# JOINT TRANSCEIVER DESIGNS FOR SECURE COMMUNICATIONS OVER MIMO RELAY 

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#### Abstract

This paper addresses the transceiver design problem for secure downlink communications over a multiple-input multiple-output (MIMO) relay system in the presence of multiple eavesdroppers. A new algorithm based on alternating optimization (AO) is first proposed to maximize the signal-to-noise ratio (SNR) of a legitimate receiver under power constraints at the base station (BS) and the relay station (RS) and a set of secrecy constraints, by using the semidefinite relaxation (SDR) technique. To reduce complexity, a simplified design algorithm based on switched relaying (SR) is also proposed, in which both the BS and the RS are equipped with a codebook of permutation matrices. Based on this codebook, we construct a number of latent transceivers, each consisting of a BS beamforming vector and an optimally scaled RS permutation matrix. We use the bisection search and second-order cone programming (SOCP) techniques to design each latent transceiver and choose the optimal one with the largest SNR. We also develop an efficient approach to construct the codebook of permutation matrices. Our results show that the SR based algorithm significantly reduces the computational complexity while maintaining a similar performance to the AO based algorithm. ${ }^{1}$


Index Terms- Beamforming, switched relaying, physical layer security.

## 1. INTRODUCTION

As a complement to traditional encryption, physical layer security [1] - [5], which exploits signal processing techniques to ensure secure communication has become an active research area. In recent years, especially, physical layer security techniques that can enhance security in communications over MIMO relay systems have attracted considerable interest [6] - [11]. Hong et al. [6] mentioned two different secrecy applications of the MIMO relays, namely: i) secrecy beamforming and precoding with trusted and untrusted relays, and ii) a secure communication system with relays as cooperative jammers. Mukherjee et al. [7] enriched Hong's theory and divided security issues in relay networks into two broad categories. For specific problems of secure communications over a MIMO relay, Ding et al. combined interference alignment with cooperative jamming in order to ensure secure transmission to the legitimate receiver in the presence of an eavesdropper in [8]. A generalized singular value decomposition (GSVD) method is used by Huang et al. in [9]

[^0]to propose a cooperative jamming (CJ) scheme for secure communications with MIMO relays. Both works [8] and [9] considered only one eavesdropper in the communication network, which is overly restrictive. By considering the case of multiple eavesdroppers, Zhang et al. exploited the beamforming and jamming technique to maximize the worst-case secrecy rate in [10] while Yang et al. optimized the relay matrix to maximize the received SNR at the destination under a set of secrecy constraints in [11]. All the mentioned studies optimize the beamforming vector and the relay matrix separately without considering the possibility of joint design.

In this paper, we jointly optimize the BS beamforming vector and the RS AF transformation matrix in order to maximize the SNR of a legitimate receiver in the presence of multiple eavesdroppers, under power constraints at the BS and the RS and a set of secrecy constraints. We firstly propose a new algorithm based on alternating optimization (AO) and using the celebrated SDR [15] technique to solve this physical layer security problem. To reduce complexity as well as the signaling overhead, a simplified algorithm based on switched relaying [16] is also proposed to solve this problem. In the SR scheme, we assume that the proposed joint design algorithm is implemented at the $\mathrm{BS}^{2}$. The BS and the RS are both equipped with a finite codebook of permutation matrices. The BS creates a number of latent transceivers based on all the elements within the codebook and determines the optimal latent transceiver as the one with the largest SNR before data transmission. An efficient approach to construct the codebook of permutation matrices is also developed in the paper. Finally, the simulation results show that the SR based algorithm significantly reduces the computational complexity while maintaining a similar performance when compared to the AO based design.

## 2. SYSTEM MODEL AND PROBLEM STATEMENT

Let us consider a MIMO relay system as depicted in Fig. 1 which consists of one BS, one RS, and one legitimate receiver denoted as node D , which is overheard by $K$ eavesdroppers. Each eavesdropper is assigned a unique index $k \in \mathcal{K} \triangleq\{1,2, \cdots, K\}$ and denoted as $E_{k} . \mathrm{D}$ and $E_{k}, \forall k \in \mathcal{K}$ are equipped with single antenna, while the BS and the RS are equipped with $N_{t}$ and $N_{r}$ antennas, respectively. We assume that no direct link between the BS and the node D is available due to severe attenuation.

