The 18th Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'07)

SUBSPACE-BASED BLIND CHANNEL ESTIMATION FOR MIMO-OFDM SYSTEMS: REDUCING THE TIME AVERAGING INTERVAL OF THE CORRELATION MATRIX

Chao-Cheng Tu
Department of ECE, McGill University
Montréal, Canada

Benoît Champagne
Department of ECE, McGill University
Montréal, Canada

ABSTRACT

Subspace-based blind channel estimation primarily exploits the orthogonality structure of the noise and signal subspaces by applying a signal-noise space decomposition to the correlation matrix of the received signal. In practice, the correlation matrix is unknown and must be estimated through time averaging over multiple symbol blocks. To this end, the wireless channel must be time-invariant over a sufficient time interval, which may pose a problem for wideband applications. In this paper, we propose a novel subspace-based blind channel estimation algorithm with a reduced time averaging interval, as obtained by exploiting the frequency correlation among adjacent OFDM subcarriers. We present simulation results of the proposed as well as referenced subspace-based methods, including Cyclic Prefix and Virtual Carriers approaches, and show that the proposed scheme is able to obtain a desired correlation matrix by reducing the number of the OFDM blocks for time averaging up to 85%.

I. INTRODUCTION

Blind channel estimation for MIMO-OFDM systems has received great attention and has become a very vital area of research in recent years. Under multichannel or multirate models, blind channel estimation by using Second Order Statistics (SOS) potentially has faster convergence rates than that by using higher order statistics [1]. Among these SOS blind approaches, subspace-based channel estimation is attractive since channel estimates can often be obtained in a closed-form from optimizing a quadratic cost function [2]. Without employing any precoding at the transmitter, a noise subspace-based method is proposed for OFDM systems by utilizing the redundancy introduced by the Cyclic Prefix (CP) [3], and it is further extended for MIMO-OFDM systems in [4]. Virtual Carriers (VC) are subcarriers that are set to zero without any information being transmitted. The presence of VC provides another useful resource that can be used for channel estimation. Such a scheme is proposed for OFDM systems [5], and it is further extended for MIMO-OFDM systems in [6].

The aforementioned approaches primarily exploit the separability of the noise and signal subspaces by applying the Eigenvalue Decomposition (EVD) on the correlation matrix of the received signal. In practice, the correlation matrix can only be estimated by averaging over multiple time samples, given the wireless channel is time-invariant during this averaging period. Since the quadratic cost function is constructed from the eigenvectors of the noise subspace obtained from the EVD, the accuracy of the eigenvectors obtained from the sampled correlation matrix dominates the performance of the estimation. Hence, the more time samples are averaged, the better the estimation performance is. However, how many samples are sufficient to obtain a sampled correlation matrix meeting a certain level of confidence? A basic rule of thumb is: the number of the time samples must be larger than or equal to the order of the correlation matrix so as to make it full rank or invertible [7]. For example, it would take at least 250 OFDM blocks (or 500 OFDM symbols) to achieve a normalized root mean square error (NRMSE) = 10^{-2} when we consider an IFFT size of 16 and SNR = 20dB for the subspace-based approaches [6]. If the size of IFFT were increased to 64, the required OFDM blocks would increase up to thousands for time averaging [8], making these subspace-based blind channel estimation approaches impractical.

In this paper, we propose a new approach for subspace-based blind channel estimation in MIMO-OFDM systems with reduced time averaging interval of the correlation matrix. This is achieved by exploiting the frequency correlation among adjacent OFDM subcarriers through the concept of subcarrier grouping [9, 10]. The proposed scheme requires only an upper bound of the channel order, and the ambiguity matrix, embedded in all the subspace-based estimation problems, can be solved by optimization. Simulation results are also presented to support our claims and designs.

The rest of this paper is organized as follows. In section II, system and channel models will be briefly described. In section III, a subspace-based blind channel estimation for MIMO-OFDM systems with reduced time averaging interval will be proposed. Simulation results of the proposed and the referenced methods will then be presented in section IV, and conclusions will be drawn in the final section.

The notation used in this paper is as follows: $E[x]$ denotes the expected value of the random variable x. $\otimes$ denotes the Kronecker product. $\mathrm{diag}(\mathbf{x})$ stands for a diagonal matrix with $\mathbf{x}$ on its main diagonal. $\bigoplus \sum_i \mathbf{X}_i = \mathrm{diag}(\mathbf{X}_i)$ denotes the direct sum of the matrices $\mathbf{X}_i$. $\mathrm{vec}(\cdot)$ is the Vec operator. $\mathrm{tr}(\mathbf{A})$ denotes the trace of the square matrix $\mathbf{A}$. $\|\mathbf{x}\|_1$ and $\|\mathbf{x}\|_2$ represents the L1 and L2 norm of the vector $\mathbf{x}$, respectively.

