

A Low-Complexity Hybrid Framework for Combining-Type Non-regenerative MIMO Relaying

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Published online: 31 January 2013
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Abstract Combining-type relay systems can benefit from distributed array gain if the signals retransmitted from different relays are superimposed coherently at the destination. For this purpose, we propose a low-complexity hybrid framework in which the non-regenerative multiple-input-multiple-output relaying matrix at each relay is generated by cascading two substructures, akin to an equalizer for the backward channel and a precoder for the forward channel. For each of these two substructures, we introduced two one-dimensional parametric families of candidate matrix transformations. The first family, non-cooperative by nature, depends only on the backward or forward channel of the same relay. The second (cooperative) family also makes use of information derived from the channels of other relays. This hybrid framework allows for the classification and comparison of all possible combinations of these substructures, including several previously investigated methods and their generalizations. The design parameters can be optimized based on individual channel realizations or on channel statistics; in the latter case, the optimum parameters can be well approximated by linear functions of the signal-to-noise ratios. The proposed methods achieve a good balance between performance and complexity: they outperform existing low-complexity strategies by a large margin in terms of both capacity and bit-error rate, and at the same time, are significantly simpler than previous near-optimal iterative algorithms.

Keywords Combining-type · Distributed array gain · Hybrid relaying · MIMO · Non-regenerative relaying

This work was supported by InterDigital Canada Ltée., the Natural Sciences and Engineering Research Council of Canada, and the Government of Québec under the PROMPT program.

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1 Introduction

Multiple-input multiple-output (MIMO) wireless relaying is a promising technology to increase system throughputs and overcome the impairments caused by multipath fading, shadowing and path-losses [14, 18, 21]. In *non-regenerative* amplify-and-forward (AF) strategies, each relay applies a linear transformation matrix to its received baseband signals before retransmitting them. These strategies incur shorter processing delays and lower implementation complexity than regenerative ones such as decode-and-forward (DF) and compress-and-forward (CF) [25].

For the one-source–one-relay–one-destination (1S-1R-1D) configuration, the optimal MIMO relaying matrix is well established in terms of various performance criteria [4, 7, 12, 15, 22, 26, 27]. Interestingly, a majority of these criteria lead to a common singular value decomposition (SVD) structure, which can scalarize the problems so that they can be readily solved using convex optimization. These optimal schemes can be extended to one-source–multiple-relays–one-destination (1S-MR-1D) systems through the use of a selection-type operation, whereby the source signals are forwarded through the single relay that offers the best link quality [5].

Another approach in the 1S-MR-1D case is to use all the relays (or a subset of them) simultaneously in order to benefit from distributed array gain [3, 18]. However, the resulting problem of designing optimal transformation matrices with constraints on the transmit power of the relays remains largely unsolved. In particular, the SVD approach does not readily extend to this case since, due to the physically separated nature of the multiple relays, their combined transformation matrix inherits a block-diagonal form. By imposing the power constraint on the received signals at the destination, instead of the transmitted signals from the relays, one can circumvent this difficulty [2, 28]. However, this cannot guarantee any optimality under the original transmit power constraints. Other existing optimal designs either consider only a total power constraint across the relays [33], or employ iterative approaches with relatively high complexity [1, 11]. As an alternative to the optimization approach, some heuristic strategies have been proposed which “borrow” ideas from MIMO transceiver design, including matched filtering (MF), zero-forcing (ZF), linear minimum mean square error (MMSE) [2, 18] and QR decomposition [24]. These methods achieve the distributed array gain and perform well in 1S-MR-1D systems [32].

In theory, the relaying matrices should be chosen so that the retransmitted signals combine coherently at the destination. To this end, we introduce a low-complexity hybrid framework in which the transformation matrix of each relay is obtained by cascading two substructures or factors, akin to an equalizer for the backward channel and a precoder for the forward channel. For each of these two substructures, we propose two different one-dimensional parametric families whose members serve as candidates. The first family, *non-cooperative* by nature, depends only on the backward or forward channel corresponding to the same relay. This family includes ZF, linear MMSE and MF as special cases [18]. The second (*cooperative*) parametric family, inspired by [2, 8], also makes use of information derived from the channels of other relays. This hybrid framework allows for the classification and comparison of all possible combinations of these substructures, including several previously investigated methods and their generalizations.

