Low-Complexity Adaptive Transceiver Techniques for K-Pair MIMO Interference Channels

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***Abstract*—In this work, we propose a low-complexity adaptive transceiver algorithm for the K-pair multiple-input multiple-output (MIMO) interference channels. The proposed algorithm is based on the joint optimization of transmit and receive vectors using the constrained constant modulus (CCM) criterion. We firstly derive CCM-based expressions for the transmit and receiver vectors. Then, we develop recursive least-squares (RLS) adaptive algorithms for their efficient implementation. Unlike earlier block-based transceivers for MIMO interference channels, the proposed algorithms have low computational complexity and can track the time-varying channels and interference as changes occur in the surrounding wireless environment. In particular, simulation results show that the proposed adaptive algorithms achieve the performance of the Sum-MSE algorithm at a much reduced complexity.**[1]

***Index Terms*- Interference alignment, MIMO interference channel, adaptive filtering, transceiver designs, constant modulus criterion.**

I. INTRODUCTION

Recently, interference alignment (IA) techniques for MIMO interference channels have been studied in [1], [2] as a means to achieve the theoretical capacity by restricting (or ”aligning”) all interference at every receiver to approximately half of the received signal space. This can be achieved through the use of specially structured signals or by a careful design of the transmit precoders. As such, IA provides an alternative framework for the joint transmitter-receiver optimization in multi-user wireless communications. It is particularly well suited for applications to coordinated multi-point transmissions (CoMP), as it can improve detection performance for a group of users at the cell edge by reducing inter-cell interference and thereby improving data rates. Further gains in performance are possible with the use of multiple antennas at the transmitters or receivers.

Considering that the potential gains of IA are only attainable in the limit of very high SNR, several researchers have recently turned their attention to the joint transmitter and receiver design by relaxing the perfect alignment constraint. Here, the aim is to achieve better capacity in the low to intermediate SNR regimes, which better represent the practical conditions of operation away from the base stations in cellular systems. In [3], novel transceiver schemes for the MIMO interference channel are proposed based on the mean square error (MSE) criterion, especially by minimizing the Sum-MSE and the maximum per user MSE. In [4], the authors propose three generalizations of IA for the MIMO interference channel, i.e. the minimum interference-plus-noise leakage (INL), the maximum signal-to-interference-plus-noise ratio (SINR) and the joint minimum MSE design algorithms, which are simulated alongside existing methods in regimes previously not considered in the literature. A gradient descent method based on maximizing the weighted sum rate is investigated in [5], where the aim is to identify a local optimal solution iteratively. In [6], a robust transceiver design for K-pair quasi-static MIMO interference channel is proposed. The authors consider a transceiver design that enforces robustness against imperfect channel state information (CSI) as well as fair performance among the users in the MIMO interference channel.

However, a short-coming of these conventional optimal approaches is that they are block-based and do not have a direct on-line implementation. That is, every time the channel estimates change, the transceiver matrices have to be recomputed from scratch via a costly iterative approach. In non-stationary environments, as in mobile radio applications, the high complexity associated to these computations render these methods impractical.

In this paper, our aim is to design robust low-complexity transceivers with good performance, for application to time-varying channels. Specifically, we develop new blind adaptive algorithms for cooperative design of the time-varying transceiver matrices in MIMO interference channels. The new algorithms are obtained based on the joint optimization (JO) of transmit and receive vectors using the constrained constant modulus (CCM) criterion [7], [8]. The final adaptive form of the new algorithms is obtained through the application of a recursive least square (RLS) approach. Compared to the prior proposed block-based transceivers for MIMO interference channels, the proposed algorithms have lower computational complexity and can track the time-varying channels and interference as changes occur in the surrounding wireless environment. Numerical results show that the proposed adaptive algorithms achieve the performance of the Sum-MSE algorithm at a significantly reduced complexity and are therefore well-suited for on-line applications in non-stationary wireless environments.

The following notations are used throughout: Superscripts $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose of their matrix argument. $\mathbf{I}$ denotes an identity matrix of appropriate dimension. $||.||^2$ denotes the Euclidean norm of its vector argument.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a generic MIMO interference channel with K multi-antenna transmitter-receiver pairs, as illustrated in Fig.

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1. For simplicity, we assume that each transmitter is equipped with the same number N_t of antennas, and that each receiver is equipped with N_r antennas. The k-th transmitter aims to send symbols to its paired receiver, where $k \in \{1, \ldots, K\}$. The k-th receiver aims to decode the symbols from its corresponding transmitter, and treats the symbols from the other transmitters as interference. The transmitters and receivers' antennas are linked by multiple wireless channels, which we assume to undergo frequency flat (e.g. Rayleigh), slow fading.

