Network-Coded Two-Way Relaying in Spectrum Sharing Systems with Quality-of-Service Requirements

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Abstract—We investigate the performance of a dual-hop two-way cognitive radio system, where the secondary users (SUs) exchange information in an underlay mode with the assistance of a half-duplex relay utilizing physical-layer network coding over finite GF(2). Moreover, we consider a practical scenario of interference from the primary users (PUs) affecting the relay and source nodes. The analysis provides a generalization of previous works as it considers an extended transmission system where the channels can consist of a combination of independent and identically distributed (i.i.d.) and independent but non-identically distributed (i.n.i.d.) Nakagami-\(m\) fading models. Also, unlike prior works, this paper focuses on the performance of both the PUs and the SUs. Closed-form expressions for the symbol error probability (SEP) and outage probability of the intended PU are obtained. In addition, we derive exact closed-form expressions for the SEP with consideration of special cases of practical interest (e.g., no interference power, interference-limited and single dominant interference cases) for the SUs. Furthermore, an upper bound on the achievable rate of the secondary system is provided. Subsequently, closed-form approximating expression for the SEP of the secondary system at high signal-to-noise ratios is obtained. Simulation results are provided and attest to the accuracy of the analytical results.

Index Terms—Cognitive radio networks, network coding, symbol error probability, two-way relaying.

I. INTRODUCTION

In the design of modern wireless communication systems, achieving higher data rates and more reliable transmissions have become pivotal goals. While some recent studies predict multi-fold increase in the data traffic by 2020 [1–3], mobile operators must currently deal with resource congestion and energy limitations of existing systems. In this context, cooperative communication has emerged as an advanced paradigm to achieve robustness and high data rate transmissions [4].

Among the many cooperative communication schemes that have been proposed in recent years, two-way relaying offers many advantages in terms of capacity increase, coverage extension and energy savings. One of the most widely embraced protocols in two-way relaying is the physical-layer network coding (PNC) in which two source nodes simultaneously transmit their information message to an intermediate relay over a multiple access channel (MAC) in the first stage, and the relay retransmits the XOR’ed version of the received messages to the source nodes over a broadcast channel (BC) in the second stage [5]. Despite a higher complexity, the PNC relaying protocol can offer lower bit error rates, which is a desirable attribute for future wireless cellular networks. For this purpose, we, henceforth, evaluate the performance of PNC relaying schemes.

Meanwhile, the spectrum resources are extremely scarce. Cognitive radio (CR) together with dynamic spectrum access (DSA) provide an advanced strategy for addressing the spectrum scarcity problem of wireless networks by allowing the sharing of resources between different classes of users [6]. One of the most common approaches for DSA in spectrum-sharing systems is in the form of an underlay scheme, whereby secondary users (SUs) are allowed to coexist with primary users (PUs) as long as the primary’s quality-of-service (QoS) is not affected. Since the underlay approach does not necessarily rely on detection of spectrum white space, it is of special interest [7]. Besides, most wireless networks operate according to a frequency reuse principle, which makes co-channel interference (CCI) a dominant factor. Hence, the transmit power of the SUs is not only dependent on the radio channel between them, but also on the interference channels from the PU to the SUs and on the primary channel as well.

Based on these considerations, much research efforts were devoted to the performance study of various schemes for relaying SU’s messages in underlay cognitive radio networks (CRNs), taking into account interference from and to the PUs. In [8–17], the purpose was to investigate the effect of primary transmissions on the performance of traditional CRNs. For instance, in [8], a closed-form expression for the outage probability (OP), under peak interference power constraint in the presence of multiple unidirectional primary transceivers, was derived assuming Rayleigh fading. The performance metrics in an amplify-and-forward (AF) CRN with best relay selection were studied in [10]. [11] presented a closed-form expression for the OP of a secondary network implementing decode-and-forward (DF) relaying. In [12], the OP of a unidirectional cognitive multiple-input multiple-output (MIMO) relaying system was analyzed. The asymptotic OP for three relay-selection strategies were obtained in [13]. The authors in [15], examined the outage performance of DF CRNs. [16] studied the performance of a multi-relay spectrum sharing system, where the diversity order was shown to be
equal to one regardless of the number of relaying nodes, and [17] investigated the impact of multiuser diversity on the outage performance of DF CRNs.

Compared to the traditional relaying considered in the above works, two-way relaying techniques can potentially double the spectral efficiency [18]. For example, [19–23] derived the OP of two-way relaying systems in the presence of CCI and additive white Gaussian noise (AWGN) at the relay(s) and the end-sources. [24] studied the problem of relay selection and optimal resource allocation for two-way AF and DF relaying in spectrum sharing systems, and [25] proposed a transmit beamforming technique for an underlay CRN, where the CR system uses part of the primary spectrum, while a MIMO secondary base station acts as a relay for the primary network. A cognitive relay precoder based on the mean square error (MSE) criterion was designed in [26], where only imperfect channel state information (CSI) was assumed available. In [27], a relay selection strategy for two-way AF relaying was presented. The OP of incremental AF and DF relaying in underlay spectrum sharing systems over Nakagami-

rate of the secondary system are derived based on Jensen’s inequality. (iv) For additional insights onto the impact of system parameters, such as fading parameters and the number of primary interferers, we derive the asymptotic SEP for different cases. The results indicate that an equal number of interferers at each SU node yields better SEP performance than the un-equal case, over the whole SNR range of interest. (v) Simulations are presented to corroborate the analysis, and to provide interesting horizons on the impact of noise, interference, fading parameters and primary outage threshold, on performance.

The rest of the paper is organized as follows: Section II introduces the system model and fading statistics. In Section III, we pursue the performance analysis of the primary network and derive an exact closed-form expression for the SEP and an upper-bound on the sum rate. The asymptotic analysis for the secondary network is developed in Section IV. Asymptotic performance analysis provided in Section V. Section VI presents a set of numerical results, and Section VII concludes the paper.

II. SYSTEM MODEL

We examine the impact of multiple primary interferers on the performance of a unidirectional primary network as well as of a CR two-way relay network. The intended primary network consists of a transmitter node, \( P \), and a receiver node, \( R \). The CRN consists of two source nodes, \( S_1 \) and \( S_2 \), which exchange information via a relay \( R \) employing PNC, as shown in Fig. 1. We assume that the direct channel between \( S_1 \) and \( S_2 \) has a negligibly small SNR due to severe fading. The amplitudes of all channels undergo flat Nakagami-

In our formulation, \( h, g \) and \( h_P \) denote the random channel coefficients from \( S_1 \) to \( R \), from \( S_2 \) to \( R \) and from the intended PU, \( P \), to its receiver \( P \), respectively. Also, \( f_{R,j}, f_{S_1,j}, f_{S_2,j} \) represent the channel coefficients from the \( j \)th interferer to \( R \), \( S_1 \) and \( S_2 \), respectively. Additionally, \( f_{P,S_1}, f_{P,S_2}, f_{P,R} \) and \( f_{P,j} \) are the flat fading coefficients from \( S_1 \) to \( P \), \( S_2 \) to \( P \), \( R \) to \( P \) and from the \( j \)th primary interferer to \( P \), respectively.

The amplitudes of all the links have Nakagami-

1Performance analysis of two-way spectrum sharing systems in Nakagami-
im environments has its own challenges. That is why many papers appeared even with the same topics as in unidirectional networks suffering from Rayleigh fading [30, 31]. There are also many works on bidirectional relay networks which are only noise limited or interference limited, or assuming Rayleigh fading [32, 33]. Therefore, none of the prior works presented such practical and comprehensive analysis of the proposed scenario.
we define the scale parameter $\beta_x = \frac{m_x}{\sum_j P_j}$, $x \in \{(S_1, j), (S_2, j), (R, j), (P, j), (P, R), (P, S_1), (P, S_2), R, g, P\}$.

