# Adaptive Linearly Constrained Minimum Variance Beamforming for Multiuser Cooperative Relaying Using the Kalman Filter

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Abstract—In this paper, we consider a wireless communication scenario with multiple source-destination pairs communicating through several cooperative amplify-and-forward relay terminals. The relays are equipped with multiple antennas that receive the source signals and transmit them to the destination nodes. We develop two iterative relay beamforming algorithms that can be applied in real-time. In both algorithms, the relay beamforming matrices are jointly designed by minimizing the received power at all the destination nodes while preserving the desired signal at each destination. The first algorithm requires the existence of a local processing center that computes the beamforming coefficients of all the relays. In the second algorithm, each relay can compute its beamforming coefficients locally with the help of some common information that is broadcasted from the other relays. This is achieved at the expense of enforcing the desired signal preservation constraints non-cooperatively. We provide two extensions of the proposed algorithms that allow the relays to control their transmission power and to modify the quality of service provided to different sources. Simulation results are presented validating the ability of the proposed algorithms to perform their beamforming tasks efficiently and to track rapid changes in the operating environment.

*Index Terms*—Adaptive signal processing, cooperative relay beamforming, Kalman filtering.

# I. INTRODUCTION

**C**OOPERATIVE relaying systems have received considerable attention in the recent years, see [1]–[3], and the references therein. The basic idea of cooperative relaying is to introduce intermediate nodes (relays) that collaboratively forward the received data from the source to the destination. Cooperative relaying brings several advantages to wireless communication systems [3]. For instance, it increases the range of communication [4] and provides spatial diversity which can be exploited by applying distributed space-time

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coding [5]. Cooperative relaying can also be used to provide spatial multiplexing in multiuser communication systems where multiple signal sources are targeting one or more destination nodes [1].

Many noncooperative multiuser zero-forcing relay beamforming algorithms have appeared in the literature, e.g., [6] and [7]. Multiuser cooperative zero-forcing relay beamforming was also proposed in [8]. All these relaying techniques use beamforming to eliminate the interference between different source-destination pairs. They require perfect knowledge of all the source-relay and relay-destination channels. This channel state information can be estimated using orthogonal pilot sequences transmitted from all the source and destination nodes. However, zero-forcing beamforming is known to be suboptimal when the signal-to-noise ratio (SNR) of the sources is relatively low as it results in increased noise power at the destination nodes [9].

In [10], we have developed a multiuser cooperative relay beamforming algorithm for wireless communication networks. In this algorithm, the beamforming matrices of the relay terminals are jointly designed such that both the noise received at each destination node and the interference caused by the sources not targeting this node are minimized. Each source signal is preserved at its targeted destination node via linear constraints. The resulting optimization problem was formulated as a convex second-order cone program (SOCP) that could be efficiently solved with polynomial complexity using interior point methods [11], [12]. However, the shortcoming of the algorithm in [10], and also of the cooperative zeroforcing beamforming algorithm in [8], is that it does not have a direct online implementation. Hence, every time one of the source-relay or relay-destination channels changes, the relay beamforming matrices have to be recomputed. This might not be computationally efficient, specially in nonstationary environments, e.g., with mobile signal sources.

In this paper, we develop iterative beamforming algorithms for cooperative amplify-and-forward MIMO-relaying wireless systems with multiple source-destination pairs. We assume that the relays can estimate their relay-destination channels with enough accuracy, for example, through training<sup>1</sup>. This assumption is well justified in e.g., outdoor wireless communication scenarios where the relays and the destination base stations are not mobile. The relay-destination channel informa-

<sup>1</sup>See also our related work on cooperative training-based adaptive beamforming for multiuser relaying wireless systems in [13].

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tion can be obtained using channel reciprocity in time division multiple access systems or by feedback from the destination nodes. We also assume that the relays can estimate and track the source-relay channels [14], [15]. We develop two adaptive relay beamforming algorithms using linearly constrained minimum variance (LCMV) beamforming. In these algorithms, the received power at all the destination nodes is minimized subject to linear constraints on the relay beamforming matrices [16]. These constraints are used to prevent the cancellation of the desired signal at each destination node. As a result, the sum of the interference and noise forwarded by the relays to the destination nodes is minimized [17]. The proposed beamforming algorithms can be applied in real time using Kalman filtering [18]. In both algorithms, we use a statespace modelling approach to solve the underlying LCMV optimization problem similar to the approach used in [19].

In the first algorithm, we assume the existence of a local processing center that is wired to the relays. The processing center receives all the required data from the relays, computes the beamforming coefficients, and feeds them back to the relays. This centralized algorithm requires a considerable amount of data exchange between the processing center and the relays every time the relay beamforming matrices are updated. In the second algorithm, each relay can estimate its beamforming coefficients locally using its received data, its local channel estimates, and some information that is broadcasted from the other relays. The relay beamforming matrices are designed such that they cooperatively minimize the received power at each destination node. However, in order to allow the decentralized computation of the beamforming coefficients, we impose the signal preservation constraints non-cooperatively, i.e., each relay beamformer is constrained such that it preserves the desired component of the received signal at each destination node due to its transmission only. In contrast, in the centralized beamforming algorithm, the desired signal component received at each destination node due to the aggregate transmission of all the relays is preserved. Hence, the distributed relay beamforming algorithm has fewer degrees of freedom for interference suppression than those available in the centralized algorithm. We also present two extensions of the proposed algorithms that allow the relays to control their transmission power and to modify the quality of service (QoS) provided to different sources. We provide numerical simulations that validate the efficacy of the proposed algorithms and their ability to track rapid changes in the operating environment.

The remainder of this paper is organized as follows. In Section II, we present the signal model and formulate the multiuser relay beamforming problem. Sections III and IV present the proposed centralized and decentralized beamforming algorithms, respectively. Section V contains the power control and QoS modification algorithms. Numerical simulation results are presented in Section VI. Finally, the paper is concluded in Section VII.

# **II. PROBLEM FORMULATION**

We consider a wireless communication scenario as depicted in Fig. 1 with J sources communicating with J destination nodes through K relay terminals, where the *j*th source is



Fig. 1. System model.

targeting the *j*th destination node only. We assume that the *k*th relay is equipped with an  $m_k$ -element antenna array that is used for receiving from the sources and transmitting to the destination nodes. The relays operate in half duplex mode, i.e., we consider a relaying strategy that uses a two-hop relaying protocol in which signal reception and transmission at the relays are time-division duplexed. At the *n*th time instant, communication between the source and destination nodes occurs in two phases. In the first phase, the sources forward their data to the relays, and in the second phase, the relays transmit the processed data to the destination nodes. Let  $h_k^{(j)}$  denote the  $m_k \times 1$  vector containing the channel coefficients (including the path loss) from the *j*th source to the *k*th relay. The  $m_k \times 1$  received signal vector at the *k*th relay terminal after the first phase of the *n*th time instant can be written as

$$\boldsymbol{x}_{k}(n) = \sum_{j=1}^{J} \sqrt{P_{j}} \boldsymbol{h}_{k}^{(j)} s_{j}(n) + \boldsymbol{n}_{k}^{(\mathrm{r})}(n)$$
(1)

where  $s_j(n)$  is the unit-power signal transmitted by the *j*th source,  $P_j$  is the transmission power of the *j*th source,  $n_k^{(r)}(n)$  is the  $m_k \times 1$  vector of white Gaussian noise with zero-mean and covariance matrix  $\sigma_k^{(r)^2} I$ , and  $(\cdot)_k^{(r)}$  refers to the *k*th relay terminal.