In the first phase, the received vector at the RS is given by $\mathbf{r}_{R}=\mathbf{H}_{1} \mathbf{p} b+\mathbf{n}_{1}$,

[^1]

Fig. 1. MIMO relay systems in the presence of multiple eavesdroppers
where $b$ denotes the transmit information symbol at a given time instant, modeled as a zero-mean Gaussian random variable with variance $E\left\{|b|^{2}\right\}=1$. $\mathbf{p} \in \mathbb{C}^{N_{t} \times 1}$ denotes the BS beamforming vector and $\mathbf{H}_{1} \in \mathbb{C}^{N_{r} \times N_{t}}$ is the channel matrix between the BS and the RS, whose elements are independent and identically distributed (i.i.d.) complex circular Gaussian variables with zero mean and unit variance, which we indicate by the standard notation $\mathcal{C N}(0,1)$, and $\mathbf{n}_{1} \in \mathbb{C}^{N_{r} \times 1}$ is the additive zero-mean complex Gaussian noise with covariance matrix $E\left[\mathbf{n}_{1} \mathbf{n}_{1}^{H}\right]=\sigma_{1}^{2} \mathbf{I}$, where $\sigma_{1}^{2}$ denotes the noise variance in the first phase (from BS to RS).

In the second phase (from RS to D), the vector $\mathbf{r}_{R} \in \mathbb{C}^{N_{r} \times 1}$ is operated by the RS AF transformation matrix $\mathbf{W} \in \mathbb{C}^{N_{r} \times N_{r}}$. The forwarded signal vector from the RS is given by -3mm

$$
\begin{equation*}
\mathbf{x}_{R}=\mathbf{W}\left(\mathbf{H}_{1} \mathbf{p} b+\mathbf{n}_{1}\right) \tag{2}
\end{equation*}
$$

For the second phase, the received signal at the legitimate receiver is given by

$$
\begin{equation*}
y_{D}=\mathbf{h}_{2}^{H} \mathbf{x}_{R}+n_{2}=\mathbf{h}_{2}^{H} \mathbf{W}\left(\mathbf{H}_{1} \mathbf{p} b+\mathbf{n}_{1}\right)+n_{2}, \tag{3}
\end{equation*}
$$

where $\mathbf{h}_{2} \in \mathbb{C}^{N_{r} \times 1}$ is the channel vector between the RS and the legitimate receiver, whose entries are i.i.d. zero mean complex circular Gaussian variables with unit variance, and $n_{2}$ denotes the additive zero mean complex Gaussian noise in the second phase, where $E\left[\left|n_{2}\right|^{2}\right]=\sigma_{2}^{2}$ denotes the second phase noise variance.

The transmit power of the BS in the first phase and that of the RS in the second phase are given by $P_{B}=E\left[\|\mathbf{p} b\|^{2}\right]=\operatorname{Tr}\left\{\mathbf{p p}^{H}\right\}$ and

$$
\begin{equation*}
P_{R}=E\left[\left\|\mathbf{x}_{R}\right\|^{2}\right]=E\left[\operatorname{Tr}\left\{\mathbf{W} \mathbf{H}_{1} \mathbf{p} \mathbf{p}^{H} \mathbf{H}_{1}^{H} \mathbf{W}^{2}+\sigma_{1}^{2} \mathbf{W} \mathbf{W}^{H}\right\}\right], \tag{4}
\end{equation*}
$$

respectively.
We adopt, as a metric of transmission reliability, the received signal-to-noise ratio (SNR) at the legitimate receiver, which is given by

$$
\begin{equation*}
\mathrm{SNR}_{D}=\frac{\left|\mathbf{h}_{2}^{H} \mathbf{W} \mathbf{H}_{1} \mathbf{p}\right|^{2}}{\sigma_{1}^{2}\left\|\mathbf{h}_{2}^{H} \mathbf{W}\right\|^{2}+\sigma_{2}^{2}} \tag{5}
\end{equation*}
$$