In the MAC stage, $S_1$ and $S_2$ send their respective messages $x_1$ and $x_2$ to $R$. During this stage, $L_P$ PUs out of the existing ones which exchange messages with their respective receivers, interfere with node $P$. In such a case, there is no need for state feedback to synchronize the primary and secondary networks. The sources and the relay are affected by $L_{S_i}$, $i \in \{1, 2\}$ and $L_R$ interferers, respectively. The interferers, which are the PUs in the proximity of the secondary network, may be i.i.d. or i.n.i.d. Under this scenario, the signal received by $R$ in the MAC stage is given by

$$y_R = \sqrt{E_{S_1} h x_1} + \sqrt{E_{S_2} g x_2} + \sum_{j=1}^{L_R} \sqrt{E_{R,j} f_{R,j} d_{R,j}} + n_R,$$

where $E_{S_i}$ is the transmit energy at $S_i$ and $S_2$, $E_{R,j}$ is the transmit energy at the $j$th interferer in the vicinity of $R$, $x_1$, $x_2$ and $d_{R,j}$ represent the unit-energy symbols transmitted from $S_1$, $S_2$ and the $j$th interferer, respectively, and $n_R \sim \mathcal{C}\mathcal{N}(0, N_0)$ represents the AWGN at $R$. The signal received by the intended primary receiver $P$, can be expressed as

$$y_{P,S} = \sqrt{E_{P} h P x_P} + \sum_{j=1}^{L_P} \sqrt{E_{P,j} f_{P,j} d_{P,j}} + \sqrt{E_{S} f_{P,S_1} x_1} + \sqrt{E_{S} f_{P,S_2} x_2} + n_P,$$

where $E_P$ denotes the transmit energy at the intended primary transmitter, $E_{P,j}$ is the transmit energy at the $j$th primary interferer in the proximity of the primary node $P$, while $x_P$ and $d_{P,j}$ represent the modulated symbols with unit energy emitted by the intended primary transmitter and $j$th interferer in the vicinity of $P$, respectively, and $n_P \sim \mathcal{C}\mathcal{N}(0, N_0)$ is the AWGN at $P$.

By employing the minimum Euclidean distance rule, the relay proceeds for joint detection of the received signal $y_R$, as expressed by [34]

$$[\bar{x}_1, \bar{x}_2] = \arg\min_{[x_1, x_2]} \left| y_R - \left( \sqrt{E_{S_1} h s_1} + \sqrt{E_{S_2} g s_2} \right) \right|,$$

where $\bar{x}_1$ and $\bar{x}_2$ are the estimates of $x_1$ and $x_2$, respectively, and $|A|=Q$ denotes the cardinality of the Q-ary constellation. The relaying node $R$ selects the best map out of a well-designed finite mapping book according to the channel condition. Then, $\bar{x}_1$ and $\bar{x}_2$ are decoded and $X_1$ and $X_2$ obtained. Next, using the PNC protocol over the finite GF(2), the relay encodes the XOR’ed version of the decoded binary symbols and produces

$$\hat{y}_R = \bar{X}_1 \oplus \bar{X}_2,$$

where $\oplus$ is the bitwise XOR operation. Then, $R$ encodes $\hat{y}_R$ and produces $x_R$ which is broadcasted to $S_1$ and $S_2$ in the BC stage. After perfect cancelation of self-interference, the received signals at the two sources and node $P$ will be

$$y_{S_1} = \sqrt{E_{R} g x_R} + \sum_{j=1}^{L_{S_1}} \sqrt{E_{S_1,j} f_{S_1,j} d_{S_1,j}} + n_{S_1},$$

$$y_{S_2} = \sqrt{E_{R} g x_R} + \sum_{j=1}^{L_{S_2}} \sqrt{E_{S_2,j} f_{S_2,j} d_{S_2,j}} + n_{S_2},$$

$$y_{R,P} = \sqrt{E_{P} h P x_P} + \sum_{j=1}^{L_P} \sqrt{E_{P,j} f_{P,j} d_{P,j}} + \sqrt{E_{R} f_{R,P} x_R} + n_P,$$

where $E_{S_1,j}$ and $E_{S_2,j}$ denote the transmit power of the $j$th interferer affecting $S_1$ and $S_2$, $d_{S_1,j}$ and $d_{S_2,j}$ are the $j$th interference unit-power symbols affecting $S_1$ and $S_2$, $n_{S_1} \sim \mathcal{C}\mathcal{N}(0, N_0)$ and $n_{S_2} \sim \mathcal{C}\mathcal{N}(0, N_0)$ are AWGN at nodes $S_1$ and $S_2$, respectively, and $E_R$ is the transmit power of the relay.

Based on (5) and (6) the received signal-to-interference plus noise ratio (SINR) at $S_1$ and $S_2$ can be expressed by

$$\gamma_{R,S_1} = \frac{E_R|g|^2}{\sum_{j=1}^{L_{S_1}} E_{S_1,j} |f_{S_1,j}|^2 + N_0},$$

$$\gamma_{R,S_2} = \frac{E_R|g|^2}{\sum_{j=1}^{L_{S_2}} E_{S_2,j} |f_{S_2,j}|^2 + N_0}.$$

Next, the performance analysis with respect to (w.r.t.) the PU is presented.

III. PERFORMANCE ANALYSIS OF THE PRIMARY NETWORK

A. Outage Probability and Power Allocation

We aim at obtaining the OP of the intended PU, based on which the power allocation of the SUs is investigated. One of the major challenges of spectrum sharing systems is that the SUs should satisfy the QoS requirements of the primary network. In our case, the reliability requirements of the PUs shall be ensured. Specifically, the OP of primary transmissions shall be guaranteed to be below a pre-defined threshold $\lambda$ [35],

Fig. 1. A two-way cognitive cooperative network in the presence of multiple primary interferers.
as in (10),

$$P_{\text{out}} \text{Pr} = \Pr \left\{ \log_2 \left( 1 + \frac{E_P|h_P|^2}{\sum_{j=1}^{L_P} E_{P,j}|f_{P,j}|^2 + E_S|f_{P,S_1}|^2 + E_S|f_{P,S_2}|^2 + N_0} \right) \leq R_P \right\} \leq \lambda, \quad (10)$$

by solving (11) w.r.t. $E_S$ using popular computing softwares, such as Matlab and Mathematica.

