A Low-Complexity Nonparametric STAP Detector

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***Abstract*—Phased array radars use space time adaptive processing (STAP) to detect targets in angle, range, and speed using an adaptive weight vector that depends mainly on the covariance matrix of the cell under test (CUT). This covariance matrix is estimated from the secondary cells surrounding the CUT under the assumption of homogeneous clutter and noise background. However, these secondary cells are often contaminated by multiple discrete interferers, targets or combination thereof, which degrade the estimation of the CUT's covariance matrix and, in turn, the detection performance. In this paper, we address the problem of detecting the nonhomogeneous secondary cells that need to be excluded from the adaptive weight calculation. We introduce a nonparametric and covariance-free alternative to the normalized adaptive matched filter (NAMF) test that does not need the tedious estimation process of the covariance matrix matrix of secondary cells nor prior knowledge about the interference distribution. Consequently, the computational complexity of the weight vector is reduced, which is of a great importance for real-time operation of radar systems. The equivalent robust performance of the proposed test compared to the NAMF test is demonstrated through simulations under different clutter scenarios and operation conditions.**

***Index Terms*—STAP, NAMF, GIP, covariance matrix estimation, K-distribution, robust statistics, depth function, NHD, SIRV, clutter.**

I. INTRODUCTION

Radar systems detect targets in range, speed, and azimuth angle. The range (fast time) domain represents the time samples received within the radar pulse repetition interval (PRI). To estimate the target Doppler (speed), coherent pulsed radars transmit a number of coherent pulses that together form a coherent pulse interval (CPI). In phased array radars, each element in the antenna array transmits the same number of coherent pulses. The space time adaptive processing (STAP) detector scans the signal in the range dimension and vectorizes the data matrix in the Doppler-angle domains. For each range cell (vector), STAP forms a weight vector depending on the covariance matrix of the cell under test (CUT). However, this covariance matrix is unknown in practice and it is estimated from the surrounding cells, known as the secondary or training cells [1].

The estimation of the CUT covariance matrix from the secondary cells is based on the assumption that the latter are homogeneous or (ideally) independent and identically distributed (iid). However, the homogeneity assumption is hardly met in real scenarios due to the presence of discrete scatterers, signal-like jammers [2], [3], multiple targets, or combinations of them. To tackle this problem, the nonhomogeneity detector (NHD) has been introduced to dentify nonhomogeneous secondary cells to be subsequently censored from the CUT covariance matrix estimation [4].

The work in [5] proposed a test known as the generalized inner product (GIP), based on the fact that the surrounding secondary cells can be considered homogeneous if they share the same covariance matrix up to a scalar. Later, the adaptive matched filter test (AMF) introduced in [6] has been applied in [7] to detect nonhomogeneous cells in Gaussian clutter environments. A normalized version of the AMF test (NAMF) was introduced in [8] and was extended to non-Gaussian clutter distributions in [9], where it has been shown to be the most robust NHD. Both of the GIP and the NAMF tests depend on estimating the matrix of the secondary cells. Based on the used covariance matrix estimator, the test is considered to be parametric or nonparametric. A covariance-free NHD that is equivalent to the GIP was introduced in [10] for Gaussian clutter. However, it is known that the GIP test is not robust especially in non-Gaussian clutter scenarios [9].

In this paper, we are concerned with the problem of detecting the nonhomogeneous secondary cells under a general clutter distribution model, including Gaussian and compound Gaussian distributions. We introduce a nonparametric and covariance-free NHD test that is equivalent (or better) in performance to the robust NAMF test, significantly reduces the computational burden. Indeed, the proposed NHD test does not require any prior knowledge of the distribution of the clutter nor estimating the covariance matrix (or its inverse) for secondary cells.

The paper is organized as follows: Section II provides the mathematical background about STAP and the clutter signal model, while Section III review the nonhomogeneity detection problem. The proposed nonparametric NHD is introduced in Section IV. The results of comparative performance evaluation for the proposed detector and the NAMF are presented and discussed in V. The work is concluded in Section VI.