During the transmission, $E_{k}, \forall k \in \mathcal{K}$ can overhear signals from both the BS and the RS. Let $\mathbf{g}_{1 k} \in \mathbb{C}^{N_{t} \times 1}$ and $\mathbf{g}_{2 k} \in \mathbb{C}^{N_{r} \times 1}$, respectively denote the complex conjugate BS- $E_{k}$ and RS- $E_{k}$ channels. The signals overheard by $E_{k}$, respectively from the BS and the RS are given by

$$
\begin{align*}
& y_{1 k}=\mathbf{g}_{1 k}^{H} \mathbf{p} b+n_{1 k}, \\
& y_{2 k}=\mathbf{g}_{2 k}^{H} \mathbf{x}_{R}+n_{2 k}=\mathbf{g}_{2 k}^{H} \mathbf{W}\left(\mathbf{H}_{1} \mathbf{p} b+\mathbf{n}_{1}\right)+n_{2 k}, \tag{6}
\end{align*}
$$

where $n_{1 k}$ and $n_{2 k}$ denote complex circular Gaussian additive noise terms with zero mean and variances $\sigma_{1 k}^{2}$ and $\sigma_{2 k}^{2}$, respectively. It is assumed that for each transmission phase, each $E_{k}$ adopts the selection diversity combining scheme ${ }^{3}$.

We aim to design the BS beamforming vector $\mathbf{p}$ and the RS AF transformation matrix $\mathbf{W}$ jointly, in order to maximize the SNR achieved by the legitimate receiver under the BS and the RS power constraints, while keeping the SNR of eavesdroppers below a certain threshold $\gamma$. The optimization problem is given by

$$
\begin{align*}
\max _{\mathbf{p}, \mathbf{W}} & \operatorname{SNR}_{D} \\
\text { s.t. } & P_{B} \leq P_{t}, P_{R} \leq P_{r},  \tag{7}\\
& \frac{\left|\mathbf{g}_{1 k}^{H} \mathbf{p}\right|^{2}}{\sigma_{1 k}^{2}} \leq \gamma, \frac{\left|\mathbf{g}_{2 k}^{H} \mathbf{W} \mathbf{H}_{1} \mathbf{p}\right|^{2}}{\sigma_{1}^{2}\left\|\mathbf{g}_{2 k}^{H} \mathbf{W}\right\|^{2}+\sigma_{2 k}^{2}} \leq \gamma, \forall k \in \mathcal{K} .
\end{align*}
$$

## 3. PROPOSED AO BASED TRANSCEIVER DESIGN

We first present the AO based transceiver design algorithm for the joint optimization of the BS beamforming vector and the RS AF transformation matrix. Let us consider the optimization of $\mathbf{p}$ while the RS AF transformation matrix $\mathbf{W}$ is fixed. We can see that the celebrated SDR technique can be applied to solve the resulting optimization problem by introducing a new variable $\mathbf{P}=\mathbf{p} \mathbf{p}^{H}$. Thus, problem (7) can be reformulated as the following problem by ignoring the rank-one constraints for $\mathbf{P}$ :

$$
\begin{align*}
\max _{\mathbf{P}} & \frac{\mathbf{h}_{2}^{H} \mathbf{W} \mathbf{H}_{1} \mathbf{P} \mathbf{H}_{1}^{H} \mathbf{W}^{H} \mathbf{h}_{2}}{\sigma_{1}^{2}\left\|\mathbf{h}_{2}^{H} \mathbf{W}\right\|^{2}+\sigma_{2}^{2}} \\
\text { s.t. } & \operatorname{Tr}\{\mathbf{P}\} \leq P_{t}, \\
& E\left[\operatorname{Tr}\left\{\left(\mathbf{W H}_{1} \mathbf{P} \mathbf{H}_{1}^{H} \mathbf{W}^{H}\right)+\sigma_{1}^{2}\left(\mathbf{W W}^{H}\right)\right\}\right] \leq P_{r},  \tag{8}\\
& \frac{\mathbf{g}_{1 k}^{H} \mathbf{P g}_{1 k}}{\sigma_{1 k}^{2}} \leq \gamma, \frac{\mathbf{g}_{2 k}^{H} \mathbf{W} \mathbf{H}_{1} \mathbf{P} \mathbf{H}_{1}^{H} \mathbf{W}^{H} \mathbf{g}_{2 k}}{\sigma_{1}^{2}\left\|\mathbf{g}_{2 k}^{H} \mathbf{W}\right\|^{2}+\sigma_{2 k}^{2}} \leq \gamma, \\
& \mathbf{P} \succeq \mathbf{0}, \forall k \in \mathcal{K} .
\end{align*}
$$

In this way, problem (7) is relaxed to a convex semidefinite program (SDP) [17], which can be efficiently solved by available software packages, e.g., SeDuMi [17].

