Min-Max MSE Transceiver with Switched Preprocessing for MIMO Interference Channels

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Abstract-In this study, we propose a robust transceiver scheme with switched preprocessing (SP) for K-user multipleinput multiple-output (MIMO) interference channels. The channel state information (CSI) available is assumed to be imperfect under norm-bounded errors (NBE). Each transmitter is provided with a codebook of permutation matrices, so that each arrangement of permutation matrices among the K transmitters will generate a group of K parallel transceivers. The optimum transceiver group within the class of all possible such groups is chosen by a suitable selection mechanism for data transmission. To design each transceiver group, we adopt a worst-case design approach to minimize the maximum per user MSE. We show that the proposed transceiver design problem can be partitioned into an alternating sequence of optimization and worst-case analysis subproblems, which involves solving Second-Order Cone Programming (SOCP) problems. Simulation results show that the performance of the proposed SP-based transceiver is significantly better than existing methods in the presence of imperfect CSL¹

Index Terms—Robust transceiver, MIMO interference channel, switched preprocessing, Min-Max, SOCP.

I. INTRODUCTION

Recently, transceiver systems for *K*-user MIMO interference channel have attracted considerable interest due to their potential to improve transmission performance and achieve higher data rates in cellular applications. To realize the potential gains of such systems in practice, it is very important to devise methods to reduce the multiuser interference which limits their performance. Interference alignment (IA) schemes are presented in [1], in the form of transmit precoders with closed-form expression, as a means to align signals from all un-intended transmitters. IA shows great potential for coordinated multi-point (CoMP) systems as it can mitigate inter-cell interference and improve cell-edge throughput. In [2], distributed IA algorithms are proposed to cancel the interference by iteratively minimizing leakage interference or maximizing signal-to-interference-plus-noise ratio (SINR).

In practice, the potential gains of IA are limited due to the realistic channel conditions or even system operation at low SNR. Moreover, the assumption of availability of complete and perfect CSI is too optimistic when we take the fluctuations of the channel and the presence of estimation errors or quantization effects into account. To overcome these limitations, several researchers have recently turned their attention to the joint transmitter and receiver design under imperfect CSI by relaxing the perfect alignment constraint. The authors in [3] propose novel transceiver schemes for the MIMO interference channel in the presence of channel estimation errors based on the MSE criterion, especially by minimizing the Sum-MSE and the maximum per user MSE. In [4], a robust transceiver design is formulated as an optimization problem to maximize the worst-case SINR among all users, which is based on alternative optimization (AO) and semi-definite relaxation (SDR) techniques. In [5], a max-min fairness linear transceiver is proposed based on the cyclic coordinate ascent strategy.

In this paper, we propose a robust switched preprocessing (SP) transceiver scheme for K-user MIMO interference channel under norm-bounded error (NBE) model. Each transmitter is provided with a size B codebook of permutation matrices². Hence, depending on the particular codebook selection made by each individual transmitter, there will be B^K possible arrangements of permutation matrices in total among the K transmitters, where each arrangement corresponds to a group of K parallel transceivers. Subsequently, the optimum transceiver group is chosen by a suitable selection mechanism for data transmission. We also develop a robust transceiver design algorithm for the construction of each transceiver group, which utilizes the worst-case concept [6], [7]. We show that this design problem can be partitioned into an alternating sequence of optimization and worst-case analysis subproblems. The first subproblem is based on minimizing the maximum per user MSE, which can be formulated as an SOCP problem that can be solved efficiently using interiorpoint methods [8]. The second one involves the computation of worst-case channel matrices with fixed transceiver, which can be solved analytically. Simulation results demonstrate that compared to other recently proposed transceiver designs, the proposed SP-based transceivers provide improved robustness against the effects of CSI errors and interference.