II. BACKGROUND

A. System Model

We propose a MIMO-OFDM system with N_T transmit and N_R receive antennas, employing N_C subcarriers. Let the mth OFDM symbol transmitted over the kth subcarrier

1-4244-1144-0/07/$25.00©2007 IEEE

be denoted as $\mathbf{x}^m[k] := \left[x_1^m[k] \ x_2^m[k] \ \cdots \ x_{N_T}^m[k]\right]^T$, where $x_q^m[k]$ is the symbol transmitted at the qth transmit antenna. Then the mth OFDM symbol transmitted over N_C subcarriers can be written as $\mathbf{x}^m := \left[\mathbf{x}^m[0]^T \ \mathbf{x}^m[1]^T \ \cdots \ \mathbf{x}^m[N_C-1]^T\right]^T$. Assume the incoming symbol streams span over N_F OFDM symbols at each epoch, then $\mathbf{x} = [\mathbf{x}^{1^T} \ \ \mathbf{x}^{2^T} \ \ \cdots \ \ \mathbf{x}^{N_F{}^T}]^T$ represents the complete set of transmitted symbols. At the receiver, let the mth received OFDM symbol over the kth subcarrier be denoted as $\mathbf{y}^m[k] := \left[y_1^m[k] \ y_2^m[k] \ \cdots \ y_{N_R}^m[k]\right]^T$, where $y_p^m[k]$ is the symbol received at the pth receive antenna. Then the mth OFDM symbol received over N_C subcarriers can be written as $\mathbf{y}^m := \left[\mathbf{y}^m[0]^T \ \mathbf{y}^m[1]^T \ \cdots \ \mathbf{y}^m[N_C-1]^T\right]^T$, and $\mathbf{y} = [\mathbf{y}^{1^T} \ \ \mathbf{y}^{2^T} \ \ \cdots \ \ \mathbf{y}^{N_F{}^T}]^T$ represents the complete set of received symbols. In the following, we assume that (1) the length of the OFDM cyclic prefix is greater than the maximum excess delay of the channel, (2) the channel is time-invariant over N_F OFDM symbols, and (3) $E[|x_q^m[k]|^2] = 1$.

B. Channel Model

We consider NLOS environments where scatterers are separated into N clusters. Let $h_{p,q,n}(t)$ be the complex channel gain at time t between the pth receive antenna and the qth transmit antenna, associated with the nth multipath $(1 \leq n \leq N)$ with a delay τ_n. Considering all the N multipaths, the response at time t between the pth receive antenna and the qth transmit antenna to an impulse applied at $t-\tau$ is given by

$$h_{p,q}(\tau,t) = \sum_{n=1}^{N} h_{p,q,n}(t)\delta(\tau-\tau_n). \tag{1}$$

The $N_R \times N_T$ complex-valued random matrix $\mathbf{H}_n(t)$ with $[\mathbf{H}_n(t)]_{p,q} := h_{p,q,n}(t)$ is zero mean with $E\left[vec\,\mathbf{H}_n(t)\ vec\,\mathbf{H}_{n'}(t)^H\right] = \mathbf{0}_{N_RN_T\times N_RN_T}$, $n \neq n'$, assuming the N clusters are uncorrelated. The equivalent frequency response from the pth receive antenna to the qth transmit antenna over the kth subcarrier at time t is then given as

$$h_{p,q}[k] = \mathcal{F}_\tau\{h_{p,q}(\tau,t)\} = \sum_{n=1}^{N} h_{p,q,n}(t)\exp\left(-j2\pi k\tau_n/TN_C\right) \tag{2}$$

for $k = 0,1,2\cdots,N_C-1$, where T is the sampling interval of OFDM symbols, and t is omitted in $h_{p,q}[k]$ for convenience. For MIMO-OFDM systems, the signal received at the pth receive antenna over the kth subcarrier and the mth OFDM symbol is given by

$$y_p^m[k] = \sqrt{\frac{E_s}{N_T}} \sum_{q=1}^{N_T} h_{p,q}[k] x_q^m[k] + n_p^m[k], \ p = 1,2,\cdots,N_R, \tag{3}$$

where E_s is the average energy evenly divided across the transmit antennas and allocated to the kth subcarrier, $n_p^m[k]$ represents the zero mean circularly symmetric complex Gaussian (ZMCSCG) noise at the pth receive antenna over the kth subcarrier and the mth OFDM symbol, and $h_{p,q}[k]$ represents the equivalent frequency response between the pth receive antenna and the qth transmit antenna, over the kth subcarrier and the mth OFDM symbol. Let $\mathbf{n}^m[k] := \left[n_1^m[k] \ n_2^m[k] \ \cdots \ n_{N_R}^m[k]\right]^T$, $\mathbf{n}^m := \left[\mathbf{n}^m[0]^T \ \mathbf{n}^m[1]^T \ \cdots \ \mathbf{n}^m[N_C-1]^T\right]^T$, and $\mathbf{n} = [\mathbf{n}^{1^T} \ \ \mathbf{n}^{2^T} \ \cdots \ \mathbf{n}^{N_F{}^T}]^T$, then the input-output relation of the MIMO-OFDM systems may be expressed by