Next, we consider the optimization of the RS AF transformation matrix $\mathbf{W}$ while assuming that $\mathbf{p}$ is fixed. Noting that $\mathbf{x}^{T} \mathbf{Y} \mathbf{z}=\operatorname{vec}\left(\mathbf{x z}^{T}\right)^{T} \operatorname{vec}(\mathbf{Y})$, the numerator in (5) can be rewritten as

$$
\begin{equation*}
\left|\mathbf{h}_{2}^{H} \mathbf{W} \mathbf{H}_{1} \mathbf{p}\right|^{2}=\left|\mathbf{u}^{T} \operatorname{vec}(\mathbf{W})\right|^{2}=\mathbf{u}^{T} \tilde{\mathbf{W}} \operatorname{conj}(\mathbf{u}) \tag{9}
\end{equation*}
$$

where $\mathbf{u}=\operatorname{vec}\left(\operatorname{conj}\left(\mathbf{h}_{2}\right) \mathbf{p}^{T} \mathbf{H}_{1}^{T}\right)$ and $\tilde{\mathbf{W}}=\operatorname{vec}(\mathbf{W}) \operatorname{vec}(\mathbf{W})^{H}$. The operator $\operatorname{vec}(\cdot)$ stacks the elements of a matrix in one long column vector while conj $(\cdot)$ denotes the conjugate of a certain matrix. Thus, the optimization objective can be reformulated as

$$
\begin{equation*}
\operatorname{SNR}_{D}=\frac{\mathbf{u}^{T} \tilde{\mathbf{W}} \operatorname{conj}(\mathbf{u})}{\sigma_{1}^{2} \sum_{j=1}^{N_{t}} \mathbf{h}_{2}^{H} \mathbf{E}_{j} \tilde{\mathbf{W}} \mathbf{E}_{j}^{H} \mathbf{h}_{2}+\sigma_{2}^{2}} \tag{10}
\end{equation*}
$$

where $\mathbf{E}_{k} \in\{0,1\}^{N_{r}^{2} \times N_{r}^{2}}$ is a linear mapping matrix such that $\mathbf{h}_{2}^{H} \mathbf{E}_{j} \operatorname{vec}(\mathbf{W})=\mathbf{h}_{2}^{H} \mathbf{W}(:, j)$, and $\mathbf{W}(:, j)$ denotes the $j$ th column of $\mathbf{W}$. Similarly, the SNR constraints of the RS- $E_{k}$ link can be reformulated as

$$
\begin{equation*}
\frac{\mathbf{v}_{k}^{T} \tilde{\mathbf{W}} \operatorname{conj}\left(\mathbf{v}_{k}\right)}{\sigma_{1}^{2} \sum_{j=1}^{N_{t}} \mathbf{g}_{2 k}^{H} \mathbf{E}_{j} \tilde{\mathbf{W}} \mathbf{E}_{j}^{H} \mathbf{g}_{2 k}+\sigma_{2 k}^{2}} \leq \gamma \tag{11}
\end{equation*}
$$

[^2]Table 1. AO based transceiver design algorithm

1. Initialize $\mathbf{W}$ and define the tolerance of accuracy $\delta$.
2. Repeat

- Solve problem (8) with fixed $\mathbf{W}$ to obtain the updated $\mathbf{p}$. Employ the rank-one recovery method if higher-rank solutions are returned by solving problem (8).
- Solve problem (13) with fixed $\mathbf{p}$ to obtain the updated $\mathbf{W}$. Employ the rank-one recovery method to obtain $\mathbf{W}$ if higher-rank solutions of $\tilde{\mathbf{W}}$ are returned by solving problem (13). If the $\mathrm{SNR}_{D}$ of the recovery solution is larger than that of the previous step, then continue; else terminate the algorithm.