The received signal vector by the kth relay at the nth time instant is linearly processed by the  $m_k \times m_k$  beamforming matrix  $W_k(n)$  and then transmitted to the destination nodes. Note that we consider an amplify-and-forward relaying scenario, and hence, the relays do not need to decode or separate the data streams of the sources.

In this work, we assume that each of the J destination nodes is equipped with a single antenna<sup>2</sup>. Let  $g_k^{(j)}$  be the  $m_k \times 1$  vector containing the complex conjugate of the channel coefficients from the kth relay to the *j*th destination node (targeted by the *j*th source). Therefore, we can write the

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<sup>&</sup>lt;sup>2</sup>The extension to the case of multiple antenna destinations where multiple sources are targeting the same destination node will be considered in our future work. In this case, the relay beamforming matrices and the destination receive beamformers have to be jointly designed.

received signal at the jth destination after the second phase of the nth time instant as

$$y_j(n) = \sum_{k=1}^{K} \boldsymbol{g}_k^{(j)^H} \boldsymbol{W}_k^H(n) \, \boldsymbol{x}_k(n) + n_j^{(d)}(n)$$
(2)

where  $n_j^{(d)}(n)$  is the white Gaussian noise with zero-mean and variance  $\sigma_j^{(d)^2}$  induced at the *j*th destination node,  $(\cdot)_j^{(d)}$ refers to the *j*th destination node, and  $(\cdot)^T$  and  $(\cdot)^H$  denote the transpose and Hermitian transpose, respectively.

The function of the relay beamforming matrices is to deliver each of the J source signals to its destination with minimum noise and interference from the other sources. We define the received signal to interference-plus-noise ratio (SINR) at the *j*th destination node as the ratio between the desired signal power and the total power of interference (caused by the sources not targeting the *j*th destination) and noise (both forwarded by the relays and generated at the destination nodes). Equation (3) at the bottom of this page provides an expression for the received SINR at the *j*th destination.

For a single source-destination pair, maximizing the received SINR at the destination can be achieved using LCMV optimization, in which we constrain the numerator and minimize the denominator of (3). In this paper, we will adopt an LCMV design approach for the relay beamforming problem with multiple source-destination pairs. We impose the following linear constraints on the relay beamforming matrices at every time instant

$$\sum_{k=1}^{K} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{h}_{k}^{(j)} = 1 \qquad \forall j = 1, \dots, J.$$
 (4)

The above constraints have the effect of preserving the desired component of the received signal at the destination nodes. They are commonly referred to as signal preservation constraints [20].

In [10], we have presented a cooperative beamforming algorithm that employs the constraints in (4) while minimizing both the interference power received at the destination nodes and the power of the noise forwarded by the relays to the destinations. For a given set of source-relay channels  $\{\boldsymbol{h}_{k}^{(j)}\}_{k,j}$  and relay-destination channels  $\{\boldsymbol{g}_{k}^{(j)}\}_{k,j}$ , this algorithm calculates the beamforming coefficients  $\{\boldsymbol{W}_{k}\}_{k}$  that solve

$$\min_{\{\boldsymbol{W}_{k}\}_{k=1}^{K}} \sum_{j=1}^{J} \sum_{i \neq j} P_{i} \Big| \sum_{k=1}^{K} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H} \boldsymbol{h}_{k}^{(i)} \Big|^{2} \\
+ \sum_{j=1}^{J} \sum_{k=1}^{K} \sigma_{k}^{(r)^{2}} \| \boldsymbol{W}_{k} \boldsymbol{g}_{k}^{(j)} \|^{2} \\
\text{s.t.} \sum_{k=1}^{K} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H} \boldsymbol{h}_{k}^{(j)} = 1 \quad \forall j = 1, \dots, J. \quad (5)$$

The above optimization problem was formulated as an SOCP that can be efficiently solved with polynomial complexity using interior point methods with a worst-case computational load of  $\mathcal{O}\left(J^{\frac{3}{2}}(M+J)(M^2+J)^2\right)$  where  $M = \sum_k m_k$ is the total number of relay antennas [10]–[12]. One of the advantages of this formulation is that any convex constraints can be easily incorporated into the problem. Examples of such constraints include power constraints on the relay terminals and QoS constraints on the received SINR at the destination nodes [21]. However, the shortcoming of the algorithm in [10], is that it does not have a direct online implementation. Hence, every time one of the source-relay or relaydestination channels changes, the beamforming matrices have to be recomputed. This might not be efficient, specially in nonstationary environments.

In this paper, we present two adaptive relay beamforming algorithms that can be implemented in real-time. We design the relay beamforming matrices such that the received power at the J destination nodes is minimized subject to the signal preservation constraints in (4). This is equivalent to minimizing, under the same constraints, the interference and noise power forwarded by the relays to the destination nodes [20], i.e., the denominator of (3) is minimized. Thus, we can write the relay beamforming problem at the nth time instant as

$$\min_{\{\boldsymbol{W}_{k}(n)\}_{k=1}^{K}} \sum_{j=1}^{J} \left| \sum_{k=1}^{K} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{x}_{k}(n) \right|^{2}$$
s.t. 
$$\sum_{k=1}^{K} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{h}_{k}^{(j)} = 1 \quad \forall j = 1, \dots, J. \quad (6)$$