II. PROPOSED TRANSMISSION SYSTEM STRUCTURE

We consider the K-user MIMO interference channel where the kth transmitter and receiver are equipped with M_k and N_k antennas respectively. We provide each transmitter with a codebook of B permutation matrices, so that there are B^K possible arrangements of permutation matrices among all the transmitters, where each arrangement will generate a group of K parallel transceivers. We can then design the B^K transceiver groups according to a chosen methodology,

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 $^{^{2}}$ A permutation matrix is a square binary matrix that has exactly one entry equal to 1 in each row and each column and 0s elsewhere; it is used here to arrange the rows of the precoder output signal vector in a particular order.

among which the optimum transceiver group is chosen by a certain mechanism, as further discussed in Section III-D. For the proposed scheme, the data is transmitted by using the selected precoding matrix first at each transmitter, then the precoded vector is processed by the selected permutation matrix, before transmission. Finally, a corresponding receive filter is employed to decode the symbols at the destination.



Fig. 1. The structure of the proposed transmission scheme.

A. Proposed System Model

The *k*th receiver aims to decode the symbols from the *k*th transmitter, and treats the symbols from other transmitters as multi-user interference (MUI). We assume that the multiple wireless channels are frequency flat, slowly fading. Focussing on the *k*th transmit-receive pair of the *n*th transceiver group, where $n \in \{1, \ldots, B^K\}$, the receive data vector $\mathbf{r}_k \in \mathbb{C}^{N_k \times 1}$ can be written as

$$\mathbf{r}_{k} = \sum_{i=1}^{K} \mathbf{H}_{ki} \mathbf{T}_{n,i} \mathbf{F}_{n,i} \mathbf{d}_{i} + \mathbf{n}_{k}$$
$$= \mathbf{H}_{kk} \mathbf{T}_{n,k} \mathbf{F}_{n,k} \mathbf{d}_{k} + \underbrace{\sum_{i=1, i \neq k}^{K} \mathbf{H}_{ki} \mathbf{T}_{n,i} \mathbf{F}_{n,i} \mathbf{d}_{i}}_{MUI} + \mathbf{n}_{k}, \quad (1)$$

where $\mathbf{d}_i \in \mathbb{C}^{S_i \times 1}$ is the vector of data symbols emitted by the *i*th transmitter and S_i denotes the number of data streams of the *i*th pair, which is chosen to meet the feasibility of degrees of freedom. In (1), $\mathbf{H}_{ki} \in \mathbb{C}^{N_k \times M_i}$ denotes the channel matrix between transmitter *i* and receiver *k*, while $\mathbf{F}_{n,i} \in \mathbb{C}^{M_i \times S_i}$ and $\mathbf{T}_{n,i} \in \mathbb{R}^{M_i \times M_i}$ respectively denote the precoder and permutation matrices corresponding to the *n*th transceiver group. In this respect, the set of matrices $\{\mathbf{H}_{ki}\mathbf{T}_{n,i}\}$ represents the collection of permuted channel matrices corresponding to the *n*th transceiver group. The term $\mathbf{n}_k \in \mathbb{C}^{N_k \times 1}$ denotes the zero mean complex circular symmetric Gaussian noise with covariance matrix $E[\mathbf{n}_k \mathbf{n}_k^H] = \sigma_n^2 \mathbf{I}$, where σ_n^2 is the noise variance. The transmitted symbol vectors \mathbf{d}_i are independent of the noise vector \mathbf{n}_k at any receiver.

At the *k*th receiver, the receive filter $\mathbf{B}_{n,k}$ of the *n*th transceiver group is then employed to detect the received signal, that is:

$$\mathbf{d}_k = Q\left\{\mathbf{B}_{n,k}\mathbf{r}_k\right\},\tag{2}$$

where $Q\{\cdot\}$ represents the quantization operation. The structure of the proposed scheme is illustrated in Fig. 1. The

main problem of transceiver design for the above MIMO interference channel is to derive the precoders $\{\mathbf{F}_{n,k}\}\$ and receive filters $\{\mathbf{B}_{n,k}\}\$ corresponding to all the transceiver groups, so as to optimize a predefined performance criterion. In this paper, we consider the time-division duplexing (TDD) mode where the transmit CSI can be obtained by channel estimation algorithms due to the reciprocity property.