II. SIGNAL MODEL

A. STAP Signal Model

Consider a pulsed Doppler radar system using a uniform linear array (ULA) antenna of N elements that are spaced at a distance d apart. The radar transmits from each antenna element M coherent pulses at a constant PRI T. The transmitted signal is assumed to be narrowband, that is, its bandwidth B satisfies $c/B \gg Nd$ where c is the speed of light [1]. The signal transmitted from each antenna element is expressed as

$$s(t) = \sum_{i=0}^{M-1} u(t - iT) \tag{1}$$

where $u(t)$ is the complex envelope of the transmitted signal. The time delay corresponding to the radar maximum unambiguous range is T, hence, the total number of range cells in a single PRI is

$$L = TB \tag{2}$$

Therefore, the CPI of a phased array doppler radar can be visualized as an $L \times M \times N$ data cube. For each range cell, the observed data defines an $M \times N$ matrix containing the received signal from each PRI and each antenna element. STAP vectorizes data matrix so obtained for each range cell, resulting in a vector $\mathbf{z} \in \mathbb{C}^J$, where $J = MN$. In this work, we consider a binary hypothesis testing whereby, for a given secondary cell,

$$H_1 : \mathbf{z} = \mathbf{x}_t + \mathbf{c} + \mathbf{n} \tag{3a}$$

$$H_0 : \mathbf{z} = \mathbf{c} + \mathbf{n} \tag{3b}$$

where H_0 and H_1 are the null and alternative hypotheses, $\mathbf{x}_t$ is the received signal from a target or a discrete interferer, $\mathbf{c}$ is the clutter signal, and $\mathbf{n}$ is the spatio-temporally additive white Gaussian noise. The received target signal is expressed as

$$\mathbf{x}_t = a\mathbf{s} \tag{4}$$

where a is the complex amplitude and $\mathbf{s}$ is the spatio-temporal target steering vector assumed to be known and normalized such that $\|\mathbf{s}\| = 1$.

For a given CUT, a STAP detector must decide whether it follows the hypothesis H_1 or H_0. To achieve this, covariance-based STAP detector estimates the covariance matrix of the CUT from from the data in surrounding, or secondary cells, and then applies a weight vector whose calculation depends on the estimated CUT covariance matrix. For the robust covariance matrix estimation, the secondary cells should be iid or homogeneous, a condition which is not satisfied in real scenarios.

B. Spherically Invariant Random Clutter Model

In this paper we are concerned with the coherent clutter model where both the in-phase and quadrature components of the received clutter signal are processed [11]. The clutter at each range cell can be modeled as a product of two components as

$$\mathbf{c} = v\mathbf{y} \tag{5}$$

where $\mathbf{y} \in \mathbb{C}^J$ has a complex Gaussian distribution with probability density function (PDF) $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ with zero mean and covariance $\mathbf{\Sigma} \in \mathbb{C}^{J\times J}$, and v is a positive scalar random variable with PDF $f(v)$, statistically independent from $\mathbf{y}$. In this case, the clutter vector $\mathbf{c}$ has the form of a spherical invariant random vector (SIRV) with a covariance matrix $\mathbf{R_c} = E(v^2)\mathbf{\Sigma}$.

Based on the SIRV model, different non-Gaussian (also called compound Gaussian) distributions of the clutter can be generated as developed in [12].

One of the most important clutter distributions is the K-distribution it provides a good fit to the envelope of the data acquired for different clutter types [13]. To express K-distribution in terms of the SIRV model, the PDF of the modulating scalar v is taken as [12]

$$f(v) = \frac{2\beta}{\Gamma(\alpha)2^\alpha} (\beta v)^{2\alpha-1} \exp\left(-\frac{\beta^2 v^2}{2}\right) \tag{6}$$

where α, β are the shape and scale parameters, respectively, and $\Gamma(\cdot)$ is the Gamma function, so that $E(v^2) = 2\alpha/\beta^2$. Detailed simulation procedures to generate coherent SIRV clutter with K-distribution and the desired covariance matrix are shown in [12].

