w^2 I_{LN} \tag{24}$$

as $N_a \to \infty$, with $E[\hat{R}_T(N_a)]$ given by (22).

4.2. Subspace decomposition

At this point, we are back into a more traditional setting and a subspace decomposition approach can be devised for the blind estimation of the channel expansion coefficients, as represented by matrix K in (15). This possibility rests on certain properties of the range space of the the filtering matrix $\mathcal{K}$ (17), which is characterized by a *generalized Sylvester resultant* structure. This structure has been extensively studied in the past, specially in the control and signal processing literature, see e.g. [9, 10, 11]. The main results of interest to us are summarized below in the form of theorems; proofs can be found in the above references. We assume that the following condition holds, i.e. $LN > B(N+M)$, so that matrix $\mathcal{K}$ is a tall matrix; we note that this implies $L > B$.

Theorem 3. Define the $L \times B$ polynomial matrix $K(z) = \sum_{m=0}^{M} K_m z^{-m}$, where $z \in \mathbb{C}$. Matrix $\mathcal{K}$ (17) is full column rank if $N \geq BM$ and if the following conditions hold:

(i) $K(z)$ is column reduced, i.e.: $\mathrm{rank}\{K_M\} = B$;

(ii) $K(z)$ is irreducible, i.e.: $\mathrm{rank}\{K(z)\} = B, \forall z \in \mathbb{C}$.

Condition (i) means that $K(z)$ has at least one $B \times B$ minor of degree BM. Let $D(z)$ denote the corresponding $B \times B$ submatrix of $K(z)$ and let $N(z)$ denote the $(L-B) \times B$ submatrix of $K(z)$ resulting from the deletion of $D(z)$. Condition (ii) implies that the matrix transfer function $N(z)D(z)^{-1}$ is irreducible, i.e. the only right common factors of $N(z)$ and $D(z)$ are unimodular matrices.

Theorem 4. Let $\mathcal{K}$ and $\mathcal{K}'$ be the filtering matrices respectively associated to coefficient matrices K and K' via the construction in (15) and (17). Under the conditions of Theorem 3, we have

$$\mathcal{R}[\mathcal{K}'] = \mathcal{R}[\mathcal{K}] \quad \text{iff} \quad K' = KA \tag{25}$$

where A is a non-singular $B \times B$ ambiguity matrix.

Proposition (25), when satisfied, ensures that the mapping from the matrix K (15) to the range space of the associated filtering matrix $\mathcal{K}$ in (17) is invertible, up to a non-singular ambiguity. Such an ambiguity is unavoidable with the subspace approach due to its inherent second-order formulation.

Let us represent matrix $E[\hat{R}_X(N_a)]$ (24) in terms of its eigenvalue decomposition, i.e.

$$E[\hat{R}_X(N_a)] = U\Lambda U^H \tag{26}$$

where $\Lambda = \mathrm{diag}(\lambda_1, \cdots, \lambda_{LN})$, with λ_i's denoting the eigenvalues arranged in non-increasing order, and $U = [U_1, \cdots, U_{LN}]$, with U_i's denoting the corresponding orthonormal eigenvectors. If the conditions of Theorem 3 are satisfied, we have: $\lambda_{B(N+M)} > \sigma_w^2$, $\lambda_i = \sigma_w^2$ for $i \in \mathcal{I}$ and

$$U_i^H \mathcal{K} = 0 \quad \text{for} \quad i \in \mathcal{I}, \tag{27}$$

where the index set $\mathcal{I} \triangleq \{B(N+M)+1, \cdots, LN\}$. Therefore, knowledge of the noise subspace $\mathcal{N}$, defined as the linear

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span of U_i for $i \in \mathcal{I}$, enables the determination of the range space of $\mathcal{K}$. In turns, according to Theorem 4, this makes it possible to estimate the channel expansion coefficient matrix K, up to an ambiguity matrix A.