B. Channel Error Model

Because of many factors such as channel estimation error, quantization error, and feedback error/delay, it is impractical to obtain perfect CSI at both transmitter and receiver. In our work, we consider the estimated CSI instead, and the channel error is assumed to follow the NBE model [7]:

$$\mathbf{H}_{ki} = \mathbf{H}_{ki} + \Delta \mathbf{H}_{ki}, \ i, k \in \{1, \dots, K\},$$
(3)

where $\widehat{\mathbf{H}}_{ki}$ denotes the estimated channel matrix between transmitter *i* and receiver *k*, and $\Delta \mathbf{H}_{ki}$ denote the corresponding channel error matrix, which is assumed to be bounded in its Euclidean norm, i.e. :

$$\|\Delta \mathbf{H}_{ki}\| \le \theta_{ki}, \ i, k \in \{1, 2, \cdots, K\}$$

$$(4)$$

where θ_{ki} is a known positive constant. Equivalently, \mathbf{H}_{ki} belongs to the uncertainty set \Re_{ki} defined as

$$\Re_{ki} = \left\{ \mathbf{H} \left| \mathbf{H} = \widehat{\mathbf{H}}_{ki} + \Delta \mathbf{H}_{ki} , \| \Delta \mathbf{H}_{ki} \| \le \theta_{ki} \right\}.$$
(5)

The shape and size of \Re_{ki} model the kind of uncertainty in the estimated CSI, which is linked to the physical phenomenon producing the CSI errors. It is worth noting that the actual errors $\Delta \mathbf{H}_{ki}$ are assumed to be unknown while the corresponding upper bounds θ_{ki} are known. This model is particularly suitable for systems where CSI is corrupted by quantization [9].

III. PROPOSED SP-BASED ROBUST TRANSCEIVER

In Sub-Section A, B and C below, we first introduce a Min-Max algorithm inspired by [3] to design the *n*th transceiver group based on the permuted channel matrices $\{\mathbf{H}_{ki}\mathbf{T}_{n,i}\}$ corresponding to the *n*th arrangement of permutation matrices among the transmitters. In Sub-Section D, a selection mechanism which is based on the Euclidean distance between the true transmit symbol vector and the noiseless pre-estimated symbol vector is introduced to choose the optimum transceiver group.

The objective of our approach is to utilize the worst-case philosophy, i.e., to minimize the worst-case maximum MSE among all users under the transmit power constraints for all possible CSI errors satisfying the norm bound. Mathematically, this problem can be formulated as

$$\begin{array}{l} \min_{\{\mathbf{B}_{n,k},\mathbf{F}_{n,k}\}} & \max_{k=1,\dots,K} \omega_{n,k} \\ \mathbf{s.t.} & \operatorname{Tr}\left(\mathbf{F}_{n,k}\mathbf{F}_{n,k}^{H}\right) \leq P_{k} \\ \mathbf{H}_{ki} \in \Re_{ki}, \, i, k = 1,\dots, K, \end{array}$$
(6)

where $\omega_{n,k}$ denotes the *k*th user's mean square error, which is given by

$$\omega_{n,k} = E[\|\widehat{\mathbf{d}}_{k} - \mathbf{d}_{k}\|^{2}] = \operatorname{Tr}[\mathbf{B}_{n,k}\sum_{i=1}^{K} (\mathbf{H}_{ki}\mathbf{V}_{n,i}\mathbf{H}_{ki}^{H})\mathbf{B}_{n,k}^{H} - \mathbf{B}_{n,k}\mathbf{H}_{kk}\mathbf{U}_{n,k} - \mathbf{U}_{n,k}^{H}\mathbf{H}_{kk}^{H}\mathbf{B}_{n,k}^{H} + \mathbf{I} + \sigma_{n}^{2}\mathbf{B}_{n,k}\mathbf{B}_{n,k}^{H}],$$
(7)

where $\mathbf{U}_{n,i} = \mathbf{T}_{n,i} \mathbf{F}_{n,i}, \mathbf{V}_{n,i} = \mathbf{U}_{n,i} \mathbf{U}_{n,i}^{H}, i \in \{1, \dots, K\}.$

Since an exact solution to this problem cannot be obtained, we propose a tractable solution approach based on the cuttingset method [10]. The proposed algorithm involves solving an alternating sequence of two subproblems. In the optimization subproblem, we solve problem (6) using fixed channel matrices. In the worst-case analysis subproblem, we compute the worst-case channel matrices with fixed precoder and receive filter.