Within this hybrid relaying framework, the design parameters of the matrix factors can be further optimized. This can be done on-line after each update of the channel matrices, or off-line based on *a priori* knowledge of channel statistics. In the latter case, the optimum parameters can be well approximated by linear functions of the signal-to-noise ratios (SNR), which reduces the implementation complexity significantly. Through simulations, we

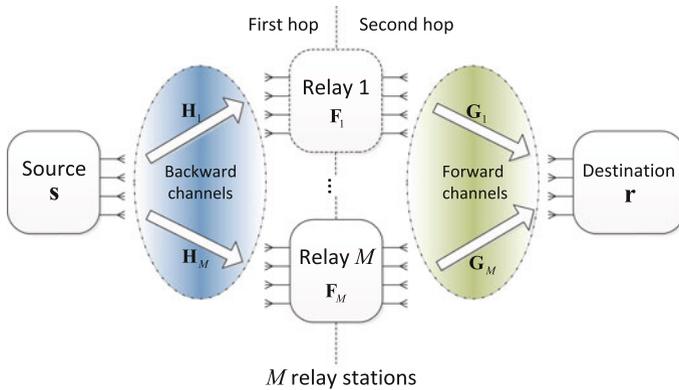


Fig. 1 A point-to-point MIMO relaying system

show that the capacity of selected hybrid schemes (with optimized parameters) comes within 1 bits/s/Hz of the upper bound achieved by the nearly capacity-optimal iterative method in [11]. In the mid-to-high SNR range, the bit-error rate (BER) performance of one hybrid method even exceeds that of the MSE-optimal iterative method. In summary, the proposed hybrid methods achieve a good balance between performance and complexity: they outperform existing low-complexity strategies by a large margin, and at the same time, are significantly simpler than previous near-optimal iterative algorithms.

The organization of this paper is as follows. Section 2 describes the system model and the underlying assumptions. Section 3 presents the new hybrid framework along with the proposed non-cooperative and cooperative matrix substructures. Suitable performance criteria and methodology for choosing their design parameters are developed in Sect. 4. The numerical results and further discussions are included in Sect. 5, followed by the conclusions in Sect. 6. The following notations are used: superscripts $*$, T , H and \dagger denote conjugate, transpose, Hermitian transpose and pseudo-inverse, respectively; \mathbf{I} is an identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean norm of its vector argument; \mathbb{R} and \mathbb{C} denote the sets of real and complex numbers.

2 System Model

Figure 1 illustrates a 1S-MR-1D MIMO relaying system in which the source forwards its message to the destination through M parallel relays. The source, destination and individual relays are equipped with N_S , N_D and N_R antennas, respectively, where we assume that $N_S = N_D$.¹ The relays work in a half-duplex mode: their antennas are used for either transmitting or receiving purposes during different time slots. We neglect the presence of the direct source-to-destination link which is typically hindered by high levels of attenuation.

We assume that the wireless channels undergo frequency non-selective block fading [29]. For now, channel state information (CSI) is assumed to be available globally. After introducing the structures of the relaying matrices, we will be able to discuss in detail how much information is needed at each node. In this work, we assume perfect synchronization between the source, relay and destination nodes. Channel estimation and timing/frequency

¹ For simplicity, each relay is equipped with the same number of antennas; however, generalization to different numbers of antennas at the relays, i.e. $N_{R,k}$, is straightforward.

synchronization are important topics in their own rights, but fall outside the scope of this work. For more details, we refer the reader to [10, 16, 20] and the references therein.