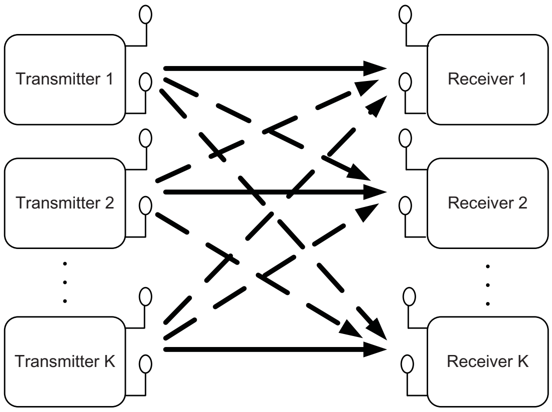

Fig. 1. MIMO interference channel

Focussing on the kth transmit-receive pair, the receive data vector $\mathbf{r}_k(i) \in \mathbb{C}^{N_r \times 1}$ at the i-th discrete-time instant can be expressed in the following form:

$$\begin{aligned}\mathbf{r}_k(i) &= \sum_{l=1}^{K} \mathbf{H}_{k,l}(i)\mathbf{f}_l(i)b_l(i) + \mathbf{n}_k(i) \\ &= \mathbf{H}_{k,k}(i)\mathbf{f}_k(i)b_k(i) + \overbrace{\sum_{l \neq k} \mathbf{H}_{k,l}(i)\mathbf{f}_l(i)b_l(i)}^{MUI} + \mathbf{n}_k(i)\end{aligned} \tag{1}$$

where $b_l(i)$ is the data symbol emitted by the l-th transmitter, $\mathbf{f}_l(i)$ denotes the $N_t \times 1$ spatial transmit filter applied to the transmitted symbol $b_l(i)$, $\mathbf{H}_{k,l}(i)$ denotes the $N_r \times N_t$ MIMO channel matrix between the l-th transmitter and the k-th receiver, and $\mathbf{n}_k(i)$ is an $N_r \times 1$ additive noise vector. In this work, we consider binary phase shift keying (BPSK) modulation where the data symbols from the K users, as represented by the indexed set $\{b_l(i)\}$, are assumed to be independent and identically distributed random variables, taking values $\{\pm 1\}$; however, extensions to other types of modulation are possible. The noise vector sequences $\mathbf{n}_k(i)$ are assumed to be temporally white, independent over the receiver index k, with zero-mean and covariance matrices $\mathbf{R}_{\mathbf{n}_k} = \sigma_k^2 \mathbf{I}$. The transmitted symbols $\{b_l(i)\}$ are independent of the noise vector $\mathbf{n}_k(i)$ at any receiver. Since the k-th transmitter aims to send symbols to its corresponding receiver, the other transmitted signals to the k-th receiver are treated as interference. Here, the acronyms MUI refer to multi-user interference.

Each receiver is equipped with a linear combiner followed by a threshold detector. Let $\mathbf{w}_k(i)$ denote the $N_r \times 1$ vector of complex weights applied to $\mathbf{r}_k(i)$ at the i-th time instant to form the linear (soft) estimate of $b_k(i)$. Then, in the case of BPSK modulation, threshold detection takes the form

$$\hat{b}_k(i) = \text{sign}\{\Re[\mathbf{w}_k^H(i)\mathbf{r}_k(i)]\} \tag{2}$$

where $\Re\{.\}$ denotes the real part of its argument.

The main problem of transceiver design for the above MIMO interference channel is to derive the precoder vectors $\{\mathbf{f}_k(i)\}$ and receiver weight vectors $\{\mathbf{w}_k(i)\}$ that optimize a predefined performance criterion. To this end, it is generally assumed that the transmitters and receivers have access to the underlying CSI comprised of the set of matrices $\{\mathbf{H}_{k,l}\}$. In practice, this information can be obtained with sufficient accuracy, for example through the use of blind channel estimation algorithms [9] or the use of sparse pilot tones at nearby frequency along with quantization and feedback. In our derivations below, we assume perfect knowledge of the CSI, while the effect of erroneous or inaccurate channel matrices is further investigated through simulations in Section VI. In this work, we seek to develop blind adaptive transceiver design algorithms that do not require the use of filter-training symbols for the derivation of the precoder and receiver weight vectors.

III. Optimal Solutions

We first formulate a global objective function and optimization criterion for the joint design of the receiver filters and transmit precoders for the multi-user MIMO interference channel problem. We then investigate certain properties of these optimal solutions that will serve as basis to derive efficient adaptive algorithms in the next section. To simplify the presentation, we temporarily drop the time dependence, i.e. index i.

A. Design Criterion

Define $\mathbf{F} = [\mathbf{f}_1, \ldots, \mathbf{f}_K]$ of size $N_t \times K$, $\mathbf{W} = [\mathbf{w}_1, \ldots, \mathbf{w}_K]$ of size $N_r \times K$. For BPSK modulation, a constant modulus objective function that captures the restoration errors of all the symbols exchanged between the transceiver pairs comprising the MIMO interference channel can be formulated as follows:

$$J_{CM}(\mathbf{F}, \mathbf{W}) = \sum_{k=1}^{K} J_k(\mathbf{F}, \mathbf{w}_k) \tag{3}$$

where we define

$$J_k(\mathbf{F}, \mathbf{w}_k) = E[(|\mathbf{w}_k^H \mathbf{r}_k(i)|^2 - 1)^2]. \tag{4}$$

In turn, the CCM problem can be formulated as follows:

$$\arg\min_{\mathbf{F},\mathbf{W}} J_{CM}(\mathbf{F}, \mathbf{W}) \tag{5}$$

subject to

$$\mathbf{w}_k^H \mathbf{H}_{k,k} \mathbf{f}_k = \gamma, \quad \forall k \in \{1, \ldots, K\}, \tag{6}$$

and $\gamma > 0$ is a constant. In practice, it is convenient to constrain the transmitted power by the K users, which can be expressed mathematically as

$$\sum_{k=1}^{K} \|\mathbf{f}_k\|^2 = \mathcal{P} \tag{7}$$

where $\mathcal{P}$ denotes the total transmitted power.