III. Nonhomogeneity Detector (NHD)

The NHD aims to detect the nonhomogeneous cells from the secondary cells used in covariance matrix estimation of the CUT, so that the homogeneity assumption of the secondary cells is valid. Therefore, the NHD tests the homogeneity of each cell, represented by vector $\mathbf{z} \in \mathbb{C}^J$, with respect to all the secondary cells, represented by the matrix $\mathbf{Z} \in \mathbb{C}^{J\times(L-1)}$, where $(L - 1)$ is the number of secondary range cells after excluding the CUT. In effect, the NHD is a STAP detector where each secondary cell is considered in turn as a CUT, while the remaining cells are considered as its secondary cells.

For an arbitrary secondary cell $\mathbf{z}$, the GIP test amounts to computing the square of the Mahalanobis distance [4], [5], that is,

$$\Lambda_{\text{GIP}} = (\mathbf{z} - \boldsymbol{\mu})^H \mathbf{R}^{-1} (\mathbf{z} - \boldsymbol{\mu}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{7}$$

where $\mathbf{R} \in \mathbb{C}^{J\times J}$ and $\boldsymbol{\mu} \in \mathbb{C}^J$ are the true covariance matrix and the mean of the secondary cell $\mathbf{z}$, respectively. The true covariance matrix of $\mathbf{z}$ is unknown in practice and an estimate $\hat{\mathbf{R}}$ is used instead; the same applies for $\boldsymbol{\mu}$. In (7), H_0 is the null hypothesis (i.e., $\mathbf{z}$ is homogeneous with respect to the remaining secondary cells), H_1 is the alternative hypothesis, and η is a threshold that is determined based on the required probability of false alarm P_f. Unfortunately, the GIP test is not robust in non-Gaussian clutter environment, as reported in [9].

A more robust test is the NAMF test given by [8]:

$$\Lambda_{\text{NAMF}} = \frac{|\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{z}|^2}{(\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{s})(\mathbf{z}^H \hat{\mathbf{R}}^{-1} \mathbf{z})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{8}$$

The covariance matrix is estimated from the remaining $(L-2)$ secondary cells (after excluding the CUT and the secondary cell $\mathbf{z}$ being tested for homogeneity) and the test in (8) is performed for each secondary cell in $\mathbf{Z}$. One of the widely used estimators of the covariance matrix is the normalized sample covariance matrix (NSCM) expressed as [14]

$$\hat{\mathbf{R}}_{\text{NSCM}} = \frac{J}{L-2} \sum_{l=1}^{L-2} \frac{\mathbf{z}_l \mathbf{z}_l^H}{\mathbf{z}_l^H \mathbf{z}_l} \tag{9}$$

where $\mathbf{z}_l$ is the l-th secondary cell in the remaining $(L - 2)$ secondary cells. The estimator in (9) is used due to its low complexity, when compared to other iterative estimators [9].

For large-dimensional systems, to evade the high computational burden of estimating the covariance matrix (and its inverse) for each secondary cell, the covariance-free equivalent of the GIP NHD detector has been introduced in [10] based on the projection depth function. A depth function $D(\mathbf{z}, F)$ provides center-outward ordering for points $\mathbf{z} \in \mathbb{C}^J$ having a cumulative distribution function (CDF) F, where the median is the deepest point [15]. Based on this ordering, outliers can be detected by their distance from the center as compared to a threshold. A related function that serves a similar purpose is the outlying function defined as [16]

$$O(\mathbf{z}, F) = \frac{1}{D(\mathbf{z}, F)} - 1 \tag{10}$$

An outlying function of particular interest to our work is the projection based outlying function defined as

$$O(\mathbf{z}, F) = \sup_{\mathbf{u}\in\mathbb{C}^J, \|\mathbf{u}\|=1} \frac{|\mathbf{u}^H\mathbf{z} - \mu(F_\mathbf{u})|}{\sigma(F_\mathbf{u})} \tag{11}$$