$$\mathbf{y} = \sqrt{\frac{E_s}{N_T}}\mathcal{H}\cdot\mathbf{x} + \mathbf{n}, \tag{4}$$

where $\mathcal{H} := \mathbf{I}_{N_F} \otimes (\bigoplus\sum_{k=0}^{N_C-1}\mathbf{H}[k])$, with diagonal blocks defined as

$$\mathbf{H}[k] = \begin{bmatrix} h_{1,1}[k] & h_{1,2}[k] & \cdots & h_{1,N_T}[k] \\ h_{2,1}[k] & h_{2,2}[k] & \cdots & h_{2,N_T}[k] \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R,1}[k] & h_{N_R,2}[k] & \cdots & h_{N_R,N_T}[k] \end{bmatrix} \in \mathbb{C}^{N_R \times N_T}. \tag{5}$$

III. Proposed Approach

By assuming the noise $\mathbf{n}$ is independent of the transmitted symbols $\mathbf{x}$, the identification of the channel $\mathcal{H}$ can be realized by first applying the EVD on the autocorrelation matrix $\mathbf{R_y} = E\left[\mathbf{y}\mathbf{y}^H\right]$, then optimizing a quadratic cost function constructed from the eigenvectors of the noise subspace. An estimate of the correlation matrix can be obtained through time averaging by

$$\hat{\mathbf{R}}_{\mathbf{y}} = \frac{1}{T_{av}} \sum_{j=1}^{T_{av}} \mathbf{y}_{(j)}\mathbf{y}_{(j)}^H, \tag{6}$$

where $\mathbf{y}_{(j)} \in \mathbb{C}^{N_RN_CN_F}$ denotes the jth epoch of the received signal $\mathbf{y}$, consisting of N_F OFDM symbols, and T_{av} is the number of OFDM blocks (or simply the number of time samples). Although a pilot-based (not blind) subspace method in the frequency domain was proposed in [11, 12], to our knowledge, a subspace-based blind channel estimation constructed from (4) has never been considered; since there are $N_TN_RN_C \geq N_TN_RL$ unknown channel coefficients to be estimated, where L denotes the channel order. Nevertheless, the number of unknowns can be reduced by exploiting the frequency correlation among adjacent OFDM subcarriers with some loss in the estimation performance; in return, the order of the correlation matrix and hence the number of time samples required for time averaging can be reduced significantly. The details are given below:

Let the frequency span of P adjacent subcarriers reside inside the coherence bandwidth of the wireless channel and let $\mathcal{S} := \{0,1,\cdots,N_C-1\}$ be the index set of the N_C subcarriers. $\mathcal{S}$ is partitioned into P disjoint subsets (assuming $(N_C/P) \in \mathbb{Z}^+$) with each subset denoted as $\mathcal{S}_p := \{s_{p,1}, s_{p,2}, \cdots, s_{p,(N_C/P)}\}$, where $s_{p,i} := p-1+(i-1)P$,

$i = 1, 2, \cdots, (N_C/P)$ for $p = 1, 2, \cdots, P$. Note that $\mathcal{S}_1 \cup \mathcal{S}_2 \cup \cdots \cup \mathcal{S}_P = \mathcal{S}$, and $\mathcal{S}_i \cap \mathcal{S}_j = \emptyset$ for $i \neq j$, where $\emptyset$ denotes the empty set. Define $\mathbf{x}_p$=$[\mathbf{x}_p^{1^T} \mathbf{x}_p^{2^T} \cdots \mathbf{x}_p^{N_F{}^T}]^T$, $\mathbf{y}_p$=$[\mathbf{y}_p^{1^T} \mathbf{y}_p^{2^T} \cdots \mathbf{y}_p^{N_F{}^T}]^T$, $\mathbf{n}_p$=$[\mathbf{n}_p^{1^T} \mathbf{n}_p^{2^T} \cdots \mathbf{n}_p^{N_F{}^T}]^T$, where

$$\begin{aligned} \mathbf{x}_p^m &:= [\, \mathbf{x}^m[s_{p,1}]^T \, \mathbf{x}^m[s_{p,2}]^T \, \cdots \, \mathbf{x}^m[s_{p,(N_C/P)}]^T \,]^T, \\ \mathbf{y}_p^m &:= [\, \mathbf{y}^m[s_{p,1}]^T \, \mathbf{y}^m[s_{p,2}]^T \, \cdots \, \mathbf{y}^m[s_{p,(N_C/P)}]^T \,]^T, \\ \mathbf{n}_p^m &:= [\, \mathbf{n}^m[s_{p,1}]^T \, \mathbf{n}^m[s_{p,2}]^T \, \cdots \, \mathbf{n}^m[s_{p,(N_C/P)}]^T \,]^T. \end{aligned}$$