B. Optimal Receive Filters

In order to investigate the solutions of this constrained optimization problem, we transform it into an unconstrained one by employing the method of Lagrange multipliers [10]. The Lagrangian function for (3)-(7) is given by:

$$\begin{aligned}\mathcal{L}(\mathbf{F},\mathbf{W},\boldsymbol{\lambda}) = \sum_{k=1}^{K} & J_k(\mathbf{F},\mathbf{w}_k)+ \\ & \sum_{k=1}^{K}\big[\lambda_k(\mathbf{w}_k^H\mathbf{H}_{k,k}\mathbf{f}_k-\gamma) \\ & +\lambda_k^*(\mathbf{f}_k^H\mathbf{H}_{k,k}^H\mathbf{w}_k-\gamma)\big] \\ & +\lambda\big(\sum_{k=1}^{K}\mathbf{f}_k^H\mathbf{f}_k-\mathcal{P}\big)\end{aligned} \tag{8}$$

where $\boldsymbol{\lambda}=[\lambda_1,...,\lambda_K]$ denotes the ordered set of Lagrange multipliers for the distortionless constraint and λ is the Lagrange multiplier associated to the power constraint. Taking the gradient of $\mathcal{L}(\mathbf{F},\mathbf{W},\boldsymbol{\lambda})$ with respect to $\mathbf{w}_k^*$ and setting the result equal to the zero vector, i.e. $\nabla_{\mathbf{w}_k^*}\mathcal{L}(\mathbf{F},\mathbf{W},\boldsymbol{\lambda})=\mathbf{0}$, we find

$$\begin{aligned}\mathbf{w}_k &= E[|z_k(i)|^2\mathbf{r}_k(i)\mathbf{r}_k^H(i)]^{-1} \\ &\quad\times\Big(E[z_k^*(i)\mathbf{r}_k(i)]-\frac{1}{2}\lambda_k\mathbf{H}_{k,k}\mathbf{f}_k\Big) \\ &=\mathbf{Q}_k^{-1}\Big(\mathbf{d}_k-\frac{1}{2}\lambda_k\mathbf{H}_{k,k}\mathbf{f}_k\Big)\end{aligned} \tag{9}$$

where $z_k(i)=\mathbf{w}_k^H\mathbf{r}_k(i)$ denotes the filtered data vector for the k-th receiver, and we define

$$\mathbf{Q}_k = E[|z_k(i)|^2\mathbf{r}_k(i)\mathbf{r}_k^H(i)] \tag{10}$$

$$\mathbf{d}_k = E[z_k^*(i)\mathbf{r}_k(i)]. \tag{11}$$

The Lagrange multiplier can be evaluated by employing the constraint in (6). Specifically, upon right multiplication of (9) by $\mathbf{f}_k^H\mathbf{H}_{k,k}^H$, we find

$$\lambda_k = \frac{2(\mathbf{f}_k^H\mathbf{H}_{k,k}^H\mathbf{Q}_k^{-1}\mathbf{d}_k-\gamma)}{\mathbf{f}_k^H\mathbf{H}_{k,k}^H\mathbf{Q}_k^{-1}\mathbf{H}_{k,k}\mathbf{f}_k}. \tag{12}$$

Finally, upon reinserting this expression into (9), we obtain

$$\mathbf{w}_k=\mathbf{Q}_k^{-1}\big(\mathbf{d}_k-\alpha_k^{-1}\mathbf{c}_k\big) \tag{13}$$

where

$$\mathbf{c}_k=\big(\mathbf{f}_k^H\mathbf{H}_{k,k}^H\mathbf{Q}_k^{-1}\mathbf{d}_k-\gamma\big)\mathbf{H}_{k,k}\mathbf{f}_k \tag{14}$$

$$\alpha_k=\mathbf{f}_k^H\mathbf{H}_{k,k}^H\mathbf{Q}_k^{-1}\mathbf{H}_{k,k}\mathbf{f}_k. \tag{15}$$

We note that this solution is implicit, as opposed to explicit, since matrix $\mathbf{Q}_k$ and vector $\mathbf{d}_k$ depend on $\mathbf{w}_k$ as well as on $\mathbf{F}$ through the received vector $\mathbf{r}_k(i)$. Nevertheless, these expressions will serve as basis in the derivation of an adaptive solution in the next section.