Note that the allowed values of $E_R$ are obtained by applying the same strategy as in evaluating the power constraint of the two sources. According to (7), and similar to the proof of Proposition 1, it can be shown that the OP of the PU’s SINR in the BC phase is given by (12), based on which the power constraint of the relay is achieved:

$$F_{P,R}(\gamma_{th}) = 1 - \sum_{j=1}^{L_P} \sum_{k=1}^{M_{P,j}} \sum_{n=0}^{m-1} \sum_{r=0}^{n} \binom{n}{r} \binom{r}{q} \times \frac{\tilde{\gamma}_{P,R}^{m_{P,j}-k+q} \Gamma(n+k-r) \Gamma(m_{P,R}+q) \alpha_{jk}}{\Gamma(n+1) \Gamma(m_{P,R}+q) \Gamma(k)} \times \tilde{\gamma}_{P,R}^{m_{P,j}-q} \exp(-\tilde{\gamma}_{P,R}^n) \leq \lambda, \quad (12)$$

where $\gamma_{th} = \frac{2R_P - 1}{\beta P} \tilde{\gamma}_{P,R} = \beta P N_0 E_R^{-1}, \alpha_{jk} = \left( \Gamma(M_{P,j} - k + 1) \right) \left( -\tilde{\gamma}_{P,R}^n \right) \psi_j(s) = \frac{\tilde{\gamma}_{P,R}^n \Gamma(n+1) \times \left( \sum_{i=1}^{L_{FP}^{P}} \left( s/\tilde{\gamma}_{P,R}^n \right) \right) \times \tilde{\gamma}_{P,R}^{m_{P,j}-k+q} \exp(-\tilde{\gamma}_{P,R}^n) \leq \lambda, \quad (11)$$

### Proposition 1

According to the preceding proposition, the OP of the PU can be obtained as

$$F_{P,S}(\gamma_{th}) = 1 - \sum_{j=1}^{L_P} \sum_{k=1}^{M_{P,j}} \sum_{n=0}^{m-1} \sum_{r=0}^{n} \binom{n}{r} \binom{r}{q} \times \frac{\tilde{\gamma}_{P,j}^n \Gamma(n+k-r) \Gamma(w+q) \theta_{ew} \sigma_{jk}}{\Gamma(n+1) \Gamma(w) \Gamma(k)} \times \tilde{\gamma}_{P,j}^n \exp(-\tilde{\gamma}_{P,j}^n) \leq \lambda, \quad (11)$$

Proof:See Appendix 1.

In the adopted power control scheme, we employ a static method to control the transmit power of the SU. As such, the SUs utilize the maximum average admissible power for transmission. Finally, for a given value of the primary OP threshold $\lambda$, the power of the secondary sources is derived by solving (11) w.r.t. $E_S$ using popular computing softwares, such as Matlab and Mathematica.

Note that the allowed values of $E_R$ are obtained by applying the same strategy as in evaluating the power constraint of the two sources. According to (7), and similar to the proof of Proposition 1, it can be shown that the OP of the PU’s SINR in the BC phase is given by (12), based on which the power constraint of the relay is achieved:

$$F_{P,R}(\gamma_{th}) = 1 - \sum_{j=1}^{L_P} \sum_{k=1}^{M_{P,j}} \sum_{n=0}^{m-1} \sum_{r=0}^{n} \binom{n}{r} \binom{r}{q} \times \frac{\tilde{\gamma}_{P,R}^{m_{P,j}-k+q} \Gamma(n+k-r) \Gamma(m_{P,R}+q) \alpha_{jk}}{\Gamma(n+1) \Gamma(m_{P,R}+q) \Gamma(k)} \times \tilde{\gamma}_{P,R}^{m_{P,j}-q} \exp(-\tilde{\gamma}_{P,R}^n) \leq \lambda, \quad (12)$$

where $\gamma_{th} = \frac{2R_P - 1}{\beta P} \tilde{\gamma}_{P,R} = \beta P N_0 E_R^{-1}$. By solving (12) w.r.t. $E_R$, the admissible power values for the relay can be obtained. We note that the outage performance of the PU is investigated for two reasons. Firstly, the OP is a key performance measure for CRN operating under time-varying and slow flat fading conditions and, secondly, it can be used to solve the power allocation problem for the SUs, as considered in this work.

**B. Symbol Error Probability**

The error rates of several modulation schemes employed in practice represented in terms of the $Q$-function as $aQ(\sqrt{2b\gamma})$, where $a$ and $b$ are modulation-specific constants [37]; for instance, $a = 1$ and $b = 1$ for binary phase-shift keying (BPSK). One method to evaluate the SEP in fading environments is to make use of the CDF-based approach, which allows us to write the average SEP of the two-way relaying system, assuming BPSK modulation, as

$$P_e = \frac{\alpha Q(\sqrt{2b\gamma})}{2 \sqrt{\pi}} \int_0^{\infty} e^{-b \gamma} F_{\gamma}(\gamma) \ d\gamma, \quad (13)$$

where $E[.]$ represents expectation over the SINR distribution.

To compute the SEP, we need to perform partial fraction expansion on (11), which results in

$$F(\gamma) = 1 - \sum_{j=1}^{L_P} \sum_{k=1}^{M_{P,j}} \sum_{n=0}^{m-1} \sum_{r=0}^{n} \binom{n}{r} \binom{r}{q} \times \frac{\gamma_{R,R}^{m_{P,j}-k+q} \Gamma(n+k-r) \Gamma(m_{P,R}+q) \alpha_{jk}}{\Gamma(n+1) \Gamma(m_{P,R}+q) \Gamma(k)} \times \gamma_{R,R}^{m_{P,j}-q} \exp(-\gamma_{R,R}^n) \leq \lambda, \quad (14)$$

\[ \sum_{l=1}^{n+k-r} \frac{q_l (\gamma + \tilde{\gamma}_{P,R}^n) - m_{P,R} \gamma_{R,R}^{m_{P,j}-q}}{\tilde{\gamma}_{P,R}^n} \]
where \( q_l = \zeta(\gamma) \) and \( \zeta(\gamma) = (\Gamma(mR,p + q + m - 1) - \Gamma(mR,p + q + m - 1))^{-1} \zeta(mR,p + q + m - 1) \delta_m \)
and \( \delta_m = \zeta(mR,p + q + m - 1) \zeta(mR,p + q + m - 1) / \beta_{R,j} \).

Then, substituting (14) into (13) and utilizing [38], the SEP of the PU can be found in closed-form as shown in (15), where \( \mathbb{G}_x,y = \Phi(n + 1/2, n - x + 3/2; y) \) and \( \Phi(x; y; z) \) is the confluent hypergeometric function of the second kind [38, Eq. (9.211.4)].

IV. PERFORMANCE ANALYSIS OF THE SECONDARY NETWORK

In this section, we investigate the SEP and achievable rate of the secondary two-way network coded relaying system. An exact SEP result at \( S_1 \) is obtained, followed by an upper bound on the achievable rate of the system. Since BPSK is easy to implement, it is fairly resistant to noise and is the most robust of all PSK modulations, especially for low data-rate applications, it has been adopted in various third-generation (3G) standards, such as European Telecommunications Standards Institute (ETSI) in Europe, the Association of Radio Industries and Business (ARIB) in Japan and various wireless LAN standards, IEEE 802.11b, RFID and Bluetooth. BPSK modulation is therefore considered in this work for the performance analysis of the secondary network.

A. Symbol Error Probability

The average SEP of the two-way relay system for BPSK modulation can be obtained from (13). To begin, notice that errors at the relay occur when the \( S_1 \) message decoded correctly by the relay node is not, or vice versa. In this case, an error at \( S_1 \) occurs when the information sent from \( R \) is erroneous but correctly detected by \( S_1 \), or when the information sent from \( R,S_1 \) is correct but decoded with error at \( S_1 \). The following proposition summarizes the SEP at \( S_1 \) for the asymmetric two-way relay channel.