Then (4) can be re-written for the pth subset as

$$\mathbf{y}_p = \sqrt{\frac{E_s}{N_T}}\, \mathcal{H}_p \cdot \mathbf{x}_p + \mathbf{n}_p, \;\; p = 1, 2, \cdots, P, \tag{7}$$

where $\mathcal{H}_p := \mathbf{I}_{N_F} \otimes \left(\bigoplus \sum_{k \in \mathcal{S}_p} \mathcal{H}[k] \right)$ is assumed to be a "tall" matrix by choosing $N_R > N_T$. The identification of $\mathcal{H}_p$ is then based on the autocorrelation matrix $\mathbf{R}_{\mathbf{y}_p} = E[\mathbf{y}_p \mathbf{y}_p^H]$, and can be written as

$$\mathbf{R}_{\mathbf{y}_p} = \mathcal{H}_p \mathbf{R}_{\mathbf{x}_p} \mathcal{H}_p^H + \mathbf{R}_{\mathbf{n}_p}, \tag{8}$$

where $\mathbf{R}_{\mathbf{x}_p} = (E_s/N_T) \cdot E[\mathbf{x}_p \mathbf{x}_p^H]$ is assumed to be of full rank, and $\mathbf{R}_{\mathbf{n}_p} = E[\mathbf{n}_p \mathbf{n}_p^H] = \sigma_n^2 \mathbf{I}$. Since the channel coefficients of adjacent P subcarriers are strongly correlated, the wireless channel can be approximated by denoting $\bar{\mathcal{H}} = \mathcal{H}_1 = \mathcal{H}_2 = \cdots = \mathcal{H}_P$, and hence an estimate of the correlation matrix can be obtained by

$$\hat{\mathbf{R}}_{\mathbf{y}_p} = \frac{1}{PT_{av}} \sum_{j=1}^{T_{av}} \sum_{p=1}^{P} \mathbf{y}_{p\,(j)} \mathbf{y}_{p\,(j)}^H, \tag{9}$$

where $\mathbf{y}_{p\,(j)} \in \mathbb{C}^{N_R N_C N_F/P}$ denotes the jth epoch of the received signal $\mathbf{y}_p$. Therefore, the number of the time samples required can be significantly reduced since the order of the correlation matrix is reduced by a factor of P, and an averaging over P subsets, which is equivalent to the frequency averaging, is applied at each time averaging interval. The parameter P may be chosen to further reduce the time averaging interval with more loss in the estimation performance.

By applying the EVD on $\hat{\mathbf{R}}_{\mathbf{y}_p}$, (9) can be expressed by: $\hat{\mathbf{R}}_{\mathbf{y}_p} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H$, where $\mathbf{U}$ is a matrix whose columns are the orthonormal eigenvectors of $\hat{\mathbf{R}}_{\mathbf{y}_p}$, and can be partitioned as

$$\mathbf{U} = [\, \mathbf{U}_s \,|\, \mathbf{U}_n \,] = [\, \mathbf{u}_1 \cdots \mathbf{u}_{d_s} \,|\, \mathbf{u}_{d_s+1} \cdots \mathbf{u}_{d_s+d_n} \,]. \tag{10}$$

The signal subspace can be denoted as $\mathrm{span}\,(\mathbf{U}_s)$, while its orthogonal complement, the noise subspace, can be denoted as $\mathrm{span}\,(\mathbf{U}_n)$, with $d_s = \mathrm{rank}\,(\bar{\mathcal{H}}) = N_T N_C N_F/P$ and $d_n = (N_R - N_T) N_C N_F/P$. $\boldsymbol{\Lambda}$ is a diagonal matrix consisting of the corresponding eigenvalues of $\mathbf{U}$, and is denoted as $\boldsymbol{\Lambda} = \mathrm{diag}\,(\lambda_1, \lambda_2, \cdots, \lambda_{d_s+d_n})$ with $\lambda_{\max} = \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{d_s+d_n} = \lambda_{\min} \geq 0$. Since $\bar{\mathcal{H}}$ and $\mathbf{U}_s$ share the same range space and are orthogonal to the range space of $\mathbf{U}_n$, we can have the following orthogonality relationship

$$\mathbf{u}_j^H \bar{\mathcal{H}} = \mathbf{0}, \;\; j = d_s + 1, \cdots, d_s + d_n. \tag{11}$$

In order for $\bar{\mathcal{H}}$ to be identifiable, d_n is chosen so that $d_n \geq N_R$, and the matrix $\bar{\mathcal{H}}$ needs to be of full column rank, or $\mathrm{rank}(\bar{\mathcal{H}}) = N_T N_C N_F/P$.