A. Min-Max Algorithm with Given Channel Matrices

The first subproblem in the proposed robust algorithm is the computation of the precoders $\mathbf{F}_{n,k}$ and receive filters $\mathbf{B}_{n,k}$ for a given set \mathcal{H} of channel matrices $\mathbf{H}_{ki}^{(j)}, j = 1, \ldots, E$, where $\mathbf{H}_{ki}^{(j)}$ is the *j*th worst-case channel matrix, and *E* is the size of the set \mathcal{H} . It is worth noting that $\mathbf{H}_{ki}^{(1)}$ is the imperfect CSI initially available, i.e. $\widehat{\mathbf{H}}_{ki}$ and *E* may be increased through each outer iteration, which will be described in the following subsections. The problem can be formulated as

$$\min_{\{\mathbf{B}_{n,k},\mathbf{F}_{n,k}\}} \max_{k=1,\dots,K, \ j=1\dots,E} \omega_{n,k}^{(j)}$$
s.t. $\operatorname{Tr}\left(\mathbf{F}_{n,k}\mathbf{F}_{n,k}^{H}\right) \leq P_{k}, \ k=1,\dots,K,$
(8)

where $\omega_{n,k}^{(j)}$ is the *k*th user's mean square error corresponding to the *j*th worst-case channel matrix. By following the iteration scheme in [3]. The optimal transmit precoding matrices can be solved from the following optimization problem

$$\begin{array}{l} \min_{\{\mathbf{F}_{n,k},t\}} & t \\ \mathbf{s.t.} & \sqrt{\omega_{n,k}^{(j)}} \leq t, \ \|\operatorname{vec}\left(\mathbf{F}_{n,k}\right)\|_{2} \leq \sqrt{P_{k}} \\ & k = 1, \dots, K, \ j = 1, \dots, E. \end{array}$$
(9)

where

$$\omega_{n,k}^{(j)} = \left\| \begin{array}{c} \delta_{n,k} \\ [\mathbf{I}_S \otimes (\mathbf{B}_{n,k} \mathbf{\Pi}_k \mathbf{H}_n^{(j)})] \operatorname{vec}(\mathbf{F}_n) - \operatorname{vec}(\mathbf{\Xi}_k) \end{array} \right\|_2^2,$$
(10)

t is an auxiliary variable that serves an upper bound on the square root of $\omega_{n,k}^{(j)}$, $\delta_{n,k} = \sigma_n \sqrt{\text{Tr}(\mathbf{B}_{n,k}\mathbf{B}_{n,k}^H)}$, $\mathbf{F}_n =$ diag { $\mathbf{F}_{n,1}, \mathbf{F}_{n,2}, \dots, \mathbf{F}_{n,K}$ } and $S = \sum_{i=1}^{K} S_i$. Furthermore, in (10), we define

$$\mathbf{H}_{n}^{(j)} = \begin{bmatrix} \mathbf{H}_{11}^{(j)} \mathbf{T}_{n,1} & \cdots & \mathbf{H}_{1K}^{(j)} \mathbf{T}_{n,K} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{K1}^{(j)} \mathbf{T}_{n,1} & \cdots & \mathbf{H}_{KK}^{(j)} \mathbf{T}_{n,K} \end{bmatrix}.$$
(11)

 $\mathbf{\Pi}_{k} = \begin{bmatrix} \mathbf{0}_{N_{k}^{-}}, \mathbf{I}_{N_{k}}, \mathbf{0}_{N_{k}^{+}} \end{bmatrix} \text{ and } \mathbf{\Xi}_{k} = \begin{bmatrix} \mathbf{0}_{S_{k}^{-}}, \mathbf{I}_{S_{k}}, \mathbf{0}_{S_{k}^{+}} \end{bmatrix}, \text{ where}$ $N_{k}^{-} = N_{k} \times \sum_{i=1}^{k-1} N_{i}, N_{k}^{+} = N_{k} \times \sum_{i=k+1}^{K} N_{i}, S_{k}^{-} = S_{k} \times \sum_{i=1}^{k-1} S_{i}, S_{k}^{+} = S_{k} \times \sum_{i=k+1}^{K} S_{i}. \text{ The operator } \otimes \text{ represents the}$