where $F_\mathbf{u}$ is the CDF of $\mathbf{u}^H\mathbf{z}$, while $\mu(F_\mathbf{u})$ and $\sigma(F_\mathbf{u})$ are the univariate location and scale measures, respectively. The projection-based outlying function has a higher breakdown value in comparison to other types of depth functions, which means higher robustness against outliers [17]. The median (MED) and median absolute deviation (MAD) have been widely used as measures of location and scale in robust statistics to detect outliers [18], [19].

The projection outlying function in (11) can be used to form a covariance-free equivalent to the GIP test [10] that provides a better performance than the GIP for Gaussian disributed clutter. However, just as the GIP, it still exhibits an unstable detection and false alarm performance in case of compound Gaussian distributed clutter as indicated in [9] and the NAMF test is shown to be the most robust NHD. In the next section we propose a nonparametric and covariance-free test based on (11) that provides the robust performance of the NAMF test.

IV. The Proposed Nonparametric NHD

Starting from Cauchy-Schwartz inequality [20] it can be shown that

$$\sup_{\mathbf{u}\in\mathbb{C}^J, \|\mathbf{u}\|=1} \left(\frac{\mathbf{u}^H\mathbf{z} - \text{MED}(\mathbf{u}^H\mathbf{Z})}{\text{MAD}(\mathbf{u}^H\mathbf{z})}\right)^2 k_f^2 = \\ (\mathbf{z} - \text{MED}(\mathbf{u}^H\mathbf{Z}))^H\mathbf{R}^{-1}(\mathbf{z} - \text{MED}(\mathbf{u}^H\mathbf{Z})) \tag{12}$$

where k_f is a scalar that depends on the CDF $F_\mathbf{u}$ (or its sample version $\hat{F}_\mathbf{u}$). The test statistic in (12) is a covariance-free alternative to the GIP in (7). A projection depth function for the steering vector $\mathbf{s}$ can also be derived $\mathbf{s}$ in the same way, that is,

$$\sup_{\|\mathbf{u}\|=1} \left(\frac{|\mathbf{u}^H\mathbf{s}|}{\text{MAD}(\mathbf{u}^H\mathbf{Z})}\right)^2 k_f^2 = \mathbf{s}^H\mathbf{R}^{-1}\mathbf{s} \tag{13}$$

In the following, we establish the main steps in deriving the proposed test. The proofs are not shown due to the lack of space.

Proposition 1. *For a range cell $\mathbf{z} \in \mathbb{C}^J$ and a target steering vector $\mathbf{s} \in \mathbb{C}^J$, the test statistic $\left(\mathbf{s}^H\mathbf{z}\right)\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}$ has the same useful signal component at its output as the test statistic $\mathbf{s}^H\mathbf{R}^{-1}\mathbf{z}$, but it has a lower interference component.*

Put simply, Proposition 1 allows to replace the vector $\mathbf{z}$ in the numerator of (8) with its projection on the target steering vector $\mathbf{s}$. It is assumed that the outputs of both tests contain two components: the useful signal component, under assumption H_1, and the interference component introduced by clutter and noise. By this modification, the new test can be expressed in terms of the projection depth function as stated in Corollary 1 below.

Corollary 1. *Let $\mathbf{s}^H\mathbf{z}\mathbf{s}$ be the projection of the received signal vector $\mathbf{z}$ on the steering vector $\mathbf{s}$, while $\mathbf{Z}$, $\mathbf{R}$, and k_f are as defined before. Then*

$$\left|(\mathbf{s}^H\mathbf{z})\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}\right|^2 = k_f^4\left|(\mathbf{s}^H\mathbf{z}) \sup_{\|\mathbf{u}\|=1}\left(\frac{|\mathbf{u}^H\mathbf{s}|}{\text{MAD}(\mathbf{u}^H\mathbf{Z})}\right)^2\right|^2 \tag{14}$$