Although $\bar{\mathcal{H}}$ can be solved from the set of homogeneous linear equations in (11); due to the limited time averaging interval, only an estimate of the noise subspace $\hat{\mathbf{U}}_n$ is available in practice. In this case, we can obtain the channel estimate $\hat{\bar{\mathcal{H}}}$ by minimizing a quadratic cost function given by

$$C(\bar{\mathcal{H}}) = \sum_{j=d_s+1}^{d_s+d_n} \|\hat{\mathbf{u}}_j^H \bar{\mathcal{H}}\|_2^2. \tag{12}$$

By partitioning $\hat{\mathbf{u}}_j$ into N_F equal segments as $\hat{\mathbf{u}}_j = [\hat{\mathbf{u}}_{j,1} \hat{\mathbf{u}}_{j,2} \cdots \hat{\mathbf{u}}_{j,N_F}]^T$, we can define a new matrix $\hat{\mathbf{V}}_j := [\hat{\mathbf{u}}_{j,1}^T \hat{\mathbf{u}}_{j,2}^T \cdots \hat{\mathbf{u}}_{j,N_F}^T]$, where $\hat{\mathbf{u}}_{j,i} \in \mathbb{C}^{1 \times N_R N_C/P}$ for $i = 1, 2, \cdots, N_F$. In addition, let us define $\bar{\mathcal{H}}' = [\mathbf{h}_1^{(p)} \; \mathbf{h}_2^{(p)} \; \cdots \; \mathbf{h}_{N_T}^{(p)}]$, where $\mathbf{h}_q^{(p)}$ is given as

$$\begin{aligned} \mathbf{h}_q^{(p)} = [&h_{1,q}[s_{p,1}] \; h_{2,q}[s_{p,1}] \; \cdots \; h_{N_R,q}[s_{p,1}] \; \cdots \\ &h_{1,q}[s_{p,(N_C/P)}] \; h_{2,q}[s_{p,(N_C/P)}] \; \cdots \; h_{N_R,q}[s_{p,(N_C/P)}]]^T \end{aligned}$$

for $q = 1, 2, \cdots, N_T$. Then minimizing the quadratic cost function in (12) is equivalent to minimizing

$$C(\bar{\mathcal{H}}') = \sum_{j=d_s+1}^{d_s+d_n} \|\bar{\mathcal{H}}'^T \hat{\mathbf{V}}_j^*\|_2^2 = \sum_{j=d_s+1}^{d_s+d_n} \|\bar{\mathcal{H}}'^T \hat{\mathbf{V}}_j^* \hat{\mathbf{V}}_j^T \bar{\mathcal{H}}'^*\|_2. \tag{13}$$

Furthermore, let $\boldsymbol{\Psi} := \sum_{j=d_s+1}^{d_s+d_n} \hat{\mathbf{V}}_j^* \hat{\mathbf{V}}_j^T \in \mathbb{C}^{(N_R N_C/P) \times (N_R N_C/P)}$, and the eigenvalues of $\boldsymbol{\Psi}$ be ordered as $\gamma_{\min} = \gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_{(N_R N_C/P)} = \gamma_{\max}$. Then from the Rayleigh-Ritz theory [13], we can have

$$\gamma_{\min} = \gamma_1 = \min_{\mathbf{w} \neq \mathbf{0}} \frac{\mathbf{w}^H \boldsymbol{\Psi} \mathbf{w}}{\mathbf{w}^H \mathbf{w}} = \min_{\mathbf{w}^H \mathbf{w} = 1} \mathbf{w}^H \boldsymbol{\Psi} \mathbf{w} \tag{14}$$

for any $\mathbf{w} \in \mathbb{C}^{(N_R N_C/P) \times 1}$. If r is a given integer with $1 \leq r \leq N_R N_C/P$, then

$$\gamma_1(\boldsymbol{\Psi}) + \cdots + \gamma_r(\boldsymbol{\Psi}) = \min_{\mathbf{Q}^H \mathbf{Q} = \mathbf{I}} \mathrm{tr}\,(\mathbf{Q}^H \boldsymbol{\Psi} \mathbf{Q}), \tag{15}$$

where $\mathbf{Q} \in \mathbb{C}^{(N_R N_C/P) \times r}$ is a matrix whose columns are the orthonormal eigenvectors corresponding to the r smallest eigenvalues of $\boldsymbol{\Psi}$.