C. Optimal Transmit Precoders

Next, taking the gradient of $\mathcal{L}(\mathbf{F},\mathbf{W},\boldsymbol{\lambda})$ with respect to $\mathbf{f}_k^*$, we first obtain

$$\begin{aligned}\nabla_{\mathbf{f}_k^*}\mathcal{L}(\mathbf{F},\mathbf{W},\boldsymbol{\lambda}) = 2\sum_{k'=1}^{K} & E[(|z_{k'}(i)|^2-1) \\ & \times(\nabla_{\mathbf{f}_k^*}|z_{k'}(i)|^2)]+\lambda_k^*\mathbf{H}_{k,k}^H\mathbf{w}_k \\ & +\lambda\mathbf{f}_k.\end{aligned} \tag{16}$$

We shall assume that under optimal operating conditions, the CM restoration error $|z_k(i)|^2-1$ is independent from the observed data $\mathbf{r}_k(i)$. Accordingly, the expectation term in (16) can be written as the product

$$E[(|z_{k'}(i)|^2-1)]E[\nabla_{\mathbf{f}_k^*}|z_{k'}(i)|^2]. \tag{17}$$

Using the definition of $z_k(i)$, the expression of $\mathbf{r}_k(i)$ in (1), and the statistical properties of the data symbols $b_k(i)$ and the noise $\mathbf{n}_k(i)$, we can show that

$$E[\nabla_{\mathbf{f}_k^*}|z_{k'}(i)|^2]=(\mathbf{w}_{k'}^H\mathbf{H}_{k',k}\mathbf{f}_k)\mathbf{H}_{k',k}^H\mathbf{w}_{k'} \tag{18}$$

Making use of (17) and (18) in (16), we obtain the desired expression for the gradient of the Lagrangian with respect to $\mathbf{f}_k^*$:

$$\begin{aligned}\nabla_{\mathbf{f}_k^*}\mathcal{L}(\mathbf{F},\mathbf{W},\boldsymbol{\lambda}) &= 2\sum_{k'=1}^{K}E[(|z_{k'}(i)|^2-1)] \\ &\quad\times(\mathbf{w}_{k'}^H\mathbf{H}_{k',k}\mathbf{f}_k)\mathbf{H}_{k',k}^H\mathbf{w}_{k'} \\ &\quad+\lambda_k^*\mathbf{H}_{k,k}^H\mathbf{w}_k+\lambda\mathbf{f}_k \\ &=2E[(|z_k(i)|^2-1)] \\ &\quad\times(\mathbf{w}_k^H\mathbf{H}_{k,k}\mathbf{f}_k)\mathbf{H}_{k,k}^H\mathbf{w}_k \\ &\quad+2\boldsymbol{\phi}(\mathbf{F},\mathbf{W})+\lambda_k^*\mathbf{H}_{k,k}^H\mathbf{w}_k+\lambda\mathbf{f}_k\end{aligned} \tag{19}$$

where we define

$$\boldsymbol{\phi}(\mathbf{F},\mathbf{W})=\sum_{\substack{k'=1\\k'\neq k}}^{K}E[(|z_{k'}(i)|^2-1)](\mathbf{w}_{k'}^H\mathbf{H}_{k',k}\mathbf{f}_k)\mathbf{H}_{k',k}^H\mathbf{w}_{k'}. \tag{20}$$

This term includes the contribution to the gradient vector in (20) from the symbols of user $k'\neq k$, i.e. multi-user interference (MUI).

Setting the gradient (19) equal to 0, we obtain

$$\mathbf{f}_k=\bar{\mathbf{Q}}_k^{-1}[\bar{\mathbf{d}}_k-\boldsymbol{\phi}(\mathbf{F},\mathbf{W})-\tfrac{1}{2}\lambda_k^*\mathbf{H}_{k,k}^H\mathbf{w}_k] \tag{21}$$

where we define

$$\bar{\mathbf{Q}}_k=E[|z_k(i)|^2]\,\mathbf{H}_{k,k}^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_{k,k}+\frac{\lambda}{2}\mathbf{I}. \tag{22}$$

$$\bar{\mathbf{d}}_k=(\mathbf{w}_k^H\mathbf{H}_{k,k}\mathbf{f}_k)\mathbf{H}_{k,k}^H\mathbf{w}_k \tag{23}$$

To evaluate the Lagrange multiplier, we proceed as above. That is, invoking (21), we have

$$\begin{aligned}\mathbf{w}_k^H\mathbf{H}_{k,k}\mathbf{f}_k=\mathbf{w}_k^H&\mathbf{H}_{k,k}\bar{\mathbf{Q}}_k^{-1}[\bar{\mathbf{d}}_k-\boldsymbol{\phi}(\mathbf{F},\mathbf{W}) \\ &-\tfrac{1}{2}\lambda_k^*\mathbf{H}_{k,k}^H\mathbf{w}_k]=\gamma\end{aligned} \tag{24}$$

from which we find

$$\lambda_k = \frac{2[\mathbf{w}_k^H \mathbf{H}_{k,k} \bar{\mathbf{Q}}_k^{-1}(\bar{\mathbf{d}}_k - \phi(\mathbf{F}, \mathbf{W})) - \gamma]}{\mathbf{w}_k^H \mathbf{H}_{k,k} \bar{\mathbf{Q}}_k^{-1} \mathbf{H}_{k,k}^H \mathbf{w}_k}. \quad (25)$$

Finally, we have

$$\mathbf{f}_k = \bar{\mathbf{Q}}_k^{-1}\big(\bar{\mathbf{d}}_k - \phi(\mathbf{F}, \mathbf{W}) - \bar{\alpha}_k^{-1}\bar{\mathbf{c}}_k\big) \quad (26)$$

where

$$\bar{\mathbf{c}}_k = \big(\mathbf{w}_k^H \mathbf{H}_{k,k} \bar{\mathbf{Q}}_k^{-1}(\bar{\mathbf{d}}_k - \phi(\mathbf{F}, \mathbf{W})) - \gamma\big)\mathbf{H}_{k,k}^H \mathbf{w}_k \quad (27)$$

$$\bar{\alpha}_k = \mathbf{w}_k^H \mathbf{H}_{k,k} \bar{\mathbf{Q}}_k^{-1} \mathbf{H}_{k,k}^H \mathbf{w}_k. \quad (28)$$