One of the advantages of the above problem formulation is that it allows the development of real-time adaptive relay beamforming algorithms. For this purpose, it is convenient to reformulate (6) more compactly as follows. Let  $w_k(n) =$ vec  $\{W_k(n)\}$  where vec  $\{\cdot\}$  is the vectorization operator that stacks the columns of a matrix on top of one another. Using the matrix identity vec  $\{ABC\} = (C^T \otimes A)$ vec  $\{B\}$  where  $\otimes$  denotes the Kronecker product of two matrices, we can write the relay beamforming design problem in (6) as

$$\min_{\{\boldsymbol{w}_{k}(n)\}_{k=1}^{K}} \qquad \sum_{j=1}^{J} \left| \sum_{k=1}^{K} \boldsymbol{a}_{k}^{(j)^{T}}(n) \boldsymbol{w}_{k}(n) \right|^{2}$$
  
s.t. 
$$\sum_{k=1}^{K} \boldsymbol{C}_{k}^{H} \boldsymbol{w}_{k}(n) = \mathbf{1}_{J}.$$
(7)

where the  $m_k^2 \times 1$  vector  $\boldsymbol{a}_k^{(j)}(n) = \boldsymbol{g}_k^{(j)} \otimes \boldsymbol{x}_k^*(n)$ , the  $m_k^2 \times J$  matrix  $\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{c}_k^{(1)}, \dots, \boldsymbol{c}_k^{(J)} \end{bmatrix}$ ,  $\boldsymbol{c}_k^{(j)} = \boldsymbol{g}_k^{(j)*} \otimes \boldsymbol{h}_k^{(j)}$ ,  $\boldsymbol{1}_J$  is the  $J \times 1$  vector containing ones, and  $(\cdot)^*$  denotes the complex conjugate operator. Throughout this work, we assume that each relay can estimate its relay-destination channels and the channels from the desired sources. Therefore, the *k*th relay terminal can compute the vectors  $\{\boldsymbol{a}_k^{(j)}(n)\}_{j=1}^J$  using its knowledge of the channel vectors of the destination nodes

$$\operatorname{SINR}_{j}(n) = \frac{P_{j} \left| \sum_{k} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{h}_{k}^{(j)} \right|^{2}}{\sum_{i \neq j} P_{i} \left| \sum_{k} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{h}_{k}^{(i)} \right|^{2} + \sum_{k} \sigma_{k}^{(r)^{2}} \left\| \boldsymbol{W}_{k}(n) \boldsymbol{g}_{k}^{(j)} \right\|^{2} + \sigma_{j}^{(d)^{2}}}.$$
(3)

and the received data vector at the *n*th time instant. It can also compute the matrix  $C_k$  using its estimate of the sourcerelay and relay-destination channels. In the next two sections, we will develop two adaptive algorithms that can be used to estimate the beamforming matrices of the relays iteratively.

## **III. CENTRALIZED ADAPTIVE BEAMFORMING**

In this section, we derive an adaptive algorithm to solve the relay beamforming problem in (7). We assume the existence of a local processing center that is connected to all the relays. The relays send their estimates of the source and destination channels to the processing center. Also, at the *n*th time instant, each relay transmits its received data vector  $\boldsymbol{x}_k(n)$  to the processing center. The processing center then computes the relay beamforming matrices and feeds them back to the relays.

Let us define the  $(\sum_k m_k^2) \times 1$  stacked beamforming vector at the *n*th time instant as  $\boldsymbol{w}(n) = [\boldsymbol{w}_1^T(n), \dots, \boldsymbol{w}_K^T(n)]^T$ . We can write the problem in (7) as

$$\min_{\boldsymbol{w}(n)} \sum_{j=1}^{J} \left| \boldsymbol{a}^{(j)^{T}}(n) \boldsymbol{w}(n) \right|^{2} \qquad \text{s.t. } \boldsymbol{C}^{H} \boldsymbol{w}(n) = \mathbf{1}_{J} \qquad (8)$$

where  $\boldsymbol{a}^{(j)}(n) = \left[\boldsymbol{a}_{1}^{(j)^{T}}(n), \ldots, \boldsymbol{a}_{K}^{(j)^{T}}(n)\right]^{T}$  and the  $\sum_{k} m_{k}^{2} \times J$ matrix  $\boldsymbol{C} = \left[\boldsymbol{C}_{1}^{H}, \ldots, \boldsymbol{C}_{K}^{H}\right]^{H}$  which is assumed to be full row-rank. We start by eliminating the linear equality constraints in (8) using the equivalent generalized sidelobe canceller implementation of the LCMV algorithm [22], [23]. Let  $\boldsymbol{w}(n) = \boldsymbol{N}\boldsymbol{v}(n) + \boldsymbol{w}_{0}$ , where the columns of the matrix  $\boldsymbol{N}$  span the null space of the matrix  $\boldsymbol{C}$ , i.e.,  $\boldsymbol{C}^{H}\boldsymbol{N} = \boldsymbol{0}$ , the  $(\sum_{k} m_{k}^{2} - J) \times 1$  vector  $\boldsymbol{v}(n)$  is the new design vector at the *n*th time instant, and  $\boldsymbol{w}_{0} = \boldsymbol{C} \left(\boldsymbol{C}^{H}\boldsymbol{C}\right)^{-1} \boldsymbol{1}_{J}$ . Therefore, we can write the beamforming design problem in (8) as

$$\min_{\boldsymbol{v}(n)} \sum_{j=1}^{J} \left| \boldsymbol{a}^{(j)^{T}}(n) \left( \boldsymbol{N} \boldsymbol{v}(n) + \boldsymbol{w}_{0} \right) \right|^{2}.$$
 (9)

We will use a state-space modelling approach to minimize the cost function in (9) similar to that used in [19]. We can write the process equation of the state-space model describing the relay beamforming design problem as

$$\boldsymbol{v}(n+1) = \boldsymbol{v}(n) + \boldsymbol{n}_v(n) \tag{10}$$

where the  $(\sum_k m_k^2 - J) \times 1$  vector  $\boldsymbol{v}(n)$  is the state vector and  $\boldsymbol{n}_v(n)$  is the process noise. The process noise allows tracking of the beamforming vector in nonstationary environments and is assumed to be white Gaussian with zero-mean and covariance  $\boldsymbol{Q} = \sigma_v^2 \boldsymbol{I}$ .

The measurement equation associated with (10) is given by

$$\boldsymbol{z}(n) = \boldsymbol{B}(n)\boldsymbol{v}(n) + \boldsymbol{n}_m(n) \tag{11}$$

where the  $J \times 1$  measurement vector  $\boldsymbol{z}(n)$  is given by

$$\boldsymbol{z}(n) = [-\boldsymbol{a}^{(1)^{T}}(n) \, \boldsymbol{w}_{0}, \dots, -\boldsymbol{a}^{(J)^{T}}(n) \, \boldsymbol{w}_{0}]^{T}, \qquad (12)$$

and the  $J imes \left( \sum_k m_k^2 - J \right)$  measurement matrix

$$\boldsymbol{B}(n) = \begin{bmatrix} \boldsymbol{N}^T \boldsymbol{a}^{(1)}(n) & \dots & \boldsymbol{N}^T \boldsymbol{a}^{(J)}(n) \end{bmatrix}^T.$$
(13)

In the above state-space model, the  $J \times 1$  vector  $\boldsymbol{n}_m(n)$  is the measurement noise and is assumed to be white Gaussian with zero-mean and covariance  $\boldsymbol{R} = \text{diag} \{\sigma_{m,1}^2, \ldots, \sigma_{m,J}^2\}$  and independent of the process noise.