4.3. Algorithm

In practice, only an estimate of the eigendecomposion (26) can be obtained based on the sample covariance matrix $\hat{R}_X(N_a)$ (19), i.e.

$$\hat{R}_X(N_a) = \hat{U}\hat{\Lambda}\hat{U}^H, \tag{28}$$

where $\hat{\Lambda}$ and $\hat{U}$ denote the corresponding estimated eigenvalue and eigenvector matrices, respectively. Once this information is available, the unknown channel parameters, represented by matrix K (15), can be obtained by solving a least squares problem derived from (27) [12, 11]. Specifically, define

$$f(K) = \sum_{i \in \mathcal{I}} \|\hat{U}_i \mathcal{K}\|^2. \tag{29}$$

where $\mathcal{K}$ is a function of K through (17). An estimate of K, say $\hat{K}$, is obtained by minimizing $f(K)$ with respect to K subject to an appropriate constraint, needed to avoid the zero solution. In this work, we impose a unitary constraint, i.e. $K^H K = I$, but other approaches are possible.

The cost function (29) can be expressed directly in terms of the matrix K, so that the desired estimate is

$$\hat{K} = \underset{K^H K = I}{\text{argmin}} \ \text{tr}\,[K^H P_{\mathcal{N}} K], \tag{30}$$

$$P_{\mathcal{N}} = \sum_{i \in \mathcal{I}} \hat{\mathcal{U}}_i \hat{\mathcal{U}}_i^H, \tag{31}$$

where $\hat{\mathcal{U}}_i$ is the block filtering matrix of size $L(M+1) \times (N+M)$ associated to eigenvector $\hat{U}_i = (\hat{U}_{i1}^T, \cdots, \hat{U}_{iN}^T)^T$ via

$$\hat{\mathcal{U}}_i = \mathcal{S}_{M+1}(\hat{U}_{i1}, \cdots, \hat{U}_{iN}). \tag{32}$$

Note that each sub-vector $\hat{U}_{in}$ has dimension $L \times 1$. The optimum solution is given by

$$\hat{K} = [\kappa_1, \ldots, \kappa_B] \tag{33}$$

where κ_b $(b = 1, \ldots, B)$ denote the B sub-dominant orthonormal eigenvectors of matrix $P_{\mathcal{N}}$.

5. SIMULATION RESULTS

We consider a linearly TV SIMO channel with $L = 4$ outputs and channel order $M = 4$ (i.e. $0 \leq m \leq 4$). The channel impulse responses $h_{n,m}$ are obtained from (7) with $B = 2$ basis functions, that is:

$$h_{n,m} = k_m^{(1)} \phi_n^{(1)} + k_m^{(2)} \phi_n^{(2)}, \quad 0 \leq n < N_a \tag{34}$$

$$\phi_n^{(1)} = 1, \quad \phi_n^{(2)} = \alpha(n - (N_a - 1)/2)) \tag{35}$$

and α is a real parameter controlling the rate of change. The transmitted symbols s_n in (1) are independent discrete complex random variables, equiprobable over a QAM constellation with zero mean and variance σ_s^2. The additive noise samples $w_n^{(l)}$ are independent complex circular Gaussian (CCG) random variables with zero mean and variance σ_n^2. The entries of the matrices $K_m = [k_m^{(1)}, k_m^{(2)}]$ are independent CCG random variables with zero-mean and unit variance.

For a given realization of the TV SIMO channel, represented by matrix K (15), the conditional SNR at the channel output, averaged over the integration time, is given by:

$$\text{SNR}(K) \triangleq \frac{1}{N_a} \sum_{n=0}^{N_a - 1} \frac{E[\|x_n - w_n\|^2]}{E[\|w_n\|^2]} = \frac{\sigma_s^2}{L\sigma_n^2} \text{tr}\,[K \Psi K^H] \tag{36}$$

In the simulations, we first generate the random matrix K and adjust σ_s^2 to obtain the desired SNR level. The performance of the estimators is evaluated in terms of the normalized mean square error (MSE) in the channel estimates. Let $\hat{h}_{n,m}$ denote the estimated TV channel impulse response obtained from (34) when using an estimate $\hat{K}$. We define:

$$\text{MSE} = E\left[\frac{\sum_n \sum_m \|\hat{h}_{n,m} - h_{n,m}\|^2}{\sum_n \sum_m \|h_{n,m}\|^2}\right]. \tag{37}$$

For each setting of the relevant parameters, we perform 500 independent experiments and obtain a sample estimate of the MSE (37). Results are presented for both the proposed algorithm and the classical algorithm [12], originally developed for the estimation of TI SIMO channels. In both cases, the value of the block size is set to $N = 10$. As is usually the case in this type of work on blind subspace-based estimation, a least squares approach is used to resolve the ambiguity (i.e complex scalar for the classical method versus 2×2 matrix A for the proposed approach).