Kronecker product and the operator $vec(\cdot)$ stacks the elements of a matrix in one long column vector. The above optimization can be efficiently solved by a standard SOCP solver, e.g. SeDuMi [11] and SDPT3 [12]. The Sum-MSE receiver [3] for the given transmit precoding matrices can be expressed as

$$\mathbf{B}_{n,k} = \mathbf{U}_{n,k}^{H} \widehat{\mathbf{H}}_{kk}^{H} \left(\sum_{i=1}^{K} \widehat{\mathbf{H}}_{ki} \mathbf{V}_{n,i} \widehat{\mathbf{H}}_{ki}^{H} + \sigma_{n}^{2} \mathbf{I} \right)^{-1}$$
(12)

In this paper, we develop a more robust receiver by taking worst-case channel matrices into account. Specifically, we consider the following Min-Max optimization problem of the receive filter matrices

$$\min_{\mathbf{B}_{n,k}\}} \max_{k=1,\dots,K, \ j=1,\dots,E} \omega_{n,k}^{(j)}.$$
 (13)

Then $\omega_{n,k}^{(j)}$ can be rewritten as

$$\omega_{n,k}^{(j)} = \left\| \operatorname{vec} \left(\mathbf{B}_{n,k} \mathbf{\Pi}_{k} \mathbf{H}_{n}^{(j)} \mathbf{F}_{n} - \mathbf{\Xi}_{k} \right) \right\|_{2}^{2} + \left\| \sigma_{n} \operatorname{vec} \left(\mathbf{B}_{n,k} \right) \right\|_{2}^{2}$$
$$= \left\| \begin{bmatrix} \sigma_{n} \operatorname{vec} \left(\mathbf{B}_{n,k} \right) \\ \left[\left(\mathbf{\Pi}_{k} \mathbf{H}_{n}^{(j)} \mathbf{F}_{n} \right)^{T} \otimes \mathbf{I}_{S_{k}} \end{bmatrix} \operatorname{vec} \left(\mathbf{B}_{n,k} \right) - \operatorname{vec} \left(\mathbf{\Xi}_{k} \right) \right\|_{2}^{2}.$$
(14)

The equality given above follows from the following property of vec (·) and Tr (·) operators for any matrices **A**, **B** and **C** of appropriate dimensions: vec (**ABC**) = ($\mathbf{C}^T \otimes \mathbf{A}$) vec (**B**). Then, the Min-Max optimization problem can be rewritten as

$$\begin{array}{l} \min_{\{\mathbf{B}_{n,k},t\}} t \\ \text{s.t.} \quad \sqrt{\omega_{n,k}^{(j)}} \le t, \ k = 1, \dots, K, \ j = 1, \dots, E. \end{array}$$
(15)

B. Computation of Worst-Case Channel Matrices

In this subsection, we consider the worst-case analysis subproblem which involves the computation of the worst-case channel matrices that belong to the uncertainty region and maximize the user MSE.

Given the precoders and receive filters, we can express the user MSE $\omega_{n,k}$ by replacing \mathbf{H}_{ki} with $\widehat{\mathbf{H}}_{ki} + \Delta \mathbf{H}_{ki}$ in (7). If the worst-case analysis subproblem can be solved exactly, then the exact robust optimal solution is considered to be possible. However, in the present problem, the exact determination of the set $\{\Delta \mathbf{H}_{ki}\}_{i,k=1}^{K}$ that maximizes the user MSE cannot be obtained. To simplify the problem, it is assumed that the channel errors are much smaller than the channel estimates, and thus the second and higher orders in $\Delta \mathbf{H}_{ki}$ can be considered to be negligible. We can approximate the kth user's MSE as

$$\omega_{n,k} = \hat{\omega}_{n,k} - \varsigma_{n,k} + \operatorname{Tr}\left[\mathbf{I} + \sigma_n^2 \mathbf{B}_{n,k} \mathbf{B}_{n,k}^H\right], \quad (16)$$

where $\hat{\omega}_{n,k}$ is obtained by replacing \mathbf{H}_{ki} with $\hat{\mathbf{H}}_{ki}$ in (7), and