3. Until the $\mathrm{SNR}_{D}$ between two adjacent iterations is less than $\delta$.
where $\mathbf{v}_{k}=\operatorname{vec}\left(\operatorname{conj}\left(\mathbf{g}_{2 k}\right) \mathbf{p}^{T} \mathbf{H}_{1}^{T}\right)$. Using the identity $\operatorname{vec}(\mathbf{A B C})=$ $\left(\mathbf{C}^{T} \otimes \mathbf{A}\right) \operatorname{vec}(\mathbf{B})[18]$, the RS power constraint can be reformulated as

$$
\begin{equation*}
\operatorname{Tr}\left\{\left(\mathbf{p}^{T} \mathbf{H}_{1}^{T} \otimes \mathbf{I}_{N_{t}}\right) \tilde{\mathbf{W}}\left(\mathbf{p}^{T} \mathbf{H}_{1}^{T} \otimes \mathbf{I}_{N_{t}}\right)^{H}\right\}+\sigma_{1}^{2} \operatorname{Tr}(\tilde{\mathbf{W}}) \leq P_{r} \tag{12}
\end{equation*}
$$

Hence, the optimization problem can be reformulated as the following SDP problem:

$$
\begin{align*}
\max _{\tilde{\mathbf{W}} \succeq \mathbf{0}} & \operatorname{SNR}_{D}  \tag{13}\\
\text { s.t. } & (11),(12), \forall k \in \mathcal{K}
\end{align*}
$$

Problem (13) can now be solved with any accuracy $\epsilon>0$ by using the bisection method to obtain $\tilde{\mathbf{W}}$. A rank-one recovery method inspired by the randomization procedure [15] will be adopted when the optimal solutions to (8) and (13) are not rank-one. The AO based iterative algorithm to solve problem (7) is summarized in Table 1, which can keep the objective nondecreasing as the iterations proceed.

## 4. PROPOSED SR BASED TRANSCEIVER DESIGN

In this section, we describe a more efficient and simpler transceiver design algorithm, namely the SR based algorithm. We equip the BS and the RS with a finite codebook of permutation matrices ${ }^{4}$, i.e., $\mathcal{T}=\left\{\mathbf{T}_{1}, \mathbf{T}_{2}, \ldots, \mathbf{T}_{B}\right\}$, where $B$ is the codebook size which satisfies $B \ll N_{r}!^{5}$. The RS AF transformation matrix is constructed by multiplying the appropriate permutation matrix from the codebook with a power scaling factor. That is to say, the optimization of $\mathbf{W}$ is replaced by that of $\beta_{l} \mathbf{T}_{l}$, where $\mathbf{T}_{l}$ and $\beta_{l}$ denote the $l$ th permutation matrix in $\mathcal{T}$ and the corresponding power scaling factor, respectively, $l \in\{1, \ldots, B\}$. Thus, the $l$ th permutation matrix gives rise to a permuted channel matrix, and creates a latent transceiver, which for each index $l$ requires the determination of the BS beamforming vector $\mathbf{p}_{l}$ and the RS power scaling factor $\beta_{l}$. We can design $B$ such latent transceivers, that is, one for each permutation matrix in $\mathcal{T}$, and choose the optimal transceiver with the largest $\mathrm{SNR}_{D}$. The proposed SR scheme works as follows:

- The BS designs the $B$ latent transceivers based on available permutation matrices within the codebook; it then determines the optimal latent transceiver with the largest $\mathrm{SNR}_{D}$.