Based on the above state-space model, a state estimator, e.g., the Kalman filter, can be used to estimate and track the design vector v(n). The estimator will yield a vector that minimizes the mean square values of the components of the measurement noise, i.e., the noise and interference power received at the J destination nodes. Hence, the cost function in (9) will be minimized. The parameters of the state-space model should be chosen as follows. The process noise variance  $\sigma_v^2$  should be selected to reflect the degree of nonstationarity of the environment. For example, setting  $\sigma_v^2 = 10^{-6}$  means that we expect each component of the vector v(n) to change by the order of  $10^{-3}$  every time instant. Also, the value of  $\sigma_{m,j}^2$  at the *n*th time instant can be calculated as the mean square value of the *m*th component of the vector  $n_m(n)$ , i.e.,

$$\sigma_{m,j}^{2}(n) = \mathbf{E}\left\{ \left| \boldsymbol{a}^{(j)^{T}}(n) \left( \boldsymbol{N}\boldsymbol{v}(n) + \boldsymbol{w}_{0} \right) \right|^{2} \right\}$$
$$= \mathbf{E}\left\{ \left| \sum_{k=1}^{K} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{x}_{k}(n) \right|^{2} \right\}$$
(14)

$$\approx P_j + \sum_{k=1}^{K} \sigma_k^{(\mathbf{r})^2} \boldsymbol{g}_k^{(j)^H} \boldsymbol{W}_k^H(n) \boldsymbol{W}_k(n) \boldsymbol{g}_k^{(j)} \quad (15)$$

where  $E\{\cdot\}$  denotes the statistical expectation and the approximation in (15) comes from the assumption that the interference from the sources that are not targeting the *j*th destination node is effectively blocked by the relay beamforming matrices. Note that the expression in (15) requires knowledge of the optimum beamforming matrices  $\{W_k(n)\}_{k=1}^K$  at every time instant. Such information may not available in practice. However, it will be shown through numerical simulations that the Kalman filtering algorithm is not very sensitive to the choice of the values of the parameters  $\{\sigma_{m,j}^2\}_{j=1}^J$ . Hence, satisfactory performance can still be obtained over a wide range of selection of these parameters.

The recursion for the estimated weight vector starts with an initial random weight vector estimate  $\hat{v}(0)$  together with its associated covariance P(0|0) = I. The weight vector estimate is updated through

$$\hat{\boldsymbol{v}}(n) = \hat{\boldsymbol{v}}(n-1) + \boldsymbol{G}(n) \left( \boldsymbol{z}(n) - \boldsymbol{B}(n) \hat{\boldsymbol{v}}(n-1) \right) \quad (16)$$

where  $\hat{v}(n)$  is the state vector estimate at the *n*th time instant and the filter gain G(n) is given by

$$G(n) = P(n|n-1)B^{H}(n)S^{-1}(n).$$
 (17)

The innovation covariance matrix S(n) and the covariance matrix of the predicted weight vector P(n|n-1) are given respectively by

$$\boldsymbol{S}(n) = \boldsymbol{B}(n)\boldsymbol{P}(n|n-1)\boldsymbol{B}^{H}(n) + \boldsymbol{R} \quad (18)$$

$$P(n|n-1) = P(n-1|n-1) + Q,$$
 (19)

and the updated state covariance matrix is given by

$$\boldsymbol{P}(n|n) = \boldsymbol{P}(n|n-1) - \boldsymbol{G}(n)\boldsymbol{S}(n)\boldsymbol{G}^{H}(n).$$
(20)

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The above iterative algorithm in (16)–(20) estimates the adaptive portion of the beamforming matrices of the Krelays jointly. The estimation process is performed at a local processing center which employs a Kalman filter having a computational complexity of  $\mathcal{O}\left(J\left(\sum_k m_k^2\right)^2\right)$  per iteration. The processing center then computes the stacked beamforming vector  $\hat{\boldsymbol{w}}(n) = \boldsymbol{N}\hat{\boldsymbol{v}}(n) + \boldsymbol{w}_0$  and feeds back the beamforming matrix  $\hat{W}_k(n)$  to the kth relay. Thus, a total number of  $\sum_k m_k^2$  coefficients are fed back from the processing center at every update of the relay beamforming matrices<sup>3</sup>. The kth relay then uses its beamforming matrix to transmit the vector  $\boldsymbol{W}_{k}(n)\boldsymbol{x}_{k}(n)$  in the second phase of the *n*th time instant to the destination nodes allowing the relays to operate in real-time provided that the processing center can compute and feedback the beamforming coefficients to the relays at a rate higher than the required update rate of the beamforming matrices. For practical values of  $\{m_k\}_{k=1}^K$  and J, the complexity of the proposed scheme is well within the reach of realtime implementation on currently available DSP hardware technology. Furthermore, the number of parameters that has to be transmitted from the kth relay to the processing center is only  $m_k$  parameters (the received signal vector). Note that the relays have to send their estimates of the source and destination channels to the processing center and update them every time these channels change. On the other hand, the processing center sends  $m_k^2$  parameters to the kth relay every time the beamforming matrices are updated.

# IV. DECENTRALIZED ADAPTIVE BEAMFORMING

The centralized beamforming algorithm presented in the previous section requires the existence of a local processing center that performs a considerable amount of data exchange with the relay terminals. This might not be feasible in some communication systems where the number of relay antenna elements is large. In this section, we will develop a decentralized adaptive beamforming algorithm that allows each relay terminal to compute its beamforming matrix locally with limited amount of data exchange with the other relays.

We assume that the kth relay terminal has access only to its received data vector  $oldsymbol{x}_k(n)$  and the estimates of its source-relay channels  $\{\boldsymbol{h}_{k}^{(j)}\}_{j=1}^{J^{n}}$ , and relay-destination channels  $\{g_k^{(j)}\}_{j=1}^J$ . Note that the joint enforcement of the signal preservation constraints in (4) requires each relay to know the channel vectors and beamforming matrices of the other relays. In order to decrease the amount of data exchanged between the relays and to facilitate the development of the decentralized beamforming algorithm, we replace the joint constraints on the beamforming matrices of the K relays in (4) by the following individual signal preservations constraints imposed on each of the K relay terminals

$$\boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{h}_{k}^{(j)} = \frac{1}{K} \quad \forall j = 1, \dots, J, k = 1, \dots, K.$$
 (21)