Therefore, the proposed nonparametric covariance-free NAMF test at each range cell is expressed as

$$\Lambda_{\text{new}} = \frac{\left|\mathbf{s}^H\mathbf{z}\right|^2 \sup_{\|\mathbf{u}\|=1}\left(\frac{|\mathbf{u}^H\mathbf{s}|}{\text{MAD}(\mathbf{u}^H\mathbf{Z})}\right)^2}{\sup_{\|\mathbf{u}\|=1}\left(\frac{|\mathbf{u}^H\mathbf{z} - \text{MED}(\mathbf{u}^H\mathbf{Z})|}{\text{MAD}(\mathbf{u}^H\mathbf{Z})}\right)^2} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{15}$$

We note, however, that the MAD of correlated data is not Fisher-consistent (the proof is omitted due to lack of space) and therefore, the sample distribution of the test in (15) is dependent on the used scale measure MAD [17]. This dependence degrades the false alarm performance of the test and, in turn, its detection performance as we will show in Section V. To address this issue, we propose decorrelating all the $(L-1)$ cells before applying (15) using the nonparametric Kendall's tau correlation estimator. For the case of a real-valued data matrix $\mathbf{X} \in \mathbb{R}^{J\times(L-1)}$, the latter is calculated as [21]

$$\hat{\psi}_{jk} = \frac{2}{(L-1)(L-2)} \sum_{i<i'} \text{sign}\left((x_{ji} - x_{ji'})(x_{ki} - x_{ki'})\right) \tag{16}$$

where $\hat{\psi}_{jk}$ is the (j,k)-th entry of the estimated correlation matrix $\hat{\mathbf{\Psi}} \in \mathbb{R}^{J\times J}$. Since, we seek to apply Kendall's tau correlation estimator to the complex secondary sample matrix $\mathbf{Z} \in \mathbb{C}^{J\times(L-1)}$, we use instead the augmented correlation matrix. Specifically [22],

$$\hat{\mathbf{\Psi}} = 2\hat{\mathbf{\Psi}}_{xx} - 2j\hat{\mathbf{\Psi}}_{xy}^T \tag{17}$$

where $\hat{\mathbf{\Psi}}_{xx}$ is the autocorrelation of the real part of $\mathbf{Z}$, $\hat{\mathbf{\Psi}}_{xy}$ is the cross correlation of the real and the imaginary parts of $\mathbf{Z} = \mathbf{X} + j\mathbf{Y}$, and $\mathbf{Z}$ is assumed to be a proper complex random matrix, i.e., $\hat{\mathbf{\Psi}}_{xy} = -\hat{\mathbf{\Psi}}_{xy}^T$. The decorrelated sample matrix is

$$\mathbf{Z}_{dec} = \hat{\mathbf{A}}^{-1}\mathbf{Z} \tag{18}$$

where $\hat{\mathbf{\Psi}} = \hat{\mathbf{A}}^H \hat{\mathbf{A}}$.

It should be emphasized that the decorrelation process is performed once for all the secondary cells and the correlation in (16) and (17) does not need to be estimated for each secondary cell as done in the NAMF or the GIP tests.

V. Performance Assessment

In this section, the performance of the proposed test is evaluated and compared to that of the NAMF test. The probability of false alarm $P_f = 0.01$ for all detection performance simulations. The average clutter-to-noise ratio (CNR) is assumed to be 20 dB. The output signal-to-interference-plus-noise ratio (SINR) is

$$\text{SINR} = |a|^2 \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} \tag{19}$$