[^3]Table 2. The design algorithm for latent transceivers

1. Initialize $t_{\text {min }}=\mathrm{SNR}_{D_{\text {min }}}$ and $t_{\text {max }}=\mathrm{SNR}_{D_{\text {max }}}$, where $\mathrm{SNR}_{D_{\text {min }}}$ and $\mathrm{SNR}_{D_{\text {max }}}$ define the range of relevant $\mathrm{SNR}_{D}$. Let $\epsilon>0$ be the desired accuracy.
2. Set $t=\left(t_{\text {min }}+t_{\text {max }}\right) / 2$.
3. Solve the SOCP feasibility problem. If the problem is feasible, then set $t_{\text {min }}=t$. Otherwise, set $t_{\max }=t$. find $\mathbf{p}_{l}, \rho_{l}$
s.t. $\quad \sqrt{t}\left\|\left[\sigma_{1}\left\|\mathbf{h}_{2}^{H} \mathbf{T}_{l}\right\|, \rho_{l} \sigma_{2}\right]\right\| \leq \mathbf{h}_{2}^{H} \mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}$,
$\left\|\mathbf{p}_{l}\right\| \leq \sqrt{P_{t}}, \Im\left(\mathbf{h}_{2}^{H} \mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right)=0$,
$\left\|\left[\left(\mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right)^{T}, \sigma_{1} \sqrt{N_{r}}\right]\right\| \leq \sqrt{P_{r}} \rho_{l}$, $\left|\mathbf{g}_{1 k}^{H} \mathbf{p}_{l}\right| \leq \sqrt{\gamma} \sigma_{1 k}$, $\left|\mathbf{g}_{2 k}^{H} \mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right| \leq \sqrt{\gamma} \sigma_{1}\left\|\mathbf{g}_{2 k}^{H} \mathbf{T}_{l}\right\|, \forall k \in \mathcal{K}$.
4. If $\left(t_{\max }-t_{\min }\right)>\epsilon$ then go to Step 2. Otherwise, return $\rho_{l}^{\star}=\rho_{l}$ and $\mathbf{p}_{l}^{\star}=\mathbf{p}_{l}$, where $\rho_{l}$ and $\mathbf{p}_{l}$ represent the last feasible solution of the upper problem and STOP.

- The BS sends the index of the optimal latent transceiver, say $l^{o}$, and the corresponding power scaling factor $\beta_{l^{\circ}}$ to the RS through signaling channels.
- Based on the signaling bits forwarded from the BS, the RS determines the corresponding AF transformation matrix $\mathbf{W}=$ $\beta_{l^{\circ}} \mathbf{T}_{l^{o}}$.

In the following, we firstly describe the design algorithm for the construction of the latent transceivers. The design method for the codebook of permutation matrices is also described in this section.

### 4.1. The design algorithm for latent transceivers

In this part, we develop a new design algorithm to construct the latent transceivers. For each permutation matrix, problem (7) now can be reformulated as

$$
\begin{align*}
\max _{\mathbf{p}_{l}, \beta_{l}} & \frac{\beta_{l}^{2}\left|\mathbf{h}_{2}^{H} \mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right|^{2}}{\sigma_{1}^{2} \beta_{l}^{2}\left\|\mathbf{h}_{2}^{H} \mathbf{T}_{l}\right\|^{2}+\sigma_{2}^{2}} \\
\text { s.t. } & \left\|\mathbf{p}_{l}\right\|^{2} \leq P_{t}, \beta_{l}^{2}\left(\left\|\mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right\|^{2}+\sigma_{1}^{2} N_{r}\right) \leq P_{r}, \\
& \frac{\mathbf{g}_{1 k}^{H} \mathbf{p}_{l} l^{2}}{\sigma_{1 k}^{2}} \leq \gamma, \frac{\beta_{l}^{2}\left|\mathbf{g}_{2 k}^{H} \mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right|^{2}}{\sigma_{1}^{2} \beta_{l}^{2}\left\|\mathbf{g}_{2 k}^{H} \mathbf{T}_{l}\right\|^{2}+\sigma_{2 k}^{2}} \leq \gamma, \forall k \in \mathcal{K} . \tag{14}
\end{align*}
$$

Firstly, we define $\rho_{l}=\frac{1}{\beta_{l}}$. Since the noise term could be very insignificant as compared to the other terms, we can ignore the effect of $\sigma_{2 k}^{2}$ to make the problem more tractable. The following form will be obtained after introducing a variable $t$ :