The signals, noises and channels are all modeled in terms their equivalent discrete-time complex baseband representations. The received signal vector $\mathbf{x}_k \in \mathbb{C}^{N_R \times 1}$ at the k th relay can be expressed as

$$\mathbf{x}_k = \mathbf{H}_k \mathbf{s} + \mathbf{w}_k, \quad k = 1, \dots, M \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N_S \times 1}$ is the source symbol vector comprised of multiple independent streams, $\mathbf{H}_k \in \mathbb{C}^{N_R \times N_S}$ is the backward channel matrix between the source and relay k , and $\mathbf{w}_k \in \mathbb{C}^{N_R \times 1}$ is an additive noise term. The signal and noise terms, \mathbf{s} and $\{\mathbf{w}_k\}$ for $k = 1, \dots, M$, are modeled as independent, circularly symmetric complex Gaussian random vectors with zero mean and covariance matrices $\mathbf{R}_s = E\{\mathbf{s}\mathbf{s}^H\} = \sigma_s^2 \mathbf{I}$ and $\mathbf{R}_{\mathbf{w}_k} = E\{\mathbf{w}_k \mathbf{w}_k^H\} = \sigma_w^2 \mathbf{I}$, respectively, where σ_s^2 is the average transmit power per antenna at the source and σ_w^2 is the average noise power induced at the individual relay antennas.

The k th relay multiplies its received noisy signal \mathbf{x}_k by a linear processing matrix $\mathbf{F}_k \in \mathbb{C}^{N_R \times N_R}$ to obtain the retransmitted signal

$$\mathbf{y}_k = \mathbf{F}_k \mathbf{x}_k. \tag{2}$$

The received signal vector at the destination, denoted by $\mathbf{r} \in \mathbb{C}^{N_D \times 1}$, takes the form of

$$\mathbf{r} = \sum_{k=1}^M \mathbf{G}_k \mathbf{F}_k \mathbf{H}_k \mathbf{s} + \sum_{k=1}^M \mathbf{G}_k \mathbf{F}_k \mathbf{w}_k + \mathbf{n}, \tag{3}$$

where $\mathbf{G}_k \in \mathbb{C}^{N_D \times N_R}$ is the forward channel matrix from relay k to the destination and $\mathbf{n} \in \mathbb{C}^{N_D \times 1}$ is the noise term induced at the destination receiver. This noise term is assumed independent from \mathbf{s} and $\{\mathbf{w}_k\}$, and modeled as a circularly symmetric complex Gaussian random vector with zero mean and covariance matrix $\mathbf{R}_n = E\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2 \mathbf{I}$, where σ_n^2 is the average noise power received at the individual destination antennas. Equation (3) can also be expressed in a ‘‘block-diagonal’’ form as

$$\mathbf{r} = \mathbf{G}\mathbf{F}\mathbf{H}\mathbf{s} + \mathbf{G}\mathbf{F}\mathbf{w} + \mathbf{n}, \tag{4}$$

where we have defined $\mathbf{G} = [\mathbf{G}_1, \dots, \mathbf{G}_M]$, $\mathbf{H} = [\mathbf{H}_1^T, \dots, \mathbf{H}_M^T]^T$, $\mathbf{w} = [\mathbf{w}_1^T, \dots, \mathbf{w}_M^T]^T$ and \mathbf{F} is a block-diagonal matrix with $\mathbf{F}_1, \dots, \mathbf{F}_M$ as main diagonal blocks. When $M = 1$, this signal model reduces to the 1S-1R-1D case.

For convenience, we introduce two important SNR parameters. The first SNR ρ_1 describes the link quality of the backward channels and is defined as the ratio of the average transmit power per source antenna to the noise power per relay antenna, i.e., $\rho_1 = \sigma_s^2 / \sigma_w^2$. The second SNR parameter ρ_2 characterizes the forward channels. Let the total transmit power of the relays be \mathcal{P} and ρ_2 is defined as the ratio of average transmit power per relay antenna to the power of the noise induced at the individual destination antennas, i.e., $\rho_2 = \mathcal{P} / (M N_R \sigma_n^2)$. Note that \mathcal{P} is consumed by the relays to transmit both the desired signal component \mathbf{s} and the additive relay noise terms $\{\mathbf{w}_k\}$. We emphasize that if one of these two SNR parameters is fixed, the system performance is upper bounded due to the corresponding noise term, even if the other SNR goes to infinity.