In all simulations, we consider the two extreme cases $\alpha = 0.1$, that represents heavy-tailed spiky clutter, and $\alpha = 100$ which represents Gaussin clutter (it is observed that the K-distribution can be approximated as a Gaussian distribution while $\alpha > 4$ [23]). The scale parameter β of the texture random variable v is allowed to randomly and independently change from one range cell to another as suggested in [23]. In the following simulations, $\beta \sim \mathcal{U}]0, 1]$ where $\mathcal{U}$ denotes the uniform distribution. The sample size of the secondary cells is fixed to $L-1 = 64$ range cells (vectors) and the dimension of the range cell is either $J = 32$ or 16. The correlation matrix of the simulated clutter is

$$\mathbf{\Psi} = \left[0.9^{|i-j|}\right], \quad 1 \leq i, j \leq J \tag{20}$$

The distance between any two adjacent antenna elements $d = 0.5\lambda$ where λ is the operating radar wavelength. The target is assumed to be at azimuth angle $\theta_t = 35°$ and normalized Doppler frequency $\bar{f}_d = f_d T = 0.3$ where f_d is the target's Doppler frequency. However, the detection performance is independent of the direction or the speed of the target.

The implementation of the supremum in (15) is practically impossible since it implies using infinite number of projections. As shown in [20], the supremum is calculated by taking the maximum over a finite number of projections. The minimum number of projections Q for a stable detection performance can not be determined analytically; however, simulations show that it can be safely set to $Q = 4J$, which is the value used here. The projection vectors are generated randomly using a uniform distribution over a hypersphere with unit radius in J-dimensional space.

Fig. 1 shows the output of the two tests, NAMF and the proposed one for $\alpha = 0.1$ and $J = 16$. Four targets are injected in secondary range cells 6, 22, 40, and 55 with different power levels. The figure shows that the outputs of the two tests are similar, but with slightly different amplitudes. To evaluate the performance of the detector statistically, Fig. 2 and Fig. 3 compare the performance of the proposed detector with that of the NAMF test (using the covariance estimator in (9)) in Gaussian and K-distributed clutter. It is observed that the proposed test improves the detection performance due to its attenuated interference as indicated in Proposition 1.

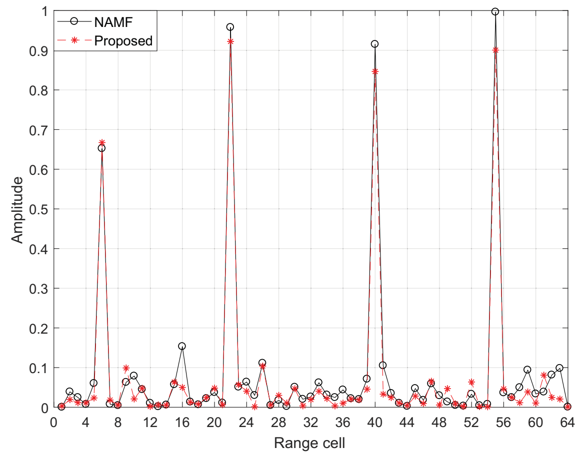

Fig. 1. NAMF and proposed tests in presence of 4 interference sources ($\alpha = 0.1$, $J = 16$)

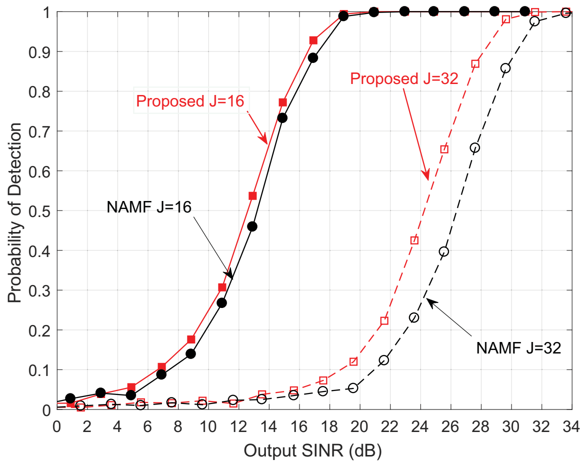

Fig. 2. Detection performance with Gaussian clutter ($\alpha = 100$)

Moreover, in both clutter distributions, one can observe that the proposed test shows a lower degradation (or sensitivity) than NAMF to an increase in the secondary cell dimension J given a fixed number of range cells L.