The above constraints non-cooperatively preserve the desired signals at the destination nodes as the contribution of each relay to the desired signals received at the destinations is fixed<sup>4</sup>. Nevertheless, we design the kth relay beamforming matrix such that we minimize the power of the signal received at the destination nodes due to the aggregate transmissions of all the relays. This allows the relays to cooperate in suppressing the interference signals at the destination nodes. Hence, we can write the distributed relay beamforming problem as

$$\min_{\{\boldsymbol{w}_{k}(n)\}_{k=1}^{K}} \sum_{j=1}^{J} \left| \sum_{k=1}^{K} \boldsymbol{a}_{k}^{(j)^{T}}(n) \boldsymbol{w}_{k}(n) \right|^{2}$$
s.t.  $\boldsymbol{C}_{k}^{H} \boldsymbol{w}_{k}(n) = \frac{1}{K} \boldsymbol{1}_{J} \quad \forall k = 1, \dots, K.$  (22)

Comparing the above decentralized problem formulationfor the K relays-with the centralized one in (8), we notice that the total number of design variables is the same, i.e.,  $\sum_{k} m_{k}^{2}$ . However, the signal preservation constraints in (22) consume KJ degrees of freedom from the relay beamforming matrices whereas the signal preservation constraints in (8) consume only J degrees of freedom. The decrease in the degrees of freedom available for beamforming is the price we have paid for preserving the desired signals at the destinations through the use of the noncooperative constraints in (21). As a result, we can expect the performance of the decentralized beamforming algorithm to be inferior to that of the centralized one especially when the SNR of the sources is high.

We start by eliminating the linear constraints in (22). Let  $\boldsymbol{w}_k(n) = \boldsymbol{N}_k \boldsymbol{v}_k(n) + \boldsymbol{w}_{0,k}$  where the columns of the matrix  $oldsymbol{N}_k$  span the null space of  $oldsymbol{C}_k$ , the  $(m_k^2-J) imes 1$  vector  $oldsymbol{v}_k(n)$ is the new design vector, and  $\boldsymbol{w}_{0,k} = \boldsymbol{C}_k \left( \boldsymbol{C}_k^H \boldsymbol{C}_k \right)^{-1} \boldsymbol{1}_J / K.$ Therefore, we can write (22) as the following unconstrained optimization problem

$$\min_{\{\boldsymbol{v}_{k}(n)\}_{k=1}^{K}} \sum_{j=1}^{J} \left| \sum_{k=1}^{K} \boldsymbol{a}_{k}^{(j)^{T}}(n) \left( \boldsymbol{N}_{k} \boldsymbol{v}_{k}(n) + \boldsymbol{w}_{0,k} \right) \right|^{2}.$$
 (23)

We will consider the design problem for the *i*th relay terminal. The *i*th relay can compute its quiescent beamforming vector  $w_{0,i}$  and the matrix  $N_i$  using its local channel state information. It employs a Kalman filter that iteratively estimates the adaptive component of its beamforming coefficients, i.e., the vector  $v_i(n)$ . The process equation for the adaptive beamforming coefficients of the *i*th relay terminal is given by

$$\boldsymbol{v}_i(n+1) = \boldsymbol{v}_i(n) + \boldsymbol{n}_{v_i}(n) \tag{24}$$

where  $n_{v_i}(n)$  is the process noise associated with the beamforming vector of the *i*th relay. It is also assumed to be white Gaussian with zero-mean and covariance  $Q_i = \sigma_{v_i}^2 I$ . In order to minimize the cost function in (23), we define the measurement equation associated with the process equation in

<sup>4</sup>Theoretically, it is possible to modify the constraint in (21) to become  $\boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H} \boldsymbol{h}_{k}^{(j)} = \beta_{k}^{(j)}$  where  $\{\beta_{k}^{(j)}\}$  are some additional optimization variables. These variables have to be constrained such that  $\sum_{k=1}^{K} \beta_{k}^{(j)} = 1$ in order to prevent the cancellation of the desired signal at the jth destination. However, enforcing this constraint and/or finding the optimal values of the parameters  $\beta_k^{(j)}$  can only be done using centralized processing.

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<sup>&</sup>lt;sup>3</sup>We note that the update rate of the relay beamforming matrices is dictated by the rate of change of the beamforming vector estimate which can be much lower than the data rate especially near convergence.

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(24) as

$$\boldsymbol{z}_{i}(n) = \boldsymbol{A}_{i}(n)\boldsymbol{N}_{i}\boldsymbol{v}_{i}(n) + \sum_{k\neq i} \boldsymbol{A}_{k}(n)\left(\boldsymbol{N}_{k}\boldsymbol{v}_{k}(n) + \boldsymbol{w}_{0,k}\right) + \boldsymbol{n}_{m}(n).$$
(25)

where the  $J \times m_l^2$  matrix

$$\boldsymbol{A}_{l}(n) = [\boldsymbol{a}_{l}^{(1)}(n), \dots, \boldsymbol{a}_{l}^{(J)}(n)]^{T}$$
(26)

and the  $J \times 1$  measurement vector  $\boldsymbol{z}_i(n)$  is given by

$$\boldsymbol{z}_i(n) = -\boldsymbol{A}_i(n)\boldsymbol{w}_{0,i}.$$
(27)

The *i*th relay can compute the matrix  $A_i(n)$  using its received data vector at the *n*th time instant and the estimates of its *J* relay-destination channels. However, it does not have access to the received data vectors or channel estimates of the other relays. In what follows, we will propose a scheme that allows each relay terminal to estimate its beamforming matrix locally with limited information exchange with the other relays. We can write the optimal adaptive beamforming vector of the *k*th relay at the *n*th time instant as

$$v_k(n) = v_k(n-1) + n_{v_k}(n-1)$$
 (28)

$$= \hat{\boldsymbol{v}}_k(n-1) + \tilde{\boldsymbol{v}}_k(n-1) + \boldsymbol{n}_{v_k}(n-1) \quad (29)$$

where (28) was obtained using the state equation (24) for the adaptive beamforming vector of the *k*th relay, and in (29), we have decomposed the optimal beamforming vector  $v_k(n-1)$  into the sum of its estimate  $\hat{v}_k(n-1)$  and the error vector  $\tilde{v}_k(n-1)$  associated with this estimate. Substituting with the above expansion for the K-1 beamforming vectors  $\{v_k(n)\}_{k\neq i}$  into (25), we can write the measurement equation associated with the beamforming vector of the *i*th relay as

$$\tilde{\boldsymbol{z}}_{i}(n) = \boldsymbol{A}_{i}(n)\boldsymbol{N}_{i}\boldsymbol{v}_{i}(n) + \tilde{\boldsymbol{n}}_{m_{i}}(n)$$
(30)

where the  $J \times 1$  modified measurement vector  $\tilde{z}_i(n)$  is given by

$$\tilde{\boldsymbol{z}}_{i}(n) = \boldsymbol{z}_{i}(n) - \sum_{k \neq i} \boldsymbol{A}_{k}(n) \left( \boldsymbol{N}_{k} \hat{\boldsymbol{v}}_{k}(n-1) + \boldsymbol{w}_{0,k} \right) \quad (31)$$

and the modified measurement noise  $\tilde{n}_{m_i}(n)$  is given by

 $\tilde{n}_{n}$ 

$$h_{i}(n) = \sum_{k \neq i} \boldsymbol{A}_{k}(n) \boldsymbol{N}_{k} \Big( \tilde{\boldsymbol{v}}_{k}(n-1) + \boldsymbol{n}_{v_{k}}(n-1) \Big) \\ + \boldsymbol{n}_{m}(n).$$
(32)