$$
\begin{array}{rl}
\max _{\mathbf{p}_{l}, \rho_{l}, t} & t \\
\text { s.t. } & t\left(\sigma_{1}^{2}\left\|\mathbf{h}_{2}^{H} \mathbf{T}_{l}\right\|^{2}+\rho_{l}^{2} \sigma_{2}^{2}\right) \leq\left|\mathbf{h}_{2}^{H} \mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right|^{2}, \\
& \left\|\mathbf{p}_{l}\right\|^{2} \leq P_{t}, t \geq 0  \tag{15}\\
& \left\|\left[\left(\mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right)^{T}, \sigma_{1} \sqrt{N_{r}}\right]\right\|^{2} \leq P_{r} \rho_{l}^{2} \\
& \left|\mathbf{g}_{1 k}^{H} \mathbf{p}_{l}\right|^{2} \leq \gamma \sigma_{1 k}^{2} \\
& \left|\mathbf{g}_{2 k}^{H} \mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}\right|^{2} \leq \gamma \sigma_{1}^{2}\left\|\mathbf{g}_{2 k}^{H} \mathbf{T}_{l}\right\|^{2}, \forall k \in \mathcal{K} .
\end{array}
$$

Since phase rotation will not change our optimization results, we can rotate the phase of $\mathbf{h}_{2}^{H} \mathbf{T}_{l} \mathbf{H}_{1} \mathbf{p}_{l}$ and then extract the root of both sides to transform the original constraints into second order cone (SOC) constraints. Thus, problem (15) can be solved with any accuracy $\epsilon>0$ by using the bisection method presented in Table 2 .

Table 3. Complexity analysis of the proposed algorithms

| Algorithms | Complexity |
| :---: | :---: |
| AO-based | $I_{1}\left(\mathcal{O}\left(m_{1} \sqrt{N_{t}}\left(N_{t}^{3}+m_{1} N_{t}^{2}+m_{1}^{2}\right)\right)+\right.$ |
|  | $\left.\mathcal{O}\left(m_{2} N_{r}\left(N_{r}^{6}+m_{2} N_{r}^{4}+m_{2}^{2}\right)\right)\right)$ |
| SR-based | $B I_{2}\left(\mathcal{O}\left(m \sqrt{6}\left(9+\left(N_{t}+1\right)^{2}+\left(N_{r}+2\right)^{2}+m^{2}\right)\right)\right)$ |

### 4.2. Codebook Design

We now propose a heuristic scheme to construct the codebook of permutation matrices. We seek to choose the permutation matrices which are more likely to result in higher power received by the legitimate receiver. Interestingly, through exhaustive simulation experiments, we observed that if the singular values of the permuted matrices $\mathbf{h}_{l}=\mathbf{h}_{2}^{H} \mathbf{T}_{l} \mathbf{H}_{1}$ are smaller, the total received power is usually larger, where $\mathbf{h}_{l}$ represents the $1 \times N_{r}$ equivalent channel matrix between the BS and the legitimate receiver.

Let $\xi^{l}$ denote the singular value of $\mathbf{h}_{l}$. We construct the codebook of permutation matrices by choosing the ones which correspond to the smallest $B$ singular values, i.e.,

$$
\begin{equation*}
\left\{\mathbf{T}_{1}, \ldots, \mathbf{T}_{B}\right\}=\arg \min _{\mathbf{T}_{l}} B\left(\xi^{l}\right) \tag{16}
\end{equation*}
$$

where $\min B(\cdot)$ returns the permutation matrices corresponding to the smallest $B$ singular values. The performance of the above codebook design approach will be presented along with the simulation results in Section 6.

## 5. COMPUTATIONAL COMPLEXITY

In this section, we compare the computational complexity of the proposed AO based and SR based algorithms. The complexity of the AO based algorithm is dominated by the solution of (8) and (13) $I_{1}$ times, where $I_{1}$ denotes the iteration number. Problem (8) involves 1 linear matrix inequality (LMI) limits of size $N_{t}$ and $m_{1}=\mathcal{O}\left(N_{t}^{2}\right)$ decision variables. Thus, the complexity of a generic interior-point method for solving problem (8) is given by $\mathcal{O}\left(m_{1} \sqrt{N_{t}}\left(N_{t}^{3}+m_{1} N_{t}^{2}+m_{1}^{2}\right)\right)$. Similarly, the complexity of solving problem (13) can be written as $\mathcal{O}\left(m_{2} N_{r}\left(N_{r}^{6}+m_{2} N_{r}^{4}+m_{2}^{2}\right)\right)$, where $m_{2}=\mathcal{O}\left(N_{r}^{4}\right)$.