Note that the above signal model is applicable to a much broader scope than multi-antenna 1S-MR-1D systems. For example, since broadband channels for single-antenna systems can also be represented by matrices, the relaying framework in this paper applies immediately to broadband single-antenna 1S-MR-1D relaying systems.

3 The Unified Hybrid Framework

The focus of this paper is to design the relay matrices $\{\mathbf{F}_k\}$ for 1S-MR-1D systems, based on the knowledge of the instantaneous channel matrices. One immediate option is to solve for matrices \mathbf{F}_k that collaboratively optimize a suitable performance criterion. However, the block-diagonal matrix \mathbf{F} in (4) complicates this problem significantly. Instead, we propose a sub-optimal, yet highly flexible hybrid framework as explained below.

One may contemplate the process of designing the relaying matrices \mathbf{F}_k in (2) as that of selecting the equivalent channel

$$\mathbf{GFH} = \sum_{k=1}^M \mathbf{G}_k \mathbf{F}_k \mathbf{H}_k, \tag{5}$$

with the purpose of maximizing the power of the received signal vector \mathbf{GFHs} , without over-amplifying the noise terms \mathbf{w}_k in (3). Intuitively, this requires a coherent signal combining of the M parallel transmissions at the destination, i.e., the matrix terms on the right-hand side of (5) are superimposed constructively, thereby leading to an M -fold distributed array gain.

Motivated by this interpretation, we propose a unified hybrid framework in which the individual relaying matrices \mathbf{F}_k are obtained by cascading two substructures (or factors), $\mathbf{A}_k \in \mathbb{C}^{N_S \times N_R}$ and $\mathbf{B}_k \in \mathbb{C}^{N_R \times N_S}$, as follows:

$$\mathbf{F}_k = \eta_k \mathbf{B}_k \mathbf{A}_k. \tag{6}$$

In light of (5), matrix \mathbf{A}_k equalizes the k th backward MIMO channel \mathbf{H}_k , generating N_S summary statistics, each of which is a signal stream impaired by noise and also interferences from other streams. Matrix \mathbf{B}_k then serves as the MIMO precoder for the k th forward channel \mathbf{G}_k , pre-canceling interstream interferences before transmitting these summary statistics through the forward channels. Finally, η_k is a positive scaling parameter introduced to satisfy the transmit power constraints

$$\mathbb{E}\{\|\mathbf{y}_k\|^2\} = \text{tr}(\mathbf{F}_k \mathbf{R}_{\mathbf{x}_k} \mathbf{F}_k^H) = P_k, \quad \forall 1 \leq k \leq M, \tag{7}$$

where $\mathbf{R}_{\mathbf{x}_k} = \mathbb{E}\{\mathbf{x}_k \mathbf{x}_k^H\} = \sigma_s^2 \mathbf{H}_k \mathbf{H}_k^H + \sigma_w^2 \mathbf{I}$. Henceforth, the scaling factor η_k in (6) satisfies

$$\eta_k = \sqrt{\frac{P_k}{\text{tr}(\mathbf{B}_k \mathbf{A}_k \mathbf{R}_{\mathbf{x}_k} \mathbf{A}_k^H \mathbf{B}_k^H)}}. \tag{8}$$

Under the above framework, the relaying strategies can be either non-cooperative or cooperative. For the non-cooperative strategies, the relaying matrix for the k th relay, \mathbf{F}_k , only depends on its own backward and forward channel matrices, i.e., \mathbf{H}_k and \mathbf{G}_k . For the cooperative strategies, at least one of the substructures also relies on some shared information related to the channels of the other relays. That is, \mathbf{F}_k depends not only on \mathbf{H}_k or \mathbf{G}_k , but also on a function of the other channel matrices, as explained below.