The covariance matrix of the modified measurement noise can be approximated as

$$\tilde{\boldsymbol{R}}_{i}(n) \approx \sum_{k \neq i} \boldsymbol{A}_{k}(n) \boldsymbol{N}_{k} \Big( \boldsymbol{P}_{k}(n-1|n-1) + \boldsymbol{Q}_{k} \Big) \boldsymbol{N}_{k}^{H} \boldsymbol{A}_{k}^{H}(n) \\ + \boldsymbol{R}$$
(33)

where  $P_k(n|n)$  is the covariance matrix of the estimated beamforming vector of the kth terminal at the nth time instant. Note that in (33), we have made the approximation that the errors in the estimated beamforming vectors of different relay terminals are uncorrelated, i.e.,  $E\left\{\tilde{v}_k(n-1)\tilde{v}_l^H(n-1)\right\} = \mathbf{0}$ for all  $k \neq l$ . This is equivalent to setting the off-diagonal submatrices of the covariance matrix of the stacked beamforming vector, P(n|n) in (20), to zero. Based on the state equation given in (24) and the modified measurement equation in (30), the *i*th relay terminal employs a Kalman filter to estimate its beamforming coefficients iteratively. The computational complexity associated with one iteration of the Kalman filter at the *i*th relay is of  $\mathcal{O} \{Jm_i^4\}$ . At each time instant, each relay computes  $J^2+J$  parameters using its received data vector, the estimates of its relay-destination channels, and its previous state estimate and covariance. These parameters are broadcasted to the other relays to be used in the next iteration. The steps of one iteration of the decentralized algorithm at the *n*th time instant for the *i*th relay beamformer can be summarized as follows:

- 1) The relay receives the data transmitted by the sources in the first phase of communication, i.e., the vector  $\boldsymbol{x}_i(n)$  is received.
- Using its previous estimate v̂<sub>i</sub>(n − 1) and its associated covariance matrix P<sub>i</sub>(n − 1|n − 1), the relay computes and broadcasts to the other relays the J×1 measurement correction vector A<sub>i</sub>(n) (N<sub>i</sub>v̂<sub>i</sub>(n − 1) + w<sub>0,i</sub>) and the J×J measurement noise covariance correction matrix A<sub>i</sub>(n)N<sub>i</sub>(P<sub>i</sub>(n − 1|n − 1) + Q<sub>i</sub>)N<sub>i</sub><sup>H</sup>A<sub>i</sub><sup>H</sup>(n).
- 3) The relay receives the measurement correction vectors  $\{A_k(n) (N_k \hat{v}_k(n-1) + w_{0,k})\}_{k \neq i}$  broadcasted by the other relays and the covariance correction matrices  $\{A_k(n)N_k (P_k(n-1|n-1) + Q_k)N_k^H A_k^H(n)\}_{k \neq i}$ .
- The relay uses the broadcasted parameters to update its modified measurement vector in (31) and the modified measurement noise covariance in (33).
- 5) Using the state-space model in (24) and (30), the relay performs one iteration of the Kalman filter to estimate the vector  $\hat{v}_i(n)$  and its associated covariance  $P_i(n|n)$ .
- The relay computes the beamforming coefficients *ŵ<sub>i</sub>(n) = N<sub>i</sub>ŵ<sub>i</sub>(n) + w<sub>0,i</sub>* and forms the beamforming matrix *Ŵ<sub>i</sub>(n)*.
- 7) The relay transmits the vector  $\hat{W}_i(n)x_i(n)$  to the destination nodes in the second phase of communication.

Note that at each iteration, each relay broadcasts only  $J^2 + J$ parameters (the measurement correction vector and its covariance) to the other relays. This is the only amount of information that has to be exchanged and there is no need to exchange the received data by the relays. The number of parameters that has to be exchanged scales gracefully with the number of source-destination pairs and is independent of the number of antennas at each relay terminal. For moderate number of source-destination pairs the number of parameters is not prohibitively large. Furthermore, if the relays are in close proximity of each other they can be connected by wires. With sufficient transmission power and enough coding, the broadcasted information can be considered error-free. It is worth mentioning that the effects of the information exchange error and quantization should be investigated. However, they fall outside the scope of this paper.

## V. PRACTICAL DESIGN CONSIDERATIONS

In this section, we derive an adaptive power control algorithm that can be used to limit the transmission power of each relay. We also extend the problem formulation in (7) to allow the relays to modify the QoS offered to the sources.

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#### A. Power Control

The relay transmission power is often bounded due to practical hardware implementation and regulation issues. However, the problem formulation in (7) does not provide any explicit constraints on the transmission power of the relays. Note that including such constraints directly in the problem formulation will hinder the development of the proposed real-time algorithms as the Kalman filter will not be able to handle the resulting second-order inequality constraints [10].

The average transmission power of the kth relay at the nth iteration,  $P_k^{(r)}(n)$ , is given by

$$P_{k}^{(\mathbf{r})}(n) = \operatorname{tr}\left\{\boldsymbol{W}_{k}^{H}(n) \left(\sum_{j=1}^{J} P_{j}\boldsymbol{h}_{k}^{(j)}\boldsymbol{h}_{k}^{(j)^{H}} + \sigma_{k}^{(\mathbf{r})^{2}}\boldsymbol{I}_{m_{k}}\right)\boldsymbol{W}_{k}(n)\right\}$$
(34)

where tr{ $\cdot$ } denotes the trace of a matrix. In order to motivate the proposed power control approach, we define the relay beamforming efficiency for the *j*th source-destination pair as the ratio between the received desired signal power and the power of the interference-plus-noise (only that forwarded from the relays) at the *j*th destination. The mathematical expression for the beamforming efficiency of the *j*th pair is given by (35) at the bottom of this page. The relay beamforming efficiency is an upper bound on the received SINR in (3) as it does not consider the noise generated at the destination. It depends only on the signal components controlled by the relays, i.e., it measures the quality of the signal forwarded by the relays to the destination nodes.

For a single source-destination pair, the LCMV design approach adopted in this paper is equivalent to maximizing the beamforming efficiency [20]. Note that the relay beamforming efficiency does not change if all the beamforming matrices  $\{W_k(n)\}_{k=1}^K$  are multiplied by a scalar. On the other hand, the received SINR increases and approaches the beamforming efficiency as the norm of the beamforming matrices increases, i.e., as the relay transmission power increases. Hence, the relay beamformer that maximizes the received SINR at a single destination node under an aggregate relay power constraint is a scaled version of the solution to the LCMV problem such that the power constraint is satisfied.