$$\varsigma_{n,k} = \operatorname{Tr}[\mathbf{U}_{n,k}^{H} \Delta \mathbf{H}_{kk}^{H} \mathbf{B}_{n,k}^{H} - \mathbf{B}_{n,k} \sum_{i=1}^{K} (\Delta \mathbf{H}_{ki} \mathbf{V}_{n,i} \widehat{\mathbf{H}}_{ki}^{H}) \mathbf{B}_{n,k}^{H}]$$
$$\mathbf{B}_{n,k} \Delta \mathbf{H}_{kk} \mathbf{U}_{n,k} - \mathbf{B}_{n,k} \sum_{i=1}^{K} (\widehat{\mathbf{H}}_{ki} \mathbf{V}_{n,i} \Delta \mathbf{H}_{ki}^{H}) \mathbf{B}_{n,k}^{H}].$$
(17)

As the last two terms in (16) are independent of CSI errors, the worst-case channel matrices maximizing the user MSE is obtained by solving

$$\min_{\{\Delta \mathbf{H}_{ki}: \| \Delta \mathbf{H}_{ki} \| \le \theta_{ki}\}_{i=1}^{K}} \varsigma_{n,k}.$$
(18)

Finally, by employing the Lagrangian multiplier method and applying the Karush-Kuhn-Tucker (KKT) conditions, we get

$$\Delta \mathbf{H}_{ki} = \frac{\mathbf{C}_{n,ki}}{\lambda_{n,ki}},\tag{19}$$

where $\mathbf{C}_{n,kk} = \mathbf{B}_{n,k}^{H} \mathbf{B}_{n,k} \widehat{\mathbf{H}}_{kk} \mathbf{V}_{n,k} - \mathbf{B}_{n,k}^{H} \mathbf{U}_{n,k}^{H}, \mathbf{C}_{n,ki} = \mathbf{B}_{n,k}^{H} \mathbf{B}_{n,k} \widehat{\mathbf{H}}_{ki} \mathbf{V}_{n,i}, i \neq k, \ \lambda_{n,ki} = \sqrt{\mathrm{Tr}(\mathbf{C}_{n,ki}\mathbf{C}_{n,ki}^{H})/\theta_{ki}^{2}}.$

It is worth noting that since the worst-case problem is based on the approximate expression for the MSE in (16), the resulting solutions of the problem are not guaranteed to be the worst channels even if the channel is in the assumed uncertainty region. However, such violations are very small as the effect of second and higher order terms of CSI error is considered to be insignificant compared to the estimated CSI.

C. Iterative Algorithm for the Robust Transceiver

We start the iterative algorithm for each transceiver group with the set \mathcal{H} of channel matrices, which initially contains only the imperfect CSI $\hat{\mathbf{H}}_{ki}$. The first subproblem deals with the solution of the iterative optimization problem in (9) and (15) for all elements of the set \mathcal{H} . The second subproblem involves the computation of the worst-case channels for the values of $\mathbf{F}_{n,k}$ and $\mathbf{B}_{n,k}$ computed in the previous subproblem. Here, to distinguish the iteration of $\mathbf{F}_{n,k}$ and $\mathbf{B}_{n,k}$ from the iteration of the two subproblems, the former iteration is denoted as inner iteration and the later as outer iteration. During the worst-case analysis subproblem in each outer iteration, the set \mathcal{H} may be expanded if the worst case MSE ω_{wor} satisfies: $\omega_{wor} - \max_{k=1,\dots,K, j=1,\dots,E} \omega_{n,k}^{(j)} \ge \underline{\omega}$. The outer iteration will be terminated if ω_{wor} satisfies: $\omega_{wor} - \max_{\substack{k=1,\dots,K, j=1,\dots,E}} \omega_{n,k}^{(j)} < \bar{\omega}$, where $\underline{\omega}$ and $\overline{\omega}$ are certain thresholds or a predefined iteration number. During the optimization subproblem, the maximum per user MSE is minimized for increasing number of worstcase channels, resulting in increasing robustness.

When the worst-case analysis subproblem has an exact solution, these outer iterations lead to the robust optimal solution [10]. For the scenario considered in this paper, the worst-case analysis is approximate, thus the iteration is not guaranteed to lead to the robust optimal solution. However, our simulations show that the proposed scheme is robust to CSI errors following the NBE model. Moreover, using warm-start [10] techniques in the outer iterations, i.e., use the previously computed precoder and receive filter to initialize the solution of the current outer iteration reduces the overall effort for our proposed scheme to converge.