To show the effect of decorrelating the data before applying the proposed test, Fig. 4 demonstrates the performance of the proposed test with and without decorrelation using Kendall's tau estimator, defined in (16) and (17), in heavy-tailed K-distributed clutter. In Fig. 4, it is observed that decorrelating the secondary cells, which is proposed in (18) for a mathematical purpose, improves the detection performance of the proposed test.

VI. Conclusion

In this paper, we introduced a covariance-free and nonparametric form of the NAMF test that does not need prior knowledge of the clutter distribution nor estimating the covariance matrix of the CUT. The new detector is based on the projection depth function, a well-known tool in robust statistics which

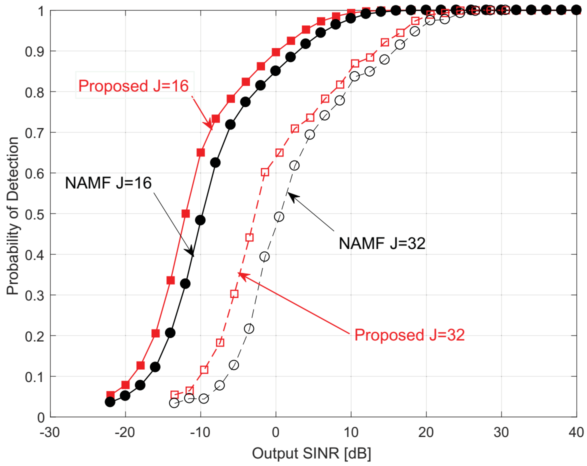

Fig. 3. Detection performance in K-distributed clutter ($\alpha = 0.1$)

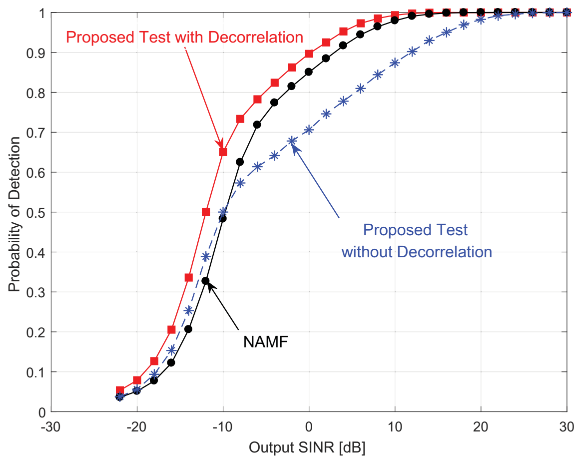

Fig. 4. Detection performance in K-distributed clutter ($\alpha = 0.1, J = 16$) with and without decorrelation

converts the multivariate problem into a univariate one using the projections of randomly generated vectors. Therefore, the new test does not need the computationally expensive process of estimating the covariance matrix (or its inverse) for each secondary cell. Using Monte Carlo simulations, the detection performance of the new test is shown to be equivalent to that of the NAMF test in highly-correlated Gaussian and compound Gaussian distributed clutter. To sum up, using tools from robust statistics, a low-complexity NHD is formed which preserves the robust performance of the NAMF test along with the nonparametric nature of these tools. These merits, make the proposed test an excellent candidate for real-time STAP radar applications.

References

[1] J. Guerci, *Space-time Adaptive Processing for Radar*. Artech House Radar Library, Artech House, 2014.

[2] A. A. Abouelfadl, F. M. Ahmed, and M. S. Soliman, "A novel noise-free jamming technique against LFM-PC search radar," in *Proc. of Int. Japan-Egypt Conf. on Electronics, Communications and Computers (JEC-ECC)*, pp. 127–130, May 2016.

[3] A. A. Abouelfadl, A. M. Samir, F. M. Ahmed, and A. H. Asseesy, "Performance analysis of LFM pulse compression radar under effect of convolution noise jamming," in *Proc. of National Radio Science Conference (NRSC)*, pp. 282–289, Feb 2016.