**Proposition 2:** Denote the instantaneous SEP at \( R \) w.r.t. links \( S_1 \) and \( S_2 \) respectively, and that at \( S_1 \) w.r.t. link \( R \) by \( P_b(\gamma_{S_1,R}) \) and \( P_b(\gamma_{S_2,R}) \), respectively. Let \( P_b(\gamma_{S_1,R}) \) be the probability that the \( S_1 \) message is decoded correctly by the relay node. Further, let \( e_{S_1,R} \) and \( e_{S_2,R} \) symbolize, respectively, that the received signal at \( S_1 \) is detected incorrectly and correctly. Then, the SEP of the asymmetric two-way network coded relaying system at \( S_1 \) is given by

\[
P_{S_1} = P_b(\gamma_{S_1,R})P_b(\gamma_{S_2,R}) + P_b(\gamma_{S_2,R})P_b(\gamma_{S_1,R})
\]

where \( \mathcal{J}_{S_1,R} = \mathcal{S}_1 \cup (\mathcal{S}_2 \setminus \{S_1\}) \) and \( \mathcal{H}_{S_1,R} = \mathcal{S}_1 \cup (\mathcal{S}_2 \setminus \{S_1\}) \). The quantity \( L_{F_x} \) denotes the number of primary interferers at node \( x \) which may have different values of \( \gamma_{S_1,R} \). In addition, \( M_{f_x} \), \( x \in \{S_1,R\} \), is the sum of the shape parameters of interferer channels affecting node \( x \) with equal values of \( \gamma_{S_1,R} \). Notice that \( P_b(\gamma_{S_1,R}) \), \( P_b(\gamma_{S_2,R}|e_{S_1,R}) \) and their corresponding CDFs and also the related functions can be obtained by replacing the subscript \( g \) by \( h \) in that of \( S_1 \). It is worth mentioning that since the relay simultaneously decodes the message information as in (3), the decoding processes are dependent of each other and we therefore used the conditional probability in obtaining \( P_{S_1} \) [39, 40].

**Proof:** See Appendix II.

For the symmetrical case when \( \gamma_{S_1,R} = \gamma_{S_2,R} \), by substituting \( P_b(\gamma_{S_1,R}|e_{S_2,R}) = P_b(\gamma_{S_2,R}|e_{S_1,R}) \) and \( P_b(\gamma_{S_1,R}|e_{S_2,R}) = P_b(\gamma_{S_2,R}|e_{S_1,R}) \) in (91) and (92), and then substituting the results in (17), \( P_b(\gamma_{S_1,R}) \) is simplified as

\[
P_b(\gamma_{S_1,R}) = 2P_b(\gamma_{S_1,R}|e_{S_2,R})[1 - P_b(\gamma_{S_2,R}|e_{S_1,R})] + [1 - P_b(\gamma_{S_2,R}|e_{S_1,R})].
\]

Next, we consider three special cases of interest where further simplifications are obtained.

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3 Obtaining the end-to-end performance metrics renders the analysis a very challenging mathematical problem. Therefore, it will be impossible to obtain any engineering insights. In addition, we recall that obtaining the performance metrics at \( S_1 \) is a standard approach adopted in the majority of works reported in the literature of two-way relaying, that enables the, otherwise tedious, analytical study of these configurations [20].
\[ J_{S_1,h,S,\lambda_{jk}} = a \left[ 1 - \sqrt{\frac{b}{\pi}} \sum_{j=1}^{L_{PS,2}} \sum_{k=1}^{M_{S_{1,j}}} \sum_{n=0}^{m_{k,n}-1} \sum_{i=0}^{n} \tilde{\gamma}_{h,S}^{m_{k,n}} \lambda_{jk} \Gamma(k+i) \Gamma(n + \frac{1}{2}) \left( \frac{\tilde{\gamma}_{S_{1,j}}}{\tilde{\gamma}_{h,S}} \right)^n \left( \frac{1}{\tilde{\gamma}_{h,S}} \right)^{n+\frac{1}{2}} \right] \cdot \] (21)

\[ H_{h,g} = a \left[ 1 - \sqrt{\frac{\pi}{b}} \sum_{n=0}^{m_{h,n}-1} \sum_{i=0}^{n} \tilde{\gamma}_{h,R}^{m_{h,n}} \rho_{k} \Gamma(k+i) \Gamma(n + \frac{1}{2}) \left( \frac{2}{\gamma_{g,R}} \right)^{k+i} \left( \frac{\tilde{\gamma}_{g,R}}{\gamma_{h,R}} \right)^{n+\frac{1}{2}} \right] \times \mathbb{G}_{k+i,\gamma_{S_{1,j}}(b+\tilde{\gamma}_{S_{1,j}})/\gamma_{h,S}} \] (22)

### 1) Interference-free Case:

**Corollary 1:** Here, we present our main results on the error probability performance of the CR dual-hop relaying system, when none of the nodes is impaired by interference \((E_{S_{2,j}} = E_{R,j} = E_I = 0 \forall j)\). As a consequence, the equations obtained in Proposition 2 are simplified as

\[ P^b(\gamma_{R,S_{1}}) = I_{h,S}, \] (24)

\[ P^b(\gamma_{S_{1,R}} | e_{S_{2,R}}) = U_{h,g}, \] (25)

\[ P^b(\gamma_{S_{1,R}} | e_{S_{2,R}}) = U_{h,g}, \] (26)

where

\[ I_{h,S} = a \left[ 1 - \sqrt{\frac{b}{\pi}} \sum_{n=0}^{m_{h,n}-1} \frac{\Gamma(n + \frac{1}{2}) \gamma_{h,S}^{m_{h,n}}}{\Gamma(n+1)(b+\tilde{\gamma}_{h,S})} \right], \] (27)

\[ U_{h,g} = a \left[ 1 - \sqrt{\frac{\pi}{b}} \sum_{n=0}^{m_{h,n}-1} \frac{\Gamma(n + \frac{1}{2}) \gamma_{h,R}^{m_{h,n}}}{\Gamma(n+1)} \right] \times \left( \frac{n}{i} \right) \left( \frac{\tilde{\gamma}_{h,R}}{2} \right)^{n-i+\frac{1}{2}} \mathbb{G}_{m_{h,n},i,\tilde{\gamma}_{h}/2}. \] (28)

### 2) Single Interferer Case:

**Corollary 2:** If only one dominant source of interference affects each node of the secondary network, the CDFs can be written as

\[ F_{\gamma_{R,S_{1}}} (\gamma) = A_{S_1,h,S}, \] (29)

\[ F_{\gamma_{S_{1,R}}} | e_{S_{2,R}} = A_{R,h,R}, \] (30)

\[ F_{\gamma_{S_{1,R}}} | e_{S_{2,R}} = B_{h,g}, \] (31)

where

\[ A_{S_1,h,S} = 1 - \sum_{n=0}^{m_{h,n}-1} \sum_{i=0}^{n} \frac{\gamma_{h,S}^{m_{h,n}}}{\Gamma(n+1)} \Gamma(m_{S_{1,1}} + i) \left( \frac{\tilde{\gamma}_{S_{1,1}}}{\gamma_{h,S}} \right)^n \left( \frac{1}{\gamma_{h,S}} \right)^{n+\frac{1}{2}} \] \times \left( \frac{n}{i} \right) \gamma_{S_{1,1}} \exp \left( -\tilde{\gamma}_{S_{1,1}} \gamma_{S_{1,1}} \right), \] (32)
\[ C_{S_1, h, S} = \alpha \left[ 1 - b^{\frac{m_h-1}{n} \sum_{n=0}^{m_h-1} \sum_{i=0}^{n} \frac{\gamma^n h_i S_{\gamma_1, l}}{(n+1) \Gamma(n+1)} \left( \frac{\gamma h_i S_{\gamma_1, l}}{\gamma h_i S} \right)^{n+\frac{1}{2}} \right] \] (37)

\[ D_{h, g} = \alpha \left[ 1 - b^{\frac{m_h-1}{n} \sum_{n=0}^{m_h-1} \sum_{i=0}^{n} \frac{\gamma^n h_i R k \Gamma(k + i) \Gamma(n + \frac{1}{2})}{(n+1) \Gamma(n+1)} \left( \frac{\gamma h_i R}{2 \gamma h_i R} \right)^{n+\frac{1}{2}} \times G_{k+i, \gamma h_i R (b+\gamma h_i), S_{\gamma_1, l} / \gamma h_i S} \right] \] (38)