In Fig. 1, we show the effect of the channel rate of change, α, on the MSE performance of the proposed algorithm and the classical algorithm. The integration time is set to $N_a = 1000$ and the average output SNR is 20dB. For the case $\alpha = 0$, which corresponds to TI channels, both methods give very similar results, with MSE values around -35dB. As α increases from 0 to 0.005, we observe a total degradation in the performance of the classical algorithm. For the proposed method, the MSE level initially increases to a value of about -25dB at $\alpha = .0004$, and then remains below this level for larger values of α. In Fig. 2, we show the effect of the integration time N_a on the MSE of the proposed method. In these experiments, we use $\alpha = 5/N_a$ and SNR = 10, 20 and 30dB. It can be seen that the MSE decreases in $1/N_a$, providing an experimental verification that the proposed method is asymptotically consistent. In Figure 3, we plot the MSE of the proposed method as a function of the output SNR for

$N_a = 1000$ and $\alpha = 0$ (TI case) and $\alpha = 0.005$. It can be observed that the MSE decreases in $1/\text{SNR}$ over the considered SNR range.

6. CONCLUSION

A new method has been proposed for the blind subspace-based identification of TV SIMO FIR channels in which the TV channel coefficients are represented via a finite basis expansion model. In contrast to earlier related works, the basis functions need not be limited to complex exponentials, and therefore do not necessitate the a priori estimation of frequency parameters. This considerably simplifies the implementation of method and provides added flexibility. The merits of the proposed technique, including asymptotic consistency, were demonstrated by numerical simulations.

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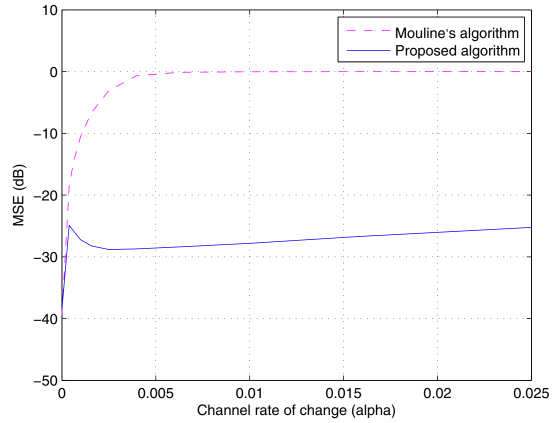

Fig. 1. MSE as a function of TV channel rate of change ($N_a = 1000$, SNR=20dB).

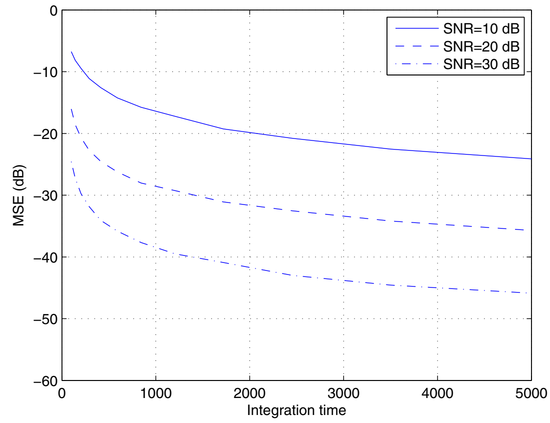

Fig. 2. MSE as a function of integration time N_a ($\alpha = 5/N_a$, SNR=10, 20 and 30dB).

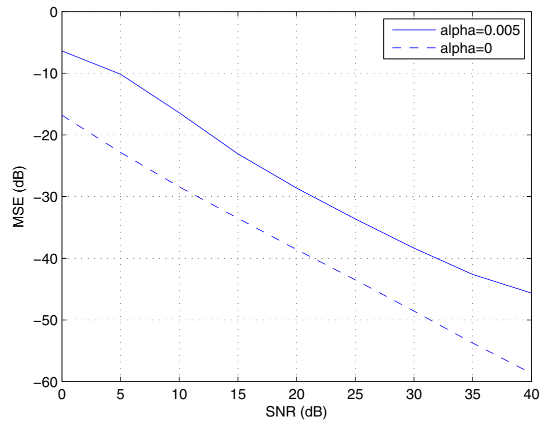

Fig. 3. MSE as a function of output SNR ($N_a = 1000$, $\alpha = 0$ and $\alpha = 0.005$) .