[4] W. L. Melvin and M. C. Wicks, "Improving practical space-time adaptive radar," in *Proc. of IEEE National Radar Conference*, pp. 48–53, May 1997.

[5] M. C. Wicks, W. L. Melvin, and P. Chen, "An efficient architecture for nonhomogeneity detection in space-time adaptive processing airborne early warning radar," in *Proc. of IEEE Radar Conference*, pp. 295–299, Oct. 1997.

[6] W.-S. Chen and I. S. Reed, "A new CFAR detection test for radar," *Digital Signal Process.*, vol. 1, no. 4, pp. 198–214, 1991.

[7] M. Rangaswamy, B. Himed, and J. H. Michels, "Performance analysis of the nonhomogeneity detector for STAP applications," in *Proc. of IEEE Radar Conference*, pp. 193–197, 2001.

[8] S. Kraut, L. L. Scharf, and L. T. McWhorter, "Adaptive subspace detectors," *IEEE Trans. Signal Process.*, vol. 49, pp. 1–16, Jan 2001.

[9] M. Rangaswamy, "Statistical analysis of the nonhomogeneity detector for non-gaussian interference backgrounds," *IEEE Trans. Signal Process.*, vol. 53, pp. 2101–2111, June 2005.

[10] G. N. Schoenig, M. L. Picciolo, and L. Mili, "Improved detection of strong nonhomogeneities for STAP via projection statistics," in *Proc. of IEEE International Radar Conference*, pp. 720–725, May 2005.

[11] A. Farina, A. Russo, and F. Scannapieco, "Radar detection in coherent Weibull clutter," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 35, pp. 893–895, Jun 1987.

[12] M. Rangaswamy, D. Weiner, and A. Ozturk, "Computer generation of correlated non-Gaussian radar clutter," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 31, pp. 106–116, Jan 1995.

[13] J. Jao, "Amplitude distribution of composite terrain radar clutter and the K-Distribution," *IEEE Trans. Antennas Propag.*, vol. 32, pp. 1049–1062, Oct. 1984.

[14] A. Younsi, A. M. Zoubir, and A. Ouldali, "Reiterative robust adaptive thresholding for nonhomogeneity detection in non-Gaussian noise," *EURASIP Journal on Advances in Signal Processing*, July 2008.

[15] Y. Zuo and R. Serfling, "General notions of statistical depth function," *Ann. Statist.*, vol. 28, pp. 461–482, April 2000.

[16] K. Mosler, *Depth Statistics*, pp. 17–34. Berlin, Heidelberg: Springer Berlin Heidelberg, 2013.

[17] Y. Zuo, "Projection-based depth functions and associated medians," *Ann. Statist.*, vol. 31, pp. 1460–1490, Oct. 2003.

[18] R. Wilcox, *Introduction to Robust Estimation and Hypothesis Testing*. Statistical Modeling and Decision Science, Elsevier Science, 2016.

[19] P. Huber, *Robust Statistics*. Wiley Series in Probability and Statistics, Wiley, 1981.

[20] S. Velasco-Forero and J. Angulo, "Random projection depth for multivariate mathematical morphology," *IEEE J. Sel. Topics Signal Process.*, vol. 6, pp. 753–763, Nov. 2012.

[21] J. Fan, H. Liu, and W. Wang, "Large covariance estimation through elliptical factor models," *ArXiv e-prints*, vol. 1507, July 2015.

[22] T. Adali, P. J. Schreier, and L. L. Scharf, "Complex-valued signal processing: The proper way to deal with impropriety," *IEEE Trans. Signal Process.*, vol. 59, pp. 5101–5125, Nov. 2011.

[23] J. H. Michels, M. Rangaswamy, and B. Himed, "Performance of parametric and covariance based STAP tests in compound-gaussian clutter," *Digital Signal Process.*, vol. 12, no. 2, pp. 307 – 328, 2002.