\begin{align*}
&\times \left( \frac{n}{i} \right) \left( \frac{\gamma^n h_i R}{\gamma h_i R \gamma + \frac{\gamma^n h_i R}{2} \gamma h_i R k + i} \right) \exp \left( -\gamma h_i R \right),
&\text{where} \quad M_{S_1} = L_{S_1} m_{S_1, j}, \quad \gamma_{S_1} = \beta_{S_1} N_0 E^{-1}_{S_1}, \quad M_R = L_{R} m_{R, j}, \quad \text{and} \quad \gamma_R = \beta_{R} N_0 E^{-1}_{R}.
&\text{Moreover, the error probability is simplified as}
&P^b(\gamma_{R, S_1} = Q_{S_1, h, S}, \) \quad (44)
&P^b(\gamma_{S_1, R} | \psi_{S_2, R}) = Q_{R, h, R}, \) \quad (45)
&P^b(\gamma_{S_1, R} | \psi_{S_2, R}) = P_{h, g}. \) \quad (46)

of the capacity region is expressed as \[ R_1 \leq \min \{ C_{R, S_2}, C_{S_1, R} \}, \] (49)
\[ R_2 \leq \min \{ C_{R, S_1}, C_{S_2, R} \}, \] (50)

where \( C_{S_1, R}, C_{R, S_2}, C_{S_2, R} \) and \( C_{R, S_1} \) are the capacity of links \( S_1, R, S_2, S_2, R, \) and \( R, S_1, \) respectively. Finding closed-form expressions for \( C_{S_1, R}, C_{R, S_2}, C_{S_2, R} \) and \( C_{R, S_1} \) requires the evaluation of \( C_{\gamma} = 0.5 \log \{ \log(1 + \gamma) \} \), which becomes intractable or computationally impractical. To circumvent this difficulty, we use an alternative approach based on Jensen's inequality, which leads to much simpler expressions. Specifically, we can show that we can obtain the following expressions:
\[ C_{R, S_2} = F_{S_2, g, s, L_{S_2}, \lambda_{S_2}, \lambda_{S_2}}, \] (51)
\[ C_{R, S_1} = F_{S_1, h, s, \lambda_{S_1}, \lambda_{S_1}}, \] (52)
\[ C_{S_1, R} = F_{R, h, r, \mu_{S_1}, \mu_{S_1}}, \] (53)
\[ C_{S_2, R} = F_{R, r, g, \mu_{S_2}, \mu_{S_2}}, \] (54)

where
\[ F_{S_2, g, s, L_{S_2}, \lambda_{S_2}, \lambda_{S_2}} = \sum_{j=1}^{L_{S_2}} \sum_{k=1}^{m_{S_2, j}} \sum_{n=0}^{\infty} \frac{\gamma^n h_i S_{\gamma_1, l}}{(n+1) \Gamma(n+1)} \left( \frac{\gamma h_i S_{\gamma_1, l}}{2 \gamma h_i S} \right)^{n+\frac{1}{2}} \times \left( \frac{n}{i} \right) \left( \frac{\gamma^n h_i S_{\gamma_1, l}}{\gamma h_i S_{\gamma_1, l} + \frac{\gamma^n h_i S_{\gamma_1, l}}{2} \gamma h_i S \gamma + i} \right) \exp \left( -\gamma h_i S_{\gamma_1, l} \right), \]
\[ \text{while} \quad \tilde{\gamma}_{S_2, j} = \beta_{S_2, j} N_0 E^{-1}_{S_2, j}, \quad \tilde{\lambda}_j = (\Gamma(M_{S_2, j} - k + 1))^{-1} \nu_j (M_{S_2, j} - k) \left( -\tilde{\gamma}_{S_2, j} \right) \quad \nu_j (s) = \tilde{\gamma}_{S_2, j} \prod_{l=1}^{M_{S_2, j}} (s / \tilde{\gamma}_{S_2, l} + 1)^{-M_{S_2, l}}. \] Here, \( L_{F_{S_2}} \) is the number of primary interferers affecting \( S_2 \) which have different values of \( \tilde{\gamma}_{S_2, j} \), and \( M_{S_2, j} \) is the summation of the shape parameters of the interferer channels at \( S_2 \) with equal values of \( \tilde{\gamma}_{S_2, j} \). A rate pair \( (R_1, R_2) \) is achievable only if the expressions in (49) and (50) are satisfied with equality.

V. ASYMPTOTIC ANALYSIS

A. Lower Bound Analysis

The performance of the two-way PNC relay system can further be quantified by analyzing the error performance based on (8) in the high SNR regime. Under this condition, which
occurs when \( E_{S1,j} \gg N_0 \) and \( E_{IR,j} \gg N_0 \), (8) and (93) in Appendix II can be upper bounded as

\[
\gamma_{R,S1} < \gamma_{up,R,S1} = \frac{E_R|h|^2}{\sum_{j=1}^{L_{SS}} |E_{S1,j}|^2},
\]

\[
\gamma_{S1,R(eS2,R)} < \gamma_{up,S1,R(eS2,R)} = \frac{E_S|h|^2}{\sum_{j=1}^{L_{RR}} |E_{R,j}|^2}.
\]

Accordingly, the CDFs of the received SINR at node \( S_1 \), can be expressed as

\[
F_{\gamma_{up,R,S1}}(\gamma) = \Xi_{S1,h,R,S1,\beta},
\]

\[
F_{\gamma_{up,Si,R(eS2,R)}}(\gamma) = \Xi_{R,h,S,\mu,jk},
\]

where

\[
\Xi_{S1,h,R,S1,\beta} = 1 - \sum_{j=1}^{L_{PS1}} \sum_{m=1}^{M_{PS1,j}} \sum_{n=0}^{m-1} \frac{\beta^n_R \lambda_{jk} \Gamma(n+k) \cdot \gamma^n}{\Gamma(n+1)} \left( \frac{\beta_R}{E_R} \right)^{(n+k)} \left( \frac{\beta_{S1,R}}{E_{S1,R}} \right)^{(n+k)}.
\]

With the aim to highlight the impact of the fading parameters on the error performance, specific asymptotic regimes are considered below:

1) **Case 1**: In this case, we characterize the impact of the amount of received interference power on the error performance of the two-way PNC relay system. To begin, consider the case when the power of the interferers are negligible compared with the useful signal power, i.e., \( E_I \ll E_S \). As such, we have the following simplification:

\[
\gamma_{S1,R(eS2,R)} < \gamma_{up,S1,R(eS2,R)} = \frac{E_S|h|^2}{2E_S|g|^2}.
\]

Then, \( F_{\gamma_{up,R(eS2,R)}}(\gamma) \) can be obtained as

\[
F_{\gamma_{up,R(eS2,R)}}(\gamma) = \xi_{h,g},
\]

where

\[
\xi_{h,g} = 1 - \sum_{n=0}^{m_{-1}} \frac{\beta^n_R \Gamma(m + n) \cdot \left( \frac{\beta_R}{E_R} \right)^n \cdot \left( \frac{\beta_{S1,R}}{E_{S1,R}} \right)^n \cdot \gamma^n}{\Gamma(n+1) \Gamma(m)}.
\]

For this case, a lower bound expression for the SEP is formulated in the following proposition.

**Proposition 3**: The lower bound on the SEP performance of the system in the asymptotically high SNR regime for Case 1 is given by (16) where

\[
P^b(\gamma_{R,S1}) = W_{S1,h,R,S1,\beta},
\]

\[
P^b(\gamma_{S1,R(eS2,R)}) = W_{R,h,S1,R,S1,\beta},
\]

\[
P^b(\gamma_{S1,R(eS2,R)}) = \Xi_{h,g},
\]

with \( W_{S1,h,R,S1,\beta} \) and \( \Xi_{h,g} \) shown on the top of next page.