Motivated by these considerations, we next propose a suboptimal adaptive algorithm that can be used to enforce individual power constraints on each relay terminal in the case where there are multiple source-destination nodes. Let the transmission power of the *k*th relay be constrained such that  $P_k^{(r)} \leq \gamma_k$  where  $\gamma_k$  represents the maximum allowed transmission power for the *k*th relay. The main steps of the power control strategy can be summarized as follows:

- 1) After the *n*th iteration, each relay estimates its average transmission power using (34).
- 2) Each relay broadcasts the power correction factor  $\zeta_k(n)$

to the other relays, which for the kth relay is defined as

$$\zeta_k(n) = P_k^{(r)}(n) / \gamma_k. \tag{36}$$

3) After receiving the power correction factors, the relays normalize their filtering estimates as follows:

$$\hat{\boldsymbol{v}}_k(n) \leftarrow \frac{1}{\sqrt{\zeta(n)}} \hat{\boldsymbol{v}}_k(n)$$
 (37)

$$\boldsymbol{P}_k(n|n) \quad \longleftarrow \quad \frac{1}{\zeta(n)} \boldsymbol{P}_k(n|n).$$
 (38)

where  $\zeta(n) = \max{\{\zeta_1(n), \zeta_2(n), \dots, \zeta_K(n)\}}$ . The relays also normalize the nonadaptive component of the beamforming vector as

$$\boldsymbol{w}_{0,k}(n) \longleftarrow \frac{1}{\sqrt{\zeta(n)}} \boldsymbol{w}_{0,k}(n).$$
 (39)

Using the above correction algorithm scales down the beamforming matrices such that transmission power of any relay does not exceed the maximum allowable limit. Furthermore, the beamforming efficiency is not affected by the above weight correction algorithm as all the beamforming matrices are scaled by the same factor, and hence, efficient relay beamforming is maintained.

## B. QoS Modification

Although the above problem formulation in (6) does not include any QoS constraints on the received signals at the destinations, we can extend the adaptive relay beamforming problem to improve the QoS received at some destination nodes. This can be achieved by introducing the scalar non-negative parameters  $\{\alpha_j\}_{j=1}^J$  and rewriting the beamforming problem as

$$\max_{\{\boldsymbol{W}_{k}(n)\}_{k=1}^{K}} \sum_{j=1}^{J} \alpha_{j} \text{SINR}_{j}(n)$$
(40)

where  $\alpha_j > 0$ ,  $\sum_{j=1}^{J} \alpha_j = J$ , and SINR<sub>j</sub>(n) is defined in (3). A high value of  $\alpha_j$  will emphasize the importance of the received SINR at the *j*th destination, and hence, the *j*th destination will receive better QoS. Using the same LCMV approach we have adopted in this paper, we can include the constants  $\{\alpha_j\}_{j=1}^{J}$  in the constraints of the LCMV problem and rewrite the problem as

$$\min_{\{\boldsymbol{w}_k(n)\}_{k=1}^K} \qquad \sum_{j=1}^J \left| \sum_{k=1}^K \boldsymbol{a}_k^{(j)^T}(n) \boldsymbol{w}_k(n) \right|^2$$
  
s.t. 
$$\sum_{k=1}^K \boldsymbol{C}_k^H \boldsymbol{w}_k(n) = \boldsymbol{\alpha}.$$
(41)

where the  $J \times 1$  vector  $\boldsymbol{\alpha}$  is given by  $\boldsymbol{\alpha} = \left[\sqrt{\alpha_1}, \dots, \sqrt{\alpha_J}\right]^T$ . A higher value for  $\alpha_j$  will provide higher gain to the desired signal of the *j*th source, and thus, its QoS will be improved.

$$\eta_{j}(n) = \frac{P_{j} \left| \sum_{k} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{h}_{k}^{(j)} \right|^{2}}{\sum_{i \neq j} P_{i} \left| \sum_{k} \boldsymbol{g}_{k}^{(j)^{H}} \boldsymbol{W}_{k}^{H}(n) \boldsymbol{h}_{k}^{(i)} \right|^{2} + \sum_{k} \sigma_{k}^{(r)^{2}} \left\| \boldsymbol{W}_{k}(n) \boldsymbol{g}_{k}^{(j)} \right\|^{2}}.$$
(35)



Fig. 2. Average received SINR versus iteration number.



Fig. 3. Average transmitted power versus iteration number.

Note that the optimization parameters in (41) do not include the parameters  $\{\alpha_j\}_{j=1}^J$ . Hence, the above centralized and decentralized algorithms derived in the previous sections can be applied directly to the modified beamforming problem in (41). The effectiveness of this approach to modify the QoS at individual destination nodes is demonstrated through numerical simulations in Section VI.

## VI. SIMULATION RESULTS

We consider a wireless communication scenario with two source-destination pairs, i.e., J = 2, where each source is communicating with a distinct destination node. The two sources are transmitting QPSK symbols<sup>5</sup>. The transmitted signals by the sources are received by K=2 relay terminals with 5 and 6 antennas each. The channels from the sources to the relays and from the relays to the destinations are modelled as Ricean flat fading with Ricean K-factor equal to 0.1 and random LOS arrival angles uniformly distributed in the interval  $[0, 2\pi]$ . The scattered component of the received signal at the relays and the destination nodes has a Laplacian power-angle-profile with random mean angle of arrival uniformly distributed in  $[0, 2\pi]$ and random angular spread uniformly distributed between  $0^{\circ}$ and  $10^{\circ}$  [24]. The noise power at the relays is selected as  $\sigma_k^{(r)^2} = 1$ , the noise power at the destination nodes is given by  $\sigma_j^{(d)^2} = 0.5$ , and the received SNRs of the sources at the relays are all equal to -5 dB, i.e.,  $P_j/\sigma_k^{(r)^2} = 10^{\frac{-5}{10}}$ . Simulation results are averaged over 1000 Monte Carlo runs.

We first investigate the convergence behaviour of the proposed algorithms under a stationary signal environment. Fig. 2 shows the average received SINR at the destination nodes given by (3)—for the proposed iterative algorithms versus time. It also shows the average received SINR of the noniterative centralized SOCP-based algorithm of [10] which we use as a benchmark to compare the performance of the proposed algorithms with. The parameters of the Kalman filters employed in the proposed algorithms are chosen as  $\sigma_v^2 = 0$ , as we consider time-invariant radio propagation channels, and  $\sigma_{m,i}^2 = 10^{-3}$ . We can clearly see from Fig. 2 that the proposed adaptive algorithms converge to yield nearly the same SINR provided by the non-iterative SOCP-based algorithm. We can also notice from Fig. 2 that the decentralized Kalman filter-based algorithm has a slower convergence rate than that of the centralized one.