In order for $\bar{\mathcal{H}}' \subseteq \mathrm{span}\,(\mathbf{Q}^*)$, $r = N_T N_C/P$ is chosen. Then minimizing (13) can be re-written by

$$\min C(\bar{\mathcal{H}}') = \min \sum_{q=1}^{N_T} \mathrm{tr}\left((\mathbf{h}_q^{(p)})^T \boldsymbol{\Psi} (\mathbf{h}_q^{(p)})^* \right), \tag{16}$$

and hence the channel estimate $\hat{\bar{\mathcal{H}}}'$ can be obtained from (15) as

$$\hat{\bar{\mathcal{H}}}' = \mathbf{Q}^* \mathbf{A}, \tag{17}$$

where $\mathbf{A} \in \mathbb{C}^{(N_T N_C/P) \times N_T}$ can be seen as an ambiguity matrix. Note that the ambiguity matrix $\mathbf{A}$ can be solved from (8) by employing the following optimization

$$\hat{\mathbf{A}} = \arg\min_{\mathbf{A}} \Big\| \text{vec}\Big(\mathcal{T}^{-1}(\mathbf{Q}^*\mathbf{A})\,\hat{\mathbf{R}}_{\mathbf{x}_p}\,(\mathcal{T}^{-1}(\mathbf{Q}^*\mathbf{A}))^H - (\hat{\mathbf{R}}_{\mathbf{y}_p} - \hat{\mathbf{R}}_{\mathbf{n}_p})\Big)\Big\|_2^2, \quad (18)$$

assuming $\hat{\mathbf{R}}_{\mathbf{x}_p}$ and $\hat{\mathbf{R}}_{\mathbf{n}_p}$ are known from additional estimations [1]. $\mathcal{T}^{-1}$ is the inverse of the matrix transformation $\mathcal{T}$, which is defined by $\mathcal{T} : \bar{\mathcal{H}} \rightarrow \bar{\mathcal{H}}'$.

IV. Simulation Results

Simulation results of the proposed as well as the referenced subspace-based methods, including CP and VC approaches for MIMO-OFDM systems, are presented in this section. We consider MIMO-OFDM systems with 2 transmit ($N_T = 2$) and 3 receive antennas ($N_R = 3$). The number of subcarriers used in the OFDM systems is 256 ($N_C = 256$). For each time epoch, the incoming symbol streams, which are independent and identically distributed (i.i.d) QPSK symbols, span over 2 OFDM symbols ($N_F = 2$). The complex-valued channel coefficients $h_{p,q,n}(t)$'s are modeled as i.i.d. and ZMCSCG random variables with $E\left[\Re[h_{p,q,n}(t)]^2\right] = E\left[\Im[h_{p,q,n}(t)]^2\right] = 1$. In addition, in order to include the subspace-based methods [4, 6] for comparisons, we consider a tapped-delay-line scenario with 2 clusters ($N = 2$) and excess delays defined by $\tau_n = (n-1)\cdot T$ for convenience. Under these circumstances, there are 10 subcarriers residing inside the coherence bandwidth ($P = 10$) if the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, while there are 100 subcarriers residing inside the coherence bandwidth ($P = 100$) if the definition is relaxed so that the frequency correlation function is above 0.5.

For the purpose of evaluating the estimation performance, the ambiguity matrices for all the methods are resolved by assuming the channel responses are known. The measures of the estimation performace considered are the root mean square error (RMSE) and the channel average bias (CAB) on a OFDM subcarrier basis, and are defined by

$$\text{RMSE}=\sqrt{\frac{1}{N_T N_R N_C N_m}\sum_{j=1}^{N_m}\sum_{p=1}^{P}\Big\|\text{vec}\Big(\mathcal{H}_{p\,(j)}-\mathcal{T}^{-1}[\hat{\bar{\mathcal{H}}}'_{(j)}]\Big)\Big\|_2^2}, \quad (19)$$

and

$$\text{CAB}=\frac{1}{N_T N_R N_C N_m}\sum_{j=1}^{N_m}\sum_{p=1}^{P}\Big\|\text{vec}\Big(\mathcal{H}_{p\,(j)} - \mathcal{T}^{-1}\,[\hat{\bar{\mathcal{H}}}'_{(j)}]\Big)\Big\|_1, \quad (20)$$

where $\hat{\bar{\mathcal{H}}}'_{(j)}$ and $\mathcal{H}_{p\,(j)}$ denotes the jth epoch of the channel estimate and channel response, respectively. We consider $N_m = 200$ Monte Carlo trials in our simulations.