The iterative optimization algorithm under NBE model is summarized in Table I for the proposed SP-based transmission scheme.

D. Selection Mechanism

In general, it can be very challenging to determine the optimal transceiver group. However the squared Euclidean distance appears to be both a simple and effective mechanism after a number of experimentations. Since the selection mechanism is only applied at the transmitter side, we employ a noiseless pre-estimated symbol vector instead of the exact received symbol vector for calculating the squared Euclidean

TABLE I Robust Min-Max algorithm for the SP-based scheme

1 for $n = 1, 2,, B^K$ %loop over transceiver groups	
2 Initialization: $\mathbf{F}_{n,k}, \ k = 1, 2, \dots, K$ and set \mathcal{H} .	
3 for the <i>i</i> th outer iteration	
4 Update the set \mathcal{H} of worst-case channel matrices	
5 for the <i>j</i> th inner iteration	
7 Compute the receive filter $\mathbf{B}_{n,k}$ according to (15).	
8 Compute the precoder $\mathbf{F}_{n,k}$ according to (9).	
9 end	
10 Compute the worst-case channel matrices according to (19), add t	0
set \mathcal{H} or stop the outer iteration according to certain thresholds.	
11 Repeat step 4-10 until the algorithm converges.	
12 Repeat step 2-11 until all transceiver groups are constructed	

distance. we use a block-based approach to conduct the selection mechanism, where the symbols transmitted by the kth user are grouped into contiguous, non-overlapping blocks of length L_k . By accumulating the values of the squared Euclidean distance in one block, we have the following selection rule

$$n_{opt} = \arg\min_{1 \le n \le B^K} \left\{ ||\mathbf{s}_j - \hat{\mathbf{s}}_j^{(n)}||^2 \right\},\tag{20}$$

where \mathbf{s}_j denotes the $\sum_{i=1}^{K} L_i \times 1$ transmit vector corresponding to the *j*th transmission block, which is given by $\mathbf{s}_j = [\mathbf{b}_{j,1}^T, \dots, \mathbf{b}_{j,K}^T]^T$. The $L_k \times 1$ vector $\mathbf{b}_{j,k} = [\mathbf{d}_{j,k,1}^T, \dots, \mathbf{d}_{j,k,L_i/S_i}^T]^T$ denotes the symbols transmitted by the *k*th user within a transmission block, where $\mathbf{d}_{j,k,m}$ denotes the data symbol emitted by the *k*th user at the *m*th transmit time of the *j*th block. In (20), $\hat{\mathbf{s}}_j^{(n)}$ denotes the $\sum_{i=1}^{K} L_i \times 1$ noiseless pre-estimated vector corresponding to the *n*th transceiver group, as given by $\hat{\mathbf{s}}_j^{(n)} = [\hat{\mathbf{b}}_{j,1}^{T(n)}, \dots, \hat{\mathbf{b}}_{j,K}^{T(n)}]^T$, where $\hat{\mathbf{b}}_{j,k}^{(n)} = [\tilde{\mathbf{d}}_{j,k,1}^{T(n)}, \dots, \tilde{\mathbf{d}}_{j,k,L_i/S_i}^{T(n)}]^T$ denotes the $L_k \times 1$ noiseless pre-estimated vector for the *k*th user of the *k* the user of the *k* the user of the *k*th user of *k* the *k* the user of *k* the

*j*th block based on the *n*th transceiver group, and the noiseless estimated vector $\tilde{\mathbf{d}}_{j,k,m}^{(n)}$ is given by

$$\tilde{\mathbf{d}}_{j,k,m}^{(n)} = \mathbf{B}_{n,k} (\sum_{i=1}^{K} \hat{\mathbf{H}}_{ki} \mathbf{T}_{n,i} \mathbf{F}_{n,i} \mathbf{d}_{j,i,m}).$$
(21)

The optimum transceiver group is chosen by minimizing the summation of the squared Euclidean distance values in one block, and it is updated once per block.