IV. Adaptive Implementation

In this Section we derive a new algorithm for adaptive transceiver design by considering the CCM criterion. We assume that the channel matrices $\mathbf{H}_{k,l}$ are known with enough accuracy. In particular, we derive a recursive least square (RL-S) algorithm for the joint adaptation of the transmit precoders and receiver weights, i.e. $\mathbf{w}_k(i)$ and $\mathbf{f}_k(i)$ respectively, where the dependence of these quantities upon the iteration index, i, is now made explicit.

A. Adaptive JO CCM-RLS Algorithm

We begin by deriving the necessary recursions to update the receiver weight vectors $\mathbf{w}_k(i)$. An exponentially weighted estimate of the matrix $\mathbf{Q}_k$ in (10) can be computed recursively as follows

$$\hat{\mathbf{Q}}_k(i) = \delta_w \hat{\mathbf{Q}}_k(i-1) + (1-\delta_w)\mathbf{v}_k(i)\mathbf{v}_k^H(i) \quad (29)$$

where we define $\mathbf{v}_k(i) = z_k(i)\mathbf{r}_k(i)$. Then, using the matrix inversion lemma [11], we can write

$$\hat{\mathbf{Q}}_k^{-1}(i) = \delta_w^{-1}\hat{\mathbf{Q}}_k^{-1}(i-1) - \delta_w^{-1}\mathbf{s}_k(i)\mathbf{v}_k^H(i)\hat{\mathbf{Q}}_k^{-1}(i-1) \quad (30)$$

where

$$\mathbf{s}_k(i) = \frac{\hat{\mathbf{Q}}_k^{-1}(i-1)\mathbf{v}_k(i)}{\delta_w + \mathbf{v}_k^H(i)\hat{\mathbf{Q}}_k^{-1}(i-1)\mathbf{v}_k(i)}. \quad (31)$$

An estimate of the cross-correlation vector in (11) can be updated through the following recursion:

$$\hat{\mathbf{d}}_k(i) = \delta_w \hat{\mathbf{d}}_k(i-1) + (1-\delta_w) z_k^*(i)\mathbf{r}_k(i) \quad (32)$$

Finally, the weight vector for the k-th receiver can be updated by using

$$\mathbf{w}_k(i) = \hat{\mathbf{Q}}_k^{-1}(i)\big(\hat{\mathbf{d}}_k(i) - \alpha_k^{-1}(i)\mathbf{c}_k(i)\big) \quad (33)$$

where $\alpha_k(i)$ and $\mathbf{c}_k(i)$ are defined as in (15) and (14), but with $\hat{\mathbf{Q}}_k(i)$, $\hat{\mathbf{d}}_k(i)$ and $\mathbf{f}_k(i)$ standing in place of $\mathbf{Q}_k$, $\mathbf{d}_k$ and $\mathbf{f}_k$, respectively.

The derivation of the recursions for updating the transmit precoder vectors $\mathbf{f}_k$ presents a difficulty related to the enforcement of the transmit power constraint and how to incorporate it into an efficient RLS-type algorithm. Indeed, due to the presence of the Lagrange multiplier λ in (22), the resulting system of equations for $\mathbf{f}_k$ cannot be solved recursively with the aid of the matrix inversion lemma. Our proposed approach to overcome this difficulty is to first obtain an adaptive RLS-type algorithm for $\mathbf{f}_k$ by relaxing the power constraint, and then incorporate the power constraint via a subsequent normalization procedure performed at each time iteration. The validity of this approach is demonstrated through numerical experiments in Section V.

Proceeding in a similar way as for the receiver weight, an exponentially weighted recursive estimate of matrix $\bar{\mathbf{Q}}_k$ in (22) is obtained as

$$\hat{\bar{\mathbf{Q}}}_k(i) = \delta_f \hat{\bar{\mathbf{Q}}}_k(i-1) + (1-\delta_f)\mathbf{u}_k(i)\mathbf{u}_k^H(i) \quad (34)$$

where we define

$$\mathbf{u}_k(i) = z_k(i)\mathbf{H}_{k,k}^H \mathbf{w}_k(i) \quad (35)$$

By using the matrix inversion lemma [11], we can write

$$\hat{\bar{\mathbf{Q}}}_k^{-1}(i) = \delta_f^{-1}\hat{\bar{\mathbf{Q}}}_k^{-1}(i-1) - \delta_f^{-1}\bar{\mathbf{s}}_k(i)\mathbf{u}_k^H(i)\hat{\bar{\mathbf{Q}}}_k^{-1}(i-1) \quad (36)$$

where

$$\bar{\mathbf{s}}_k(i) = \frac{\hat{\bar{\mathbf{Q}}}_k^{-1}(i-1)\mathbf{u}_k(i)}{\delta_f + \mathbf{u}_k^H(i)\hat{\bar{\mathbf{Q}}}_k^{-1}(i-1)\mathbf{u}_k(i)}. \quad (37)$$