Fig. 1 shows the RMSE measure of the proposed and referenced subspace-based methods, which is a function of the number of the OFDM blocks (each OFDM block is consitituted of 2 OFDM symbols) employed for obtaining a sampled correlation matrix when the SNR = 20dB. Fig. 2 shows the corresponding CAB measure. As expected, the estimation performance of all the methods is improved when the number of the OFDM blocks is increased for time averaging. For the referenced methods, we consider a fixed degree of freedom equals to 8 and 16, respectively. When comparing the same method with different degrees of freedom, either CP or VC method with a higher degree of freedom outperforms. In addition, with the same degree of freedom, the CP method outperforms the VC method since the dimension of the CP's eigenvectors is larger, imposing more constraints on the channel estimates, which also coincides with the results from [5, 6]. For the proposed methods, we consider $P = 1, 2, 4, \cdots, 128$. In general, we find that the number of the time samples required is reduced when P is large, while the estimation performance is also reduced due to the channel approximations over the P adjacent subcarriers. On the contrary, the number of the time samples required is increased when P is small, while the estimation performace is also improved. To achieve the best tradeoff, we note that P should be chosen to have the frequency spans of these P subcarriers residing around the coherence bandwidth, with the definition of the correlation function above 0.9.

Fig. 3 shows the RMSE measure of the proposed and referenced subspace-based methods, which is a function of the SNR when the number of the OFDM blocks employed for obtaining a sampled correlation matrix is fixed to 10 (i.e., 20 OFDM symbols). Fig. 4 shows the corresponding CAB measure. In this case, we can observe that the number of the time averaging interval, rather than the SNR, dominates the estimation performance. For the proposed methods, the best tradeoff is achieved when $P = 16$. We also note that the proposed methods outperform the referenced methods with any degrees of freedom. While the estimation performance is closely related to the ambiguity matrix, it should be mentioned that most of the ambiguity matrices are obtained with the aid of pilot symbols; however, the ambiguity matrix of our proposed methods can be resolved through an optimization with an additional complexity.

V. Conclusion

To the best of our knowledge, less than 16 OFDM subcarriers is considered in most simulations of the related works. Hence, the length of the time averaging interval did not become a serious issue. However, this issue arises when we consider a practical scenario such as the number of the OFDM subcarriers is up to 128, and the wireless channel is time-invariant for only a few OFDM symbols. Under these circumstances, the traditional subspace-based methods suffer from an extremely slow convergence rate, making them impractical. In this paper, we proposed a novel subspace-based estimation method with a faster convergence rate, mainly by exploiting the frequency correlation among adjacent subcarriers through the concept of subcarrier grouping. Within reasonable time averaging interval, the simulation results were shown to support the proposed methods by achieving both a higher convergence rate and a better estimation accuracy. We also proposed a method to obtain the ambiguity matrix without inserting any pilot symbols at the transmitter side.

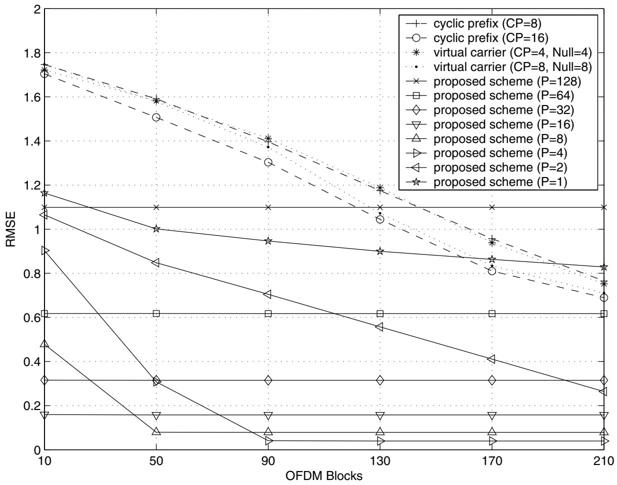

Figure 1: RMSE versus T_{av} (SNR=20dB).

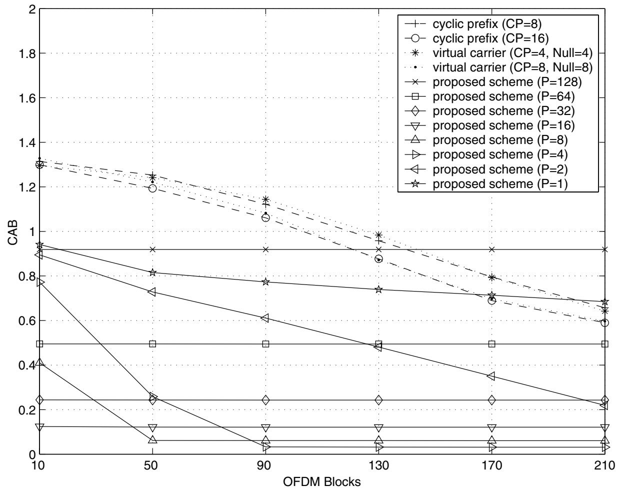

Figure 2: CAB versus T_{av} (SNR=20dB).