**Proof**: The proof is similar to that of Proposition 2.

2) **Case 2**: Consider the case where the power of the useful signal is much smaller than that of the interferers, i.e., \( E_S \ll E_I \). For this case, \( \gamma_{S1,R(eS2,R)} \) can be approximated as

\[
\gamma_{S1,R(eS2,R)} < \gamma_{up,S1,R(eS2,R)} = \frac{E_S|h|^2}{\sum_{j=1}^{L_{RR}} |E_{R,j}|^2}.
\]

Here, \( \gamma_{up,S1,R(eS2,R)} = \gamma_{up,S1,R(eS2,R)} \), which implies that \( F_{\gamma_{up,R(eS2,R)}}(\gamma) \) can be expressed as

\[
F_{\gamma_{up,R(eS2,R)}}(\gamma) = F_{\gamma_{S1,R(eS2,R)}}(\gamma).
\]

One may conclude that in this case increasing the SNR has no impact on the average SEP. In fact, since \( E_S \ll E_I \), the quality of the secondary links is much worse than that of the primary-to-secondary links. Therefore, the performance of the secondary relay network does not improve by increasing the SNR. The diversity order in this case is equal to 0.

**B. Simplified Analysis**

Since the derived expressions are complex, herein, we present simplified closed-form formulae for the SEP based on a linearization approach as in [44]. Using Taylor series, the behavior of the PDF of the SINR around the origin can be expanded as follows:

\[
F_{\gamma_{R,S1}}(\gamma) \approx Z_{S1,h,S,\lambda_{jk}},
\]

\[
F_{\gamma_{S1,R(eS2,R)}}(\gamma) \approx Z_{R,h,S,\lambda_{jk}},
\]

\[
F_{\gamma_{S1,R(eS2,R)}}(\gamma) \approx T_{h,g},
\]

where

\[
Z_{S1,h,S,\lambda_{jk}} = \frac{\gamma_{h,g} m_h}{\Gamma(m + 1)} \sum_{j=1}^{L_{PS1}} \sum_{m=1}^{M_{PS1,j}} \sum_{i=0}^{m_h} \left( m_h \right) \lambda_{jk} \Gamma(k + i) \Gamma(k) \Gamma(k+S1,j),
\]

\[
T_{h,g} = \sum_{n=0}^{m_{-1}} \eta_{m,g} \left( \frac{\gamma_{h,g} m_h}{\Gamma(m + 1)} \sum_{j=1}^{L_{PS1}} \sum_{m=1}^{M_{PS1,j}} \sum_{i=0}^{m_h} \left( m_h \right) \lambda_{jk} \Gamma(k + i) \Gamma(k) \Gamma(k+S1,j),
\]

\[
Z_{S1,h,S,\lambda_{jk}} = \frac{\gamma_{h,g} m_h}{\Gamma(m + 1)} \sum_{j=1}^{L_{PS1}} \sum_{m=1}^{M_{PS1,j}} \sum_{i=0}^{m_h} \left( m_h \right) \lambda_{jk} \Gamma(k + i) \Gamma(k) \Gamma(k+S1,j),
\]

\[
T_{h,g} = \sum_{n=0}^{m_{-1}} \eta_{m,g} \left( \frac{\gamma_{h,g} m_h}{\Gamma(m + 1)} \sum_{j=1}^{L_{PS1}} \sum_{m=1}^{M_{PS1,j}} \sum_{i=0}^{m_h} \left( m_h \right) \lambda_{jk} \Gamma(k + i) \Gamma(k) \Gamma(k+S1,j),
\]

and \( \eta_{m,g} = \left( \frac{\gamma_{h,g} m_h}{\Gamma(m + 1)} \right) \). Accordingly, using the CDF-based approach as in Proposition 2, we obtain

\[
P^b(\gamma_{R,S1}) = \frac{a \Gamma(m + 1/2)}{b m^{3/2}} Z_{S1,h,S,\lambda_{jk}},
\]

\[
P^b(\gamma_{S1,R(eS2,R)}) = \frac{a \Gamma(m + 1/2)}{b m^{3/2}} Z_{R,h,S,\lambda_{jk}},
\]

\[
P^b(\gamma_{S1,R(eS2,R)}) = a \left[ 1 - \sqrt{b/\pi} \sqrt{\gamma_{h,g} m_h} \right],
\]
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\[ W_{S_1,h,R,\lambda_j} = a \left[ 1 - \sqrt{b/\pi} \sum_{j=1}^{L_S_i} \sum_{k=1}^{M_{S_i,j}} \sum_{n=0}^{m_j-1} \lambda_j k \Gamma(n+k) \Gamma(n+1/2) \frac{(E_{S_1,j})^k}{\beta S_{1,j}} \right] \times \Phi \left( n + 1, 3/2 - k, b \beta S_{1,j} R / (\beta h E_{S_1,j}) \right). \]

\[ \lambda_{g,h} = a \left[ 1 - \sqrt{b/\pi} \sum_{n=0}^{m_j-1} \frac{\eta m_j \Gamma(m_j + n) \Gamma(n + 1/2)}{\Gamma(n+1) \Gamma(m_g)} \Phi \left( n + 1, 3/2 - m_g, b \beta h / (2 \beta h) \right) \right]. \]

Finally, by substituting the results into (16), a simpler closed-form expression for the average SEP of the system can be obtained.

VI. NUMERICAL RESULTS AND DISCUSSION

Monte-Carlo simulations are performed to validate the analytical results. For ease, we denote the number of interferers affecting the secondary nodes by \( L = [L_{S_1}, L_{S_2}, L_R] \), and that impacting the PU by \( L = [L_P] \). In the following simulation evaluations, \( \gamma_{th} \) is set to 3dB and the noise power are normalized to be 0dB. Also, in all figures the horizontal axis is the primary transmit SNR, \( \gamma_{tp} \).

The outage and error performance comparison between the analytical results and the simulation results corresponding to the intended PU is illustrated in Fig. 2 and Fig. 3 respectively, for different values of \( L \) and fading parameter \( m = 2 \). In these figures, the useful power of the PU and the interference power profile satisfy \( E_{P} - E_{P,j} = 30dB \), where \( E_{P,j} \) is the transmit energy at the \( j \)th \( (j \in \{1,2,\ldots,L_P\}) \) primary interferer in the proximity of node P. First, the agreement between the plots from the analysis and those from simulations confirm the accuracy of the analysis. As observed, the interference form the other PUs’ has an adverse influence on the outage and the SEP of the intended PU. It is evident that the OP and SEP improve with increasing received SNR at the PU. The transmit power of the SU is controlled for the target reliability at the primary. Therefore, as the primary OP threshold \( \lambda \) decreases, the performance of the SUs would degrade.

Figs. 4 to 8 show the SEP versus SNR for the two-way PNC cognitive relaying system, for Case 1, i.e., when \( E_I \ll E_S \). To examine the accuracy of the expressions in Corollaries 1-3, Fig. 4 shows the results for the i.i.d. case with the corresponding lower bound and asymptotic results, as well as the SEP obtained through simulations. Two sets of plots are presented, for a primary OP threshold \( \lambda = 0.1 \) and 0.01, while \( m = 2 \). Fig. 5 depicts a similar set of results for \( \lambda = 0.01 \) and \( m = 1,2 \). As observed, the analytical results yield an excellent match across the entire SNR range.