An estimate of the cross-correlation vector (23) can be updated through the recursion:

$$\begin{aligned}\hat{\bar{\mathbf{d}}}_k(i) = \delta_f \hat{\bar{\mathbf{d}}}_k(i-1) + (1-\delta_f) z_k^*(i)\mathbf{H}_{k,k}^H \\ \times \mathbf{w}_k(i)\mathbf{w}_k^H(i)\mathbf{H}_{k,k}\mathbf{f}_k(i).\end{aligned} \quad (38)$$

A recursive estimate of $\phi(\mathbf{F}, \mathbf{W})$ is given by

$$\begin{aligned}\hat{\phi}(i) = {} & \delta_f \hat{\phi}(i-1) \\ & + (1-\delta_f)\sum_{\substack{k'=1 \\ k' \neq k}}^{K} (|z_{k'}(i)|^2 - 1) \\ & \times (\mathbf{w}_{k'}^H(i)\mathbf{H}_{k',k}\mathbf{f}_k(i))\mathbf{H}_{k',k}^H \mathbf{w}_{k'}(i).\end{aligned} \quad (39)$$

Finally, the transmit precoder for the k-th receiver can be updated by using the following expression,

$$\mathbf{f}_k(i) = \hat{\bar{\mathbf{Q}}}_k^{-1}(i)\big(\hat{\bar{\mathbf{d}}}_k(i) - \hat{\phi}(i) - \bar{\alpha}_k^{-1}(i)\bar{\mathbf{c}}_k(i)\big) \quad (40)$$

where $\bar{\alpha}_k(i)$ and $\bar{\mathbf{c}}_k(i)$ are obtained from (28) and (27) with obvious modifications, i.e. using $\hat{\bar{\mathbf{Q}}}_k(i)$, $\hat{\bar{\mathbf{d}}}_k(i)$, $\hat{\phi}(i)$ and $\mathbf{w}_k(i)$ in place of $\bar{\mathbf{Q}}_k$, $\bar{\mathbf{d}}_k$, $\phi(\mathbf{F}, \mathbf{W})$ and $\mathbf{w}_k$, respectively.

After the transmit precoders of all the users have been updated at the i-th time instant, we employ the following expression to control the transmit power:

$$\mathbf{f}_k(i) \leftarrow \sqrt{\frac{\mathcal{P}}{\beta}}\mathbf{f}_k(i) \quad (41)$$

where the arrow denotes an overwrite operation and

$$\beta = \sum_{k=1}^{K} \|\mathbf{f}_k(i)\|^2. \quad (42)$$

The proposed blind CCM algorithm with adaptive RLS implementation is summarized in table I.

TABLE I
PROPOSED ADAPTIVE JO CCM-RLS ALGORITHM

Initialize $\mathbf{f}_k(0)$, $\mathbf{w}_k(0)$, $\hat{\mathbf{Q}}_k^{-1}(0)$, $\hat{\hat{\mathbf{Q}}}_k^{-1}(0)$,
    $\hat{\mathbf{d}}_k(0)$ and $\hat{\hat{\mathbf{d}}}_k(0)$.
Set the forgetting factor δ_w, δ_f.
For $i = 1, 2, \ldots$ (time iteration)
  For $k = 1 : K$ (update receiver weigths)
    Compute filtered data vector $z_k(i)$ and vector $\mathbf{v}_k(i)$
    Update $\hat{\mathbf{Q}}_k^{-1}(i)$ by using (30) and (31)
    Update $\hat{\mathbf{d}}_k(i)$ by using (32)
    Compute the new CCM receiver weight $\mathbf{w}_k(i)$ by using (33)
  End
  For $k = 1 : K$ (update transmit precoders)
    Compute vector $\mathbf{u}_k(i)$ (35)
    Update $\hat{\hat{\mathbf{Q}}}_k^{-1}(i)$ by using (36) and (37)
    Update $\hat{\hat{\mathbf{d}}}_k(i)$ by using (38)
    Compute $\hat{\phi}(i)$ by using (39)
    Compute the new CCM transmitter $\mathbf{f}_k(i)$ by using (40)
  End
  Apply power normalization as in (41) and (42)
End