IV. SIMULATIONS

In this section, we illustrate the performance of the proposed SP-based robust transceiver schemes and compare them with existing transceiver algorithms for K-user MIMO interference channel through simulation. We assume that the system has K = 3 transmission pairs where each transmitter and receiver is equipped with 4 antennas $(M_k = N_k = 4, \forall k)$. Each transmitter sends two data streams $(S_k = 2, \forall k)$. The channel model used in the simulations is a quasi-static flat fading channel with Rayleigh distribution. The channels vary only in between consecutive transmission blocks, where each block contains $L_k = 128, \forall k$ symbols. The quadrature phase-shift keying (QPSK) is employed. In our simulations, each transmitter has the same transmit power constraint $(P_k = P_t = 4, \forall k)$.

Referring to (5) we assume that $\theta_{ki} = \theta, \forall i, k$. The SNR is defined as $\frac{P_t}{\sigma_n^2}$. For fair performance comparison, all the schemes apply the linear MMSE receive filters (expect the robust schemes) in the simulations.

Among the proposed techniques, we consider: (1) Robust Min-Max: the proposed robust transceiver with Min-Max precoder and Sum-MSE receiver. (2) Robust Min-Max Min-Max: the proposed robust transceiver with Min-Max precoder and Min-Max receiver. (3) Robust Min-Max Min-Max with SP: the proposed robust SP-based transceiver scheme with Min-Max optimization algorithm. (4) *B*-entry codebook: for the SP-based scheme, the codebook of each transmitter contains *B* permutation matrices.

Fig. 2 shows the average BER performance versus input SNR of the proposed SP-based transceiver scheme and the existing transceiver algorithms, namely, the explicit IA algorithm proposed in [1], the distributed IA and the distributed Max-SINR algorithm proposed in [2], the robust Sum-MSE minimization and Min-Max algorithm proposed in [3]. All the iterative algorithms employ 16 iterations and the right singular matrices are used for initialization. In particular, in Fig. 2 (a) we consider the case with perfect CSI while in Fig. 2 (b) the algorithms are implemented in the presence of CSI errors following the NBE model. The results of Fig. 2 (a) show that the best performance is achieved by the proposed SP-based Min-Max transceiver algorithm with the 2-entry codebook, followed by the Min-Max Min-Max algorithm, the Min-Max algorithm, the Sum-MSE minimization algorithm, the distributed Max-SINR algorithm, the distributed IA algorithm, and the explicit IA algorithm.



Fig. 2. Average BER performance versus SNR for the proposed SP-based transceiver scheme and the existing transceiver algorithms

In Fig. 2 (b), the components of the CSI error matrices $\Delta \mathbf{H}_{ki}$ are generated as independent and identically distributed complex Gaussian random variables with zero mean and variance $\sigma_e^2 = 0.002$ with norm bound $\theta_{ki} = \theta = 0.15, \forall i, k, \omega = 2.5 \times 10^{-3}, \overline{\omega} = 9 \times 10^{-4}$. The robust transceivers employ no more than 10 outer iterations. Since we use the previously computed precoders and receive filters to initialize the solution of the current outer iteration, we reduce the inner iteration number from 16 to 4 for all outer iterations except the first

one. From the results, we can see that the proposed SP-based robust scheme achieves the best performance. In particular, the proposed SP-based robust transceiver with 2-entry codebook³ can save over 5dB in comparison with non-robust algorithms, at the average BER level of 1×10^{-3} . The results show the superior ability of the proposed SP-based scheme to handle channel uncertainties and multiuser interference.

V. CONCLUSION

In this paper, we have presented a robust transceiver scheme based on switched preprocessing for K-user MIMO interference channel. For the construction of transceiver groups, we have developed a robust transceiver algorithm based on the worst-case concept. We have showed that the algorithm can be partitioned into an alternating sequence of two subproblems. We also have designed a selection mechanism to select the optimum transceiver group based on minimizing the squared Euclidean distance between the true transmit symbol vector and the noiseless pre-estimated receive vector. The simulation results have shown that the proposed robust transceiver scheme can significantly outperform existing transceiver algorithms in the presence of imperfect CSI.

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³ In the simulation, the 2-entry codebook consists of	$ \begin{array}{c} 1\\ 0\\ 0\\ \end{array} $	0 1 0	0 0 1	0 0 0	
and $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$	[0	0	0	1.]