Fig. 6 illustrates corresponding sets of results for the i.n.i.d. case, for \( m = 1 \) and 2 and with \( \lambda = 0.01 \). Similar conclusions as above can be drawn. For completeness, the case of Rayleigh fading is shown. From Fig. 5 and Fig. 6, it can be deduced that when the interferers’ channels are i.n.i.d., the performance improvement is significant compared with the case when the interferers’ channels are i.i.d.

Figs. 7 and 8 depict the results when the channels consist of a combination of i.i.d. and i.n.i.d. Nakagami-\( m \) fading to examine Corollaries 1-3, for \( m = 1,2 \) and \( \lambda = 0.1,0.01 \). In addition, the average SEP performance under Case 2, i.e. when \( E_S \ll E_I \), is a horizontal line matched exactly at 1/2 shown in Figs. 4 to 8. As observed for these two practical Cases 1 and 2, when \( E_S \ll E_I \) the secondary network is almost in outage and
exhibits poor error performance (see the parts of the curves before the cutoff points), while when $E_I \ll E_S$ (Case 1) and assuming that the interference power is increasing with the transmit power of the relay and the secondary sources, the SEP improvement is visible only in the medium SNR range. With an increase in the transmit power $E_S$, the error probability reaches a floor at high SNR. Therefore, the PNC cognitive relaying system is more vulnerable to noise than to interference for low and moderate SNRs, whereas it is more susceptible to interference at high SNR.

Additionally, some interesting observations are drawn from Figs. 4 to 8, as summarized next: (i) There is a close match between the asymptotic results and the simulations, even for low SNRs. Besides, in the low-to-medium SNR range, as the SNR increases the SEP performance improves because the dominant factor is the AWGN. (ii) The performance of the interference-free system ($L = [0, 0, 0]$) as well as that of the single-interferer case ($L = [1, 1, 1]$) are included as benchmark. In these two special cases, the simulation results are in good agreement with the analytical ones (Corollary 1 and 2, respectively). (iii) For the special scenario where the SUs terminals transmit with the same power characteristics as the interfering terminals, implying that the interference-to-noise ratio (INR) and the SNR tend to infinity simultaneously as the additive noise power becomes negligible, the presence of interference at the secondary nodes induces a floor level at high SNR in the SEP performance, which is reflected in a zero diversity order (as indicated by the slope of the curves), while for the interference-free case error floors do not occur. This demonstrates that the use of interference cancellation is crucial for attaining the beneficial effects of diversity. (iv) It can be concluded from the results in Figs. 4-8 that the number of interfering signals has no effect on the SEP in the low SNR range, whereas a degradation can be seen as the SNR increases. (v) As expected, there is a significant improvement in performance as the fading parameters ($m$) and the primary’s OP threshold ($\lambda$) increase. (vi) Since both the source nodes and the relay experience interferences from the primary side, the floor point on the error performance is reached at lower SNR values.

We now turn our attention to Fig. 9, which illustrates the achievable rate performance of the system for different distributions of interferers, when their total is constant (here fixed to 27), and different values of $\lambda$, for $m = 2$. It can clearly be seen that with the increase of $\lambda$ a better performance is achieved. The gap between the simulation and analytical results is due to the use of the Jensen’s inequality in
the derivation of expressions (51)-(54). The unmarked curve shows the rate $R_1$ of the system when $C_{\gamma R,S_1}$, $C_{\gamma R,S_2}$, $C_{\gamma S_1}$, and $C_{\gamma S_2}$ are computed by simulation (for clarity, only the case of $L = [27, 0, 0]$ is shown). As expected, an equal number of interferers at each node of the secondary network gives better performance. Also, the worse performance occurs for $L = [27, 0, 0]$, i.e. when only $S_1$ is affected by interference.

VII. CONCLUSION

This paper considered a traditional primary network coexisting with a two-way cognitive relay network where two SU source nodes communicate with each other through a relay using a PNC protocol while sharing the spectrum with multiple PUs. We investigated the effects of interference created by multiple primary transceivers and by the CRN on the performance of both a target PU and the SUs. The desired signals were assumed to be subject to Nakagami-$m$ fading. Furthermore, it was assumed that there is an arbitrary number of interferers subject to both i.i.d. and i.n.i.d. Nakagami-$m$ fading, with each interfering signal having a different power and undergoing a different amount of fading. Exact closed-form expressions for the OP and SEP of a target PU were derived. For the SUs, closed-form expressions for the SEP and its lower bound, as well as an upper bound on the achievable rates were derived. Cases of interference-free and single interference reception at the SUs were studied by deriving new expressions for the average SEP valid for BPSK modulation. Simple asymptotic expressions for the error performance were also developed. It was shown that interference at the secondary layer leads to floor levels in the SEP, which occur because the higher the SNR the higher the interference on information-bearing link. The simulation results indicate that the fading parameters have significant impact on the OP and SEP performances. Furthermore, for low SNR values, the error performance is not sensitive to the number of co-channel interfering signals. Comparisons with simulation results showed that the newly developed analytical expressions for the average SEP accurately predict the system’s performance.

APPENDIX I

PROOF OF PROPOSITION 1

According to (10) and making the change of variables $x = \frac{E_P}{N_0} |h_P|^2$, $y = \sum_{j=1}^{L_{FP}} |f_{P,j}|^2$ and $z = E_S(|f_{P,S_1}|^2 + |f_{P,S_2}|^2)$, the PDFs of these RVs are given by $f_X(x) = \tilde{\gamma}^m P \frac{\Gamma(m-1)}{(m-1)!} \exp(-\tilde{\gamma} px)$, $f_Y(y) = \sum_{j=1}^{L_{FP}} \frac{\alpha_j k_j}{(k_j)!} y^{k_j-1} \exp(-\tilde{\gamma} j y)$ and $f_Z(z) = \sum_{l=1}^{L_{FS}} \sum_{w=1}^{M_{PS}} \theta_{lw} \frac{1}{(w)!} z^{w-1} \exp(-\tilde{\gamma} P S_1 z)$.

According to (10), the CDF of the primary SINR is obtained as

$$F_P(\gamma) = \mathbb{E}_y[\mathbb{E}_w[Pr(x \leq \gamma (y + w + z + \sigma_P^2) | y, z, w)]]$$

$$= \int_0^\infty \int_0^\infty \int_0^\infty f_X(x) f_Y(y) f_Z(z) dx dy dz$$

$$= \int_0^\infty \int_0^\infty F_x(\gamma (y + z + 1)) f_Y(y) f_Z(z) f_W(w) dw dz$$

$$= \int_0^\infty \int_0^\infty \left[1 - \sum_{n=0}^{m-1} \left(\tilde{\gamma} P (y + z + 1)^\gamma \right)^n \frac{\Gamma(n+1)}{\Gamma(n+1)} \right] f_Y(y) f_Z(z) dz$$

$$= \int_0^\infty \int_0^\infty \left[1 - \sum_{n=0}^{m-1} \sum_{r=0}^{n} \frac{\tilde{\gamma} P r^n (y + z + 1)^\gamma}{\Gamma(n+1)} \right] f_Y(y) f_Z(z) dz$$

$$= \int_0^\infty \int_0^\infty \left[1 - \sum_{n=0}^{m-1} \sum_{r=0}^{n} \frac{\tilde{\gamma} P r^n (y + z + 1)^\gamma}{\Gamma(n+1)} \right] f_Y(y) f_Z(z) dz$$

$$= \int_0^\infty \int_0^\infty \left[1 - \sum_{n=0}^{m-1} \sum_{r=0}^{n} \frac{\tilde{\gamma} P r^n (y + z + 1)^\gamma}{\Gamma(n+1)} \right] f_Y(y) f_Z(z) dz$$

$$= \int_0^\infty \int_0^\infty f_Y(y) f_Z(z) dz$$

where $\mathbb{E}[\cdot]$ represents expectation. (80) can be split into two separate integrals as follows

$$I_0 = \int_0^\infty \int_0^\infty f_Y(y) f_Z(z) dz = 1$$

$$I_1 = \sum_{j=1}^{m-1} \sum_{r=0}^{n} \frac{\tilde{\gamma} P r^n (y + z + 1)^\gamma}{\Gamma(n+1)}$$

$$\times \exp(-\tilde{\gamma} P r y - z^\gamma) y^{n-r} \exp(-\tilde{\gamma} P r y)$$

$$\times f_Y(y) f_Z(z) dz$$

$$\times f_Y(y) f_Z(z) dz$$

$$\times f_Y(y) f_Z(z) dz$$
Finally, the CDF of the primary SINR is expressed as shown in (83). In terms of (83), the OP of the SINR of the PU can be directly expressed as

$$P_{\text{out}}^{\text{pri}} = P_{\text{f}}(\gamma_{\text{th}}).$$

(84)