B. Convergence Analysis

In this part, we discuss the convergence of the proposed adaptive algorithm. Although the overall CCM function is not jointly convex on all the input quantities, by adjusting the parameter γ it is convex over each of the transmit and receive filters [7], [8]. Note that for the i-th iteration we first update the vector $\mathbf{w}_k(i)$ by fixing $\mathbf{f}_k(i-1)$. Then, the vector $\mathbf{f}_k(i)$ is computed by fixing $\mathbf{w}_1(i) \ldots \mathbf{w}_K(i)$. From the algorithm in table I, we can see that $\mathbf{w}_k(i)$ is a function of $\mathbf{f}_k(i-1)$, and $\mathbf{f}_k(i)$ is a function of $\mathbf{w}_1(i) \ldots \mathbf{w}_K(i)$, which can be given by $\mathbf{w}_k(i) = \mathcal{F}\{\mathbf{f}_k(i-1)\}$ and $\mathbf{f}_k(i) = \mathcal{G}\{\mathbf{w}_1(i) \ldots \mathbf{w}_K(i)\}$. Since we optimize one of the weighting vectors by fixing the others, we obtain the value of the CCM function for the i-th received symbol $\mathcal{L}^{(i)}\big(\mathcal{F}\{\mathbf{f}_k(i-1)\}, \mathbf{f}_k(i-1)\big) = \min_{x \in X} \mathcal{L}^{(i)}\big(x, \mathbf{f}_k(i-1)\big)$ and $\mathcal{L}^{(i)}\big(\mathbf{w}_k(i), \mathcal{G}\{\mathbf{w}_1(i) \ldots \mathbf{w}_K(i)\}\big) = \min_{y \in Y} \mathcal{L}^{(i)}\big(\mathbf{w}_k(i), y\big)$, where $\mathcal{F}\{\mathbf{f}_k(i-1)\} \in X$ and $\mathcal{G}\{\mathbf{w}_1(i) \ldots \mathbf{w}_K(i)\} \in Y$. The optimum solutions are searched from X and Y, respectively. We obtain $\mathcal{L}^{(i)}\big(\mathcal{F}\{\mathbf{f}_k(i-1)\}, \mathbf{f}_k(i-1)\big) \leq \mathcal{L}^{(i-1)}\big(\mathcal{F}\{\mathbf{f}_k(i-2)\}, \mathbf{f}_k(i-1)\big)$ and $\mathcal{L}^{(i)}\big(\mathbf{w}_k(i), \mathcal{G}\{\mathbf{w}_1(i) \ldots \mathbf{w}_K(i)\}\big) \leq \mathcal{L}^{(i)}\big(\mathbf{w}_k(i), \mathcal{G}\{\mathbf{w}_1(i-1) \ldots \mathbf{w}_K(i-1)\}\big) = \mathcal{L}^{(i)}\big(\mathcal{F}\{\mathbf{f}_k(i-1)\}, \mathbf{f}_k(i-1)\big)$. Based on the above inequalities, we find that the CCM cost function $\mathcal{L}^{(i)}$ is not increasing and lower bounded to 0. The proposed algorithm is therefore able to converge to a local minimum as the number of received symbols increases.

C. Computational Complexity

We now evaluate the computational complexity of the proposed CCM-RLS algorithm when used over MIMO interference channels. To this end, we count the number of additions and multiplications required for each time iteration of the algorithm, where the results are summarized in Table II. For reference, we also present the corresponding figures for the Sum-MSE algorithm [3], [4]. The complexity of the Sum-MSE algorithm is cubic in the number of transmit or receive antennas due to the presence of matrix inversion [11]. It also includes a parameter M, which is the average number of required iteration by this algorithm. In particular, for a configuration with $N_r = 8$, $N_t = 3$, $K = 4$ and $M = 20$, the numbers of multiplications and additions for the Sum-MSE transceiver are 90000 and 86400, respectively. The numbers of multiplications and additions for the blind CCM-RLS algorithm are 2221 and 1231, respectively. Compared to the Sum-MSE algorithm, the proposed blind CCM-RLS algorithm reduces the computational complexity significantly.

V. SIMULATIONS

In this section, we evaluate the performance of the proposed JO CCM-RLS transceiver and compare it with the Sum-MSE transceiver [4] and the fixed transmit precoding [12] with the blind CCM-RLS receiver [7]. We adopt a Monte-carlo simulation approach and conduct several experiments in order to verify the effectiveness of the proposed technique. We assume that the MIMO channel is flat fading with Rayleigh distribution. The sequence of channel coefficients is computed according to Clarke model [14]. The normalized Doppler frequency is $f_dT = 5 \times 10^{-5}$. In order to take into account the effect of imperfect CSI, we assume that the estimated MIMO channel are given by $\hat{\mathbf{H}}_{k,l}(i) \approx \mathbf{H}_{k,l}(i) + \triangle\mathbf{H}_{k,l}(i)$, where $\triangle\mathbf{H}_{k,l}(i)$ is a complex estimation error matrix which is assumed to be Gaussian distributed with zero mean and $E[\triangle\mathbf{H}_{k,l}(i)\triangle\mathbf{H}_{k,l}^H(i)] = p^2 N_t \mathbf{I}$, where p^2 denotes the variance of each element in the error matrix [13]. We define the input SNR$= \mathcal{P}/\sigma_k^2$.

For the proposed algorithm, we initialize $\mathbf{f}_k(0)$ with a normalized random vector, set $\mathbf{w}_k(0), \hat{\mathbf{d}}_k(0)$ and $\hat{\hat{\mathbf{d}}}_k(0)$ to the zero vector, and initialize $\hat{\mathbf{Q}}_k^{-1}$ and $\hat{\hat{\mathbf{Q}}}_k^{-1}$ with $\delta\mathbf{I}$, $\delta = 0.01$. Among the proposed schemes, we consider:

- JO CCM-RLS: the proposed joint adaptive RLS algorithm based on the CCM criterion for joint transceiver optimization.
- Fixed CCM-RLS: the fixed precoder [12] with the blind adaptive RLS receiver based on the CCM criterion [7].
- Sum-MSE: the block-based transceiver algorithm based on minimizing the sum MSE [4].