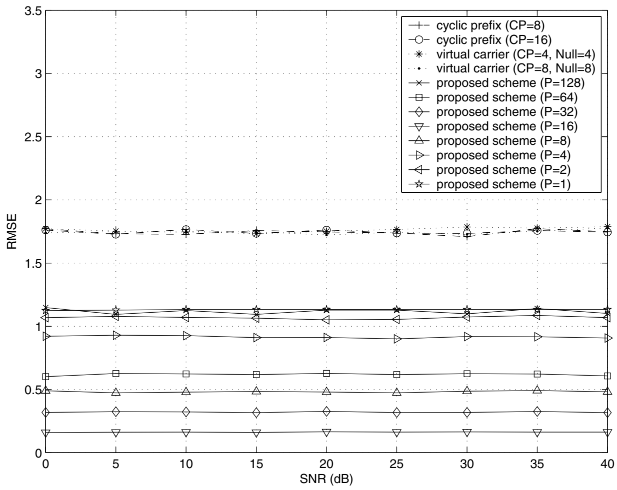

Figure 3: RMSE versus SNR (T_{av} = 10).

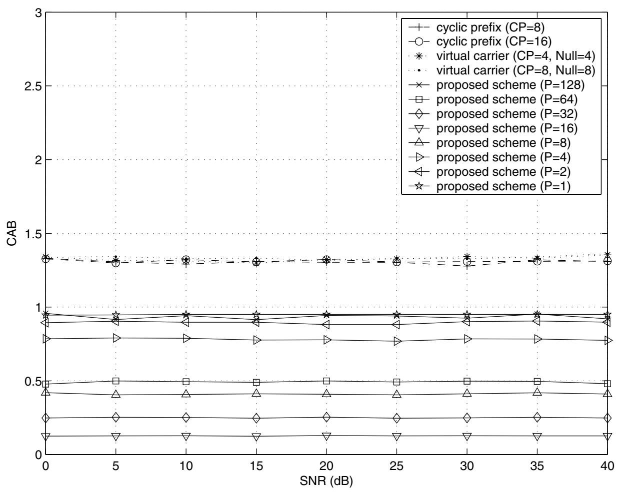

Figure 4: CAB versus SNR (T_{av} = 10).

REFERENCES

[1] L. Tong and S. Perreau, "Multichannel blind identification: from subspace to maximum likelihood methods," *IEEE Proc.*, vol. 86, no. 10, pp. 1951–1968, Oct. 1998.

[2] E. Moulines, P. Duhamel, J.-F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, no. 2, pp. 516–525, Feb. 1995.

[3] X. Cai and A. N. Akansu, "A subspace method for blind channel identification in OFDM systems," in *Proc. IEEE ICC*, vol. 2, no. 18-22, June 2000, pp. 929–933.

[4] W. Bai, C. He, L.-G. Jiang, and H.-W. Zhu, "Blind channel estimation in MIMO-OFDM systems," in *Proc. IEEE Globecom*, vol. 1, no. 17-21, Nov. 2002, pp. 317–321.

[5] C. Li and S. Roy, "Subspace-based blind channel estimation for OFDM by exploiting virtual carriers," *IEEE Trans. Wireless Commun.*, vol. 2, no. 1, pp. 141–150, Jan. 2003.

[6] C. Shin and E. J. Powers, "Blind channel estimation for MIMO-OFDM systems using virtual carriers," in *Proc. IEEE Globecom*, vol. 4, no. 29, Nov. 2004, pp. 2465–2469.

[7] X. Mestre and M. A. Lagunas, "Finite sample size effect on minimum variance beamformers: Optimum diagonal loading factor for large arrays," *IEEE Trans. Signal Processing*, vol. 54, no. 1, pp. 69–82, Jan. 2006.

[8] C. Shin, R. W. Heath, Jr., and E. J. Powers, "Blind channel estimation for MIMO-OFDM systems," *IEEE Trans. Veh. Technol.*, vol. 56, no. 2, pp. 670–685, Mar. 2007.

[9] Z. Liu, Y. Xin, and G. B. Giannakis, "Space-time-frequency coded OFDM over frequency-selective fading channels," *IEEE Trans. Signal Processing*, vol. 50, no. 10, pp. 2465–2476, Oct. 2002.

[10] A. F. Molisch, M. Z. Win, and J. H. Winters, "Space-time-frequency (STF) coding for MIMO-OFDM systems," *IEEE Commun. Lett.*, vol. 6, no. 9, pp. 370–372, Sept. 2002.

[11] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Pilot-based channel estimation for OFDM systems by tracking the delay subspace," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 315–325, Jan. 2004.

[12] M. Cicerone, O. Simeone, and U. Spagnolini, "Channel estimation for MIMO-OFDM systems by using modal analysis/filtering," *IEEE Trans. Commun.*, vol. 54, no. 11, pp. 2062–2074, Nov. 2006.

[13] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.