To ensure that the primary’s communications is reliable, the corresponding OP shall remain below a threshold $\lambda$ and we must have

$$P_{\text{out}}^{\text{pri}} \leq \lambda \Rightarrow P_{\text{out}}^{\text{pri}} - \lambda \leq 0.$$  

(85)

Finally, by solving (85) w.r.t. $E_S$, the power constraint of $S_1$ and $S_2$ can be achieved as shown in (11).

**APPENDIX II**

**PROOF OF PROPOSITION 2**

To begin, the CDF of $\gamma_{R,S_1}$ in (8) is obtained as

$$F_{\gamma_{R,S_1}}(\gamma) = K_{S_1,h,S_1,\lambda,k},$$

(86)

where

$$K_{S_1,h,S_1,\lambda,k} = 1 - \sum_{j=1}^{\text{LS}_S} \sum_{1=1}^{\text{MS}_S} \sum_{i=0}^{n-1} \frac{\gamma_{h,S_1,\lambda,k}^{i+1}}{\Gamma(n+1)\Gamma(k)} \sum_{j=0}^{n-1} \frac{n}{i} \frac{\gamma_{j,S_1}}{(\gamma_{j,S_1} + \gamma_{S_1,j})^{k+1}} \exp(-\gamma_{h,S_1}).$$

(87)

By substituting (86) into (13), with [45, Eq. 9.211.4], and doing some manipulations, we arrive at (18). Next, we derive $P^b(\gamma_{R})$. We have

$$P^b(\gamma_{R}) = P^b(\gamma_{S_1,R}|e_{S_2,R}) P^b(\gamma_{S_2,R})$$

$$+ P^b(\gamma_{S_2,R}|e_{S_2,R}) P^b(\gamma_{S_1,R}).$$

(88)

$P^b(\gamma_{S_1,R})$ and $P^b(\gamma_{S_2,R})$ can be further expressed as

$$P^b(\gamma_{S_1,R}) = P^b(\gamma_{S_1,R}|e_{S_2,R}) P^b(\gamma_{S_2,R}),$$

(89)

$$P^b(\gamma_{S_2,R}) = P^b(\gamma_{S_2,R}|e_{S_2,R}) P^b(\gamma_{S_1,R}).$$

(90)

With the help of (89) and (90), we obtain (91) and (92). To find $P^b(\gamma_{S_1,R}|e_{S_2,R}), P^b(\gamma_{S_2,R}|e_{S_2,R}), P^b(\gamma_{S_1,R}|e_{S_2,R})$ and $P^b(\gamma_{S_2,R}|e_{S_2,R})$, we make use of the CDF-based approach. To this end, the SINRs $\gamma_{S_1,R,e_{S_2,R}}, \gamma_{S_2,R,e_{S_2,R}}, \gamma_{S_1,R,e_{S_2,R}}$ and $\gamma_{S_2,R,e_{S_2,R}}$ need to be obtained. According to (3), these SINRs at the relay are given by

$$\gamma_{S_1,R,e_{S_2,R}} = \frac{E_S|h|^2}{\sum_{j=1}^{\text{LS}_S} |E_{R,j}| |f_{j,R}|^2 + N_0},$$

(93)

$$\gamma_{S_1,R,e_{S_2,R}} = \frac{E_S|h|^2}{2E_S|g|^2 + \sum_{j=1}^{\text{LS}_S} |E_{R,j}| |f_{j,R}|^2 + N_0}.$$

(94)

and their corresponding CDFs can be obtained as

$$F_{\gamma_{S_1,R,e_{S_2,R}}}(\gamma) = C_{R,h,R,M,j,k},$$

(95)

$$F_{\gamma_{S_1,R,e_{S_2,R}}}(\gamma) = N_{h,g},$$

(96)

where

$$N_{h,g} = 1 - \sum_{n=0}^{\text{LS}_S} \sum_{k=1}^{\text{MS}_S} \frac{\gamma_{h,R}^n}{\Gamma(n+1)\Gamma(k)} \sum_{j=0}^{n-1} \frac{n}{i} \frac{\gamma_{j,R}}{(\gamma_{j,R} + \gamma_{R,j})^{k+1}} \exp(-\gamma_{h,R}).$$

(97)

Substituting (95), (96), $F_{\gamma_{S_1,R,e_{S_2,R}}}(\gamma)$ and $F_{\gamma_{S_2,R,e_{S_2,R}}}(\gamma)$ into (13), we reach the equations shown in Proposition 2.

**REFERENCES**


\[ F_P(\gamma) = 1 - \sum_{j=1}^{L} \sum_{k=1}^{M_j} \sum_{t=1}^{L_E} \sum_{o=1}^{M_j} \gamma^{n} \sum_{r=0}^{n} \sum_{q=0}^{r} \frac{\gamma_p \Gamma(n + k - r) \Gamma(t + q) \theta_{iw} \alpha_{jk} \left( \frac{n}{r} \right) \left( \frac{r}{q} \right)}{\Gamma(n + 1) \Gamma(\gamma)} \exp(-\gamma_p \gamma). \] (83)

\[ P_b^b(\gamma_{S_1 R}) = \frac{P_b^b(\gamma_{S_1 R} | e_{S_2 R}) + P_b^b(\gamma_{S_2 R} | e_{S_1 R}) \left( P_b^b(\gamma_{S_1 R} | e_{S_2 R}) - P_b^b(\gamma_{S_1 R} | e_{S_2 R}) \right)}{1 - \left[ P_b^b(\gamma_{S_1 R} | e_{S_2 R}) - P_b^b(\gamma_{S_1 R} | e_{S_2 R}) \right] \left[ P_b^b(\gamma_{S_1 R} | e_{S_2 R}) - P_b^b(\gamma_{S_2 R} | e_{S_1 R}) \right]}. \] (91)

\[ P_b^b(\gamma_{S_2 R}) = \frac{P_b^b(\gamma_{S_2 R} | e_{S_1 R}) + P_b^b(\gamma_{S_1 R} | e_{S_2 R}) \left( P_b^b(\gamma_{S_2 R} | e_{S_1 R}) - P_b^b(\gamma_{S_2 R} | e_{S_1 R}) \right)}{1 - \left[ P_b^b(\gamma_{S_1 R} | e_{S_2 R}) - P_b^b(\gamma_{S_1 R} | e_{S_2 R}) \right] \left[ P_b^b(\gamma_{S_2 R} | e_{S_1 R}) - P_b^b(\gamma_{S_2 R} | e_{S_1 R}) \right]}. \] (92)

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