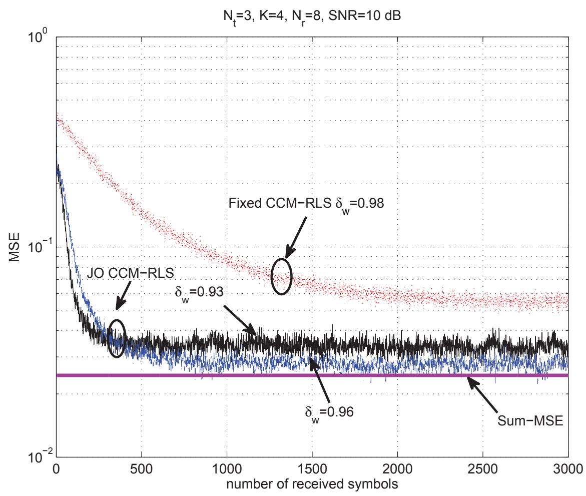

Fig. 2. MSE performance versus number of received symbols in MIMO interference channels. SNR= 10dB. $f_dT = 5 \times 10^{-5}$. $N_t = 3, N_r = 8, K = 4$. $\delta_f = 0.998$

TABLE II
COMPUTATIONAL COMPLEXITY OF ALGORITHMS

Algorithm	Number of operations per time iteration	
	Multiplications	Additions
JIO CCM-RLS	$K(7N_t^2+5N_r^2+2N_tN_r+11N_r+10N_t)$ $+2KN_t+1$	$K(4N_t^2+3N_r^2+2N_tN_r+3N_r+2N_t-1)$ $+KN_t-1$
Sum-MSE [3], [4]	$KM\big(2N_tN_rK+(N_t^2+N_r^2)K+2N_t^3+N_r^3$ $+N_t^2+N_r^2+2\big)$	$KM\big(2N_tN_rK+(N_t^2+N_r^2)K+2N_t^3+N_r^3$ $-(N_t+N_r)K+N_t^2+N_r^2+1\big)$

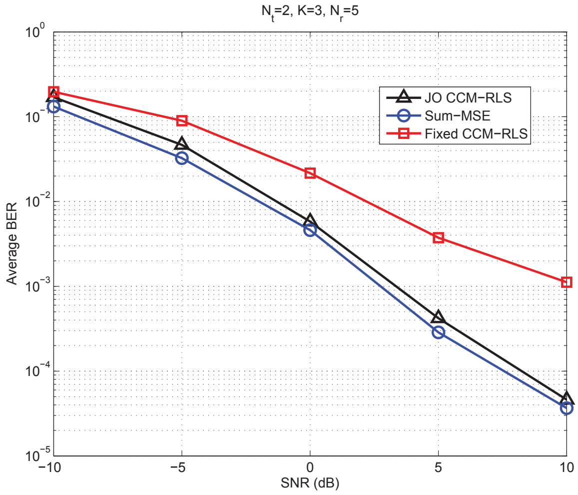

Fig. 3. Steady-state averaged BER performance versus SNR in MIMO interference channels. $f_dT = 5\times10^{-5}$. $N_t = 2, N_r = 5, K = 3$. $\delta_f = 0.998$, $\delta_w = 0.98$.

Fig. 2 shows the MSE performance versus the number of received symbols for the proposed JO CCM-RLS transceiver and the conventional schemes. We consider a system configuration with $N_t = 3$, $K = 4$, $N_r = 8$. Firstly, we can see that the proposed JO CCM-RLS algorithm converges much faster than the conventional CCM-RLS receiver with a fixed transmit precoder. Then, the proposed CCM-RLS transceiver with $\delta_w = 0.96$ and $\delta_f = 0.998$ converges to a value closed to the performance of the Sum-MSE algorithm. Here, we set SNR$=$ 10dB and $p^2 = 0.05$. The conventional Sum-MSE algorithm requires 20 iterations to obtain the optimal solutions for each transmitted symbol. The proposed method is 40 times more efficient compared to the Sum-MSE algorithm, as suggested by the figures in Section IV.

Fig. 3 illustrates the steady-state averaged bit error rate (BER) versus the input SNR, where we set $p^2 = 0.05$. We consider a system configuration with $N_t = 2$, $K = 3$, $N_r = 5$. From the results, we can see that the best performance is achieved by the Sum-MSE transceiver, followed by the proposed JO CCM-RLS transceiver and the conventional CCM-RLS receiver with a fixed precoder. It is worth mentioning that compared to the Sum-MSE algorithm, the proposed algorithm requires an increase of 0.5dB in transmitted power to maintain a BER at the level of 10^{-3}. The performance of the proposed adaptive algorithm achieves the performance of the Sum-MSE algorithm at a significantly reduced complexity, which verifies the effectiveness of the proposed technique. In particular, compared to the conventional CCM-RLS scheme, the proposed algorithm can save up to 4 dB at the BER level of 10^{-2}.

VI. CONCLUSION

In this paper, we have proposed a low-complexity adaptive transceiver algorithm based on the joint optimization using the constrained constant modulus criterion for MIMO interference channels. We have derived the optimal solutions for the transmit and receive vectors, and developed recursive least-squares adaptive algorithms for their efficient implementation. The computational complexity analysis of the proposed adaptive algorithm and the conventional Sum-MSE algorithm has been carried out. The simulation results have shown that the performance of the proposed adaptive algorithm comes close to that of the Sum-MSE algorithm at a reduced complexity. We remark that our proposed algorithms also can be extended to other forms of digital modulation.

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