The complexity of the SR based algorithm is dominated by the solution of problem (15) $I_{2}$ times, where $I_{2}$ is the iteration number. Problem (15) involves 3 SOC constraints, including 1 SOC of dimension 3, 1 SOC of dimension $N_{t}+1$ and 1 SOC of dimension $N_{r}+2$. The number of variables is $m=\mathcal{O}\left(N_{t}+1\right)$.

We summarize the computational complexity of the two algorithms in Table 3 for comparison. It is of interest to investigate the asymptotic complexity of the proposed algorithms when $N_{t}, N_{r}$ and $K$ are large, i.e., when we let $N_{r}=N_{t}=K \longrightarrow \infty$. We further assume that $I_{1}=I_{2}=I$ for simplicity. Under these conditions, one can verify that the complexities of the proposed AO based and SR based algorithms in Table 3 are $2 I N_{t}^{13}$ and $3 \sqrt{6} B I N_{t}^{3}$, respectively. Since the value of codebook size $B$ is usually small, the SR based algorithm has lower complexity compared with the AO based algorithm.

## 6. SIMULATIONS

In this section, we evaluate the performance of the proposed SR based and AO based algorithms. In all experiments, the number of antennas is $N_{t}=N_{r}=4$. The maximum transmit power is normalized as $P_{t}=P_{r}=1$, and the noise variances of the legitimate links are adjusted so that the input $\mathrm{SNR} \triangleq \frac{P_{t}}{\sigma_{1}^{2}}=\frac{P_{r}}{\sigma_{2}^{2}}$. The noise variances of all the eavesdroppers through the BS- $E_{k}$ and the RS- $E_{k}$ links are set to $\sigma_{1 k}^{2}=\sigma_{2 k}^{2}=0.01$ while the threshold $\gamma$ is set as $0 d B$. All
the results are obtained by averaging 1000 independent Monte Carlo runs.


Fig. 2. The $\mathrm{SNR}_{D}$ achieved at the legitimate receiver versus the number of eavesdroppers $(\mathrm{SNR}=20 \mathrm{~dB})$.


Fig. 3. The $\mathrm{SNR}_{D}$ achieved at the legitimate receiver versus the codebook size $(\mathrm{SNR}=20 \mathrm{~dB}, K=2)$.

Fig. 2 shows that the performance of the SR based algorithm can be very close to that of the AO based algorithm in terms of the achieved $\mathrm{SNR}_{D}$. The number of signaling bits for the SR based algorithm is much less than that for the AO based algorithm since only the index of the optimal transceiver and the power scaling factor have to be sent to the RS, instead of the whole RS AF transformation matrix. Fig. 3 presents the $\mathrm{SNR}_{D}$ achieved at the legitimate receiver as a function of codebook size $B$, which shows that the SR based algorithm equipped with the well designed codebook works better than the SR based algorithm equipped with a randomly generated codebook.

## 7. CONCLUSION

In this paper, we have considered the joint transceiver design problem for secure communication over a MIMO relay with multieavesdroppers. We have proposed AO based and SR based algorithms for the joint optimization of the BS beamforming vector and the RS AF transformation matrix. Specifically, the SR based algorithm can achieve similar performance compared with the AO based algorithm with reduced complexity and overhead. We have also developed an efficient approach for the permutation matrix codebook design. The simulation results have shown satisfactory performance of the proposed algorithms. Robust design algorithms against channel state information (CSI) errors will be considered in the future.

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[^1]:    ${ }^{2}$ In cellular systems, it is preferable to implement most of the signal processing operations at the BS rather than the RS, due to the fact that the BS is more powerful while the RS is expected to have a simple structure and low energy consumption [12] - [14].

[^2]:    ${ }^{3}$ In this paper, selection diversity combining is assumed at each $E_{k}$ due to its operational simplicity. However, the proposed algorithm can be extended to other types of combiners, such as the optimal maximum ratio combiner.

[^3]:    ${ }^{4}$ A permutation matrix is a square binary matrix that has exactly one entry equal to 1 in each row and each column, while all the other entries are equal to 0 .
    ${ }^{5}$ The total number of permutation matrices at the RS is $N_{r}$ !. It is not realistic to use all the permutation matrices as the codebook when $N_{r}$ is large.