Fig. 2 also shows the received SINR versus iteration number for the centralized and distributed adaptive beamforming algorithms with the power control modification proposed in Section V. The transmission power constraint factors are selected as  $\gamma_1 = \gamma_2 = 40$ . As one would expect, constraining the transmission power of the relays reduces the received SINR. Fig. 3 shows the average relay transmission power for one relay versus the iteration number for different algorithms. We can clearly see that the proposed adaptive power control algorithms can effectively limit the transmission power of the relays over time. In fact, it can be verified that the power-constrained adaptive algorithms have almost the same beamforming efficiency as the unconstrained ones, and hence, efficient relay transmission is maintained by the power control algorithm.

Next, we explore the sensitivity of the proposed algorithms to the choice of the measurement noise covariance parameters  $\{\sigma_{m,i}^2\}_{i=1}^2$ . We select  $\sigma_{m,1}^2 = \sigma_{m,2}^2 = \sigma_m^2$ . We declare the convergence of the Kalman filter at the  $n_c$ th time instant if  $\|\hat{v}(n_c) - \hat{v}(n_c - 1)\|_{\infty} \leq 5 \times 10^{-4}$  where  $\|\cdot\|_{\infty}$  denotes the infinity norm of a vector. Fig. 4 shows the average received SINR after convergence of the Kalman filters versus the value of the parameter  $\sigma_m^2$ . We can see from Fig. 4 that the performance of the two proposed algorithms does not severely degrade over a large range of the parameter  $\sigma_m^2$ . In particular, the Kalman filtering algorithms show improved steady state performance for  $\sigma_m^2 \in [10^{-10}, 1]$ . It is worth mentioning that for very small values of  $\sigma_m^2$ , i.e.,  $\sigma_m^2 < 10^{-10}$ , the Kalman filter considers the measurement vector a perfect measurement, as a result, the convergence speed of the filter decreases substantially. On the other hand, for high values of  $\sigma_m^2$ , i.e.,

<sup>&</sup>lt;sup>5</sup>The proposed algorithms do not make use of any properties of the signal constellation. Hence, it is possible to use any constellation type and we are not limited to QPSK.



Fig. 4. Average received SINR versus  $\sigma_m^2$ .



Fig. 5. Average number of iterations required for convergence versus  $\sigma_m^2$ .

 $\sigma_m^2 > 1$ , the cost function is not sufficiently minimized and the performance of the proposed algorithms deteriorates. Fig. 5 shows the average number of iterations required for convergence of each algorithm, i.e., the value of  $n_c$ , versus the value of  $\sigma_m^2$ . We can see from Fig. 5 that the decentralized algorithm requires more iterations for convergence than the centralized one due to the approximations in its state-space model in (33).

Next, we investigate the performance of the proposed algorithms for different values of the SNR of the sources. The SNRs of all the sources are kept equal and are varied between -20 and 5 dB. The value of  $\sigma_m^2$  is chosen as  $10^{-3}$ . Fig. 6 shows the average received SINR of the two sources after convergence of the Kalman filter versus different values of the SNR of the sources. We can see from Fig. 6 that the proposed centralized beamforming algorithm has good performance for all values of the SNR. On the other hand, at high SNR, above 0 dB, the performance of the decentralized beamforming algorithm degrades. This can be attributed to the decrease in the degrees of freedom due to enforcing the signal



Fig. 6. Average received SINR versus the SNR of the sources.



Fig. 7. Average received SINR of each source versus the parameter  $\alpha_1$ .

preservation constraints noncooperatively. Another reason for the deterioration of the performance of the decentralized beamformer at high SNR is due the assumption we have made in (33) that the errors in the estimated beamforming vectors of different terminals are uncorrelated. In fact, as the SNR of the sources increases, the relay beamforming matrices focus more on suppressing the interference received at the destination nodes than on reducing the received noise power. Since the interference suppression is accomplished by the relay terminals cooperatively, the errors in the beamforming vectors of different relays are more correlated as the SNRs of the sources increase. This leads to performance degradation of the distributed Kalman filtering beamforming algorithm as these correlations are not modelled in the state-space model.

In the next simulation, we explore the effect of changing the parameters  $\{\alpha_j\}_{j=1}^2$  in (41) on the QoS of the two sources. We consider the same signal environment considered in the previous examples. The SNRs of the two sources is chosen as -5 dB. The value of the parameter  $\alpha_1$  is varied between 0.2 and 1.8 and the value of  $\alpha_2 = J - \alpha_1 = 2 - \alpha_1$ . Fig. 7 shows



Fig. 8. Average received SINR versus iteration number.

the average received SINR for each source at its destination node versus the parameter  $\alpha_1$ . We can see that as the value of  $\alpha_1$  increases, the received SINR of the first source increases at the expense of the received SINR of the second source. This simulation shows the efficacy of using the parameters  $\{\alpha_j\}_{j=1}^J$  in (40) in improving the QoS of some sources.

Finally, we consider a nonstationary signal environment. The simulation setup is similar to the one we have considered in the previous simulation in terms of the configuration of the sources, relays, and destination nodes. The source-relay channels and relay-destination channels are fixed during the first 500 time instants. At iteration time 500, we suddenly switch to a new, independent set of channel realizations that remain in use until the end of the simulation experiment at iteration 1000. This type of experiments is commonly used in the adaptive filtering literature to evaluate the ability of an algorithm to track rapid changes in the underlying signal environment, e.g., [25]. The SNRs of the sources are all equal to -5 dB. The parameters of our Kalman filter-based algorithms are selected as  $\sigma_v^2 = 10^{-8}$  and  $\sigma_m^2 = 10^{-3}$ . Fig. 8 displays the average received SINR of the two sources versus iteration number. We can clearly see the capability of the proposed beamformers to readapt to the new signal environment and rapidly converge back to yield satisfactory performance.

# VII. CONCLUSION

We have presented two adaptive cooperative beamforming algorithms for MIMO-relaying wireless systems with multiple source-destination pairs. The beamforming matrices of the relays are jointly designed by minimizing the total power received at all the destination nodes subject to linear constraints that preserve the desired signal at each destination. Both algorithms are based on Kalman filtering and can be applied iteratively in real-time. In the first algorithm, a local processing center computes the beamforming coefficients of all the relays which requires a significant amount of communication between the processing center and the relays. In the second algorithm, each relay can compute its beamforming coefficients locally using its received data, its relay-destination channel estimate, and some information that is broadcasted by the other relays. We have also extended the proposed algorithms to allow the relays to control their transmission power and to modify the QoS provided to different sources. Simulation results have been presented that validate the ability of the proposed algorithms to yield a performance comparable to that of the non-iterative centralized SOCP-based algorithm at low and medium SNRs.

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