Table 1 Special cases of the non-cooperative hybrid framework

λ_a	λ_b	\mathbf{A}_k	\mathbf{B}_k	Previous methods
0	0	ZF	ZF	ZF relaying [18]
ρ_1^{-1}	ρ_2^{-1}	MMSE	MMSE	Linear MMSE [5, 18]
∞	∞	MF	MF	MF [3, 5, 18]
ρ_1^{-1}	0	MMSE	ZF	2-step MMSE[2, 8]
$\{0, \rho_1^{-1}, \infty\}$	$\{0, \rho_2^{-1}, \infty\}$	ZF/MMSE/MF		Hybrid [32]

3.1 Non-cooperative Approach

Here, each one of the substructures \mathbf{A}_k and \mathbf{B}_k is selected from a corresponding one-dimensional parametric family of matrices. That is, we let

$$\mathbf{A}_k^{\text{NC}} = \left(\lambda_a \mathbf{I} + \mathbf{H}_k^H \mathbf{H}_k \right)^{-1} \mathbf{H}_k^H, \tag{9a}$$

$$\mathbf{B}_k^{\text{NC}} = \mathbf{G}_k^H \left(\lambda_b \mathbf{I} + \mathbf{G}_k \mathbf{G}_k^H \right)^{-1}, \tag{9b}$$

where λ_a and λ_b are real, nonnegative design parameters. For instance, by choosing λ_a equal to ∞ , 0 or $1/\rho_1$, \mathbf{A}_k^{NC} is proportional to \mathbf{H}_k^H , \mathbf{H}_k^\dagger or $(\mathbf{I} + \rho_1 \mathbf{H}_k^H \mathbf{H}_k)^{-1} \mathbf{H}_k^H$, respectively.² In turn, these matrices correspond to the MF, ZF and MMSE substructures which were studied in previous works [3, 5, 18]. A similar argumentation can be made about \mathbf{B}_k^{NC} . The superscript NC in (9a) means that these substructures are non-cooperative by nature, for they are determined only by the local backward or forward channels, \mathbf{H}_k or \mathbf{G}_k .

By cascading \mathbf{A}_k^{NC} and \mathbf{B}_k^{NC} as in (6), we can obtain a non-cooperative hybrid relaying strategy which includes several previous methods as special cases, as summarized in Table 1. By varying each one of the design parameters λ_a and λ_b in (9a) from zero to infinity, we generalize these previously proposed methods to other intermediate situations of interest.

3.2 Cooperative Approach

Next, we extend the proposed hybrid framework by considering cooperative strategies where the design of the relaying matrices \mathbf{F}_k explicitly takes into account the combining nature of the signal transmission in 1S-MR-1D systems. This is achieved by exploiting some shared information (but not necessarily all the channel matrices). To this end, we propose alternative parametric families of matrix transformations for \mathbf{A}_k and \mathbf{B}_k :

$$\mathbf{A}_k^{\text{C}} = \left(\lambda_a \mathbf{I} + \sum_{j=1}^M \mathbf{H}_j^H \mathbf{H}_j \right)^{-1} \mathbf{H}_k^H, \tag{10a}$$

$$\mathbf{B}_k^{\text{C}} = \mathbf{G}_k^H \left(\lambda_b \mathbf{I} + \sum_{j=1}^M \mathbf{G}_j \mathbf{G}_j^H \right)^{-1}, \tag{10b}$$

² As λ_a goes to infinity, \mathbf{A}_k^{NC} approaches $\lambda_a^{-1} \mathbf{H}_k^H$ asymptotically; when $\lambda_a = 1/\rho_1$, $\mathbf{A}_k^{\text{NC}} = \rho_1 (\mathbf{I} + \rho_1 \mathbf{H}_k^H \mathbf{H}_k)^{-1} \mathbf{H}_k^H$. In both cases, the resulting scalar factor can be absorbed by η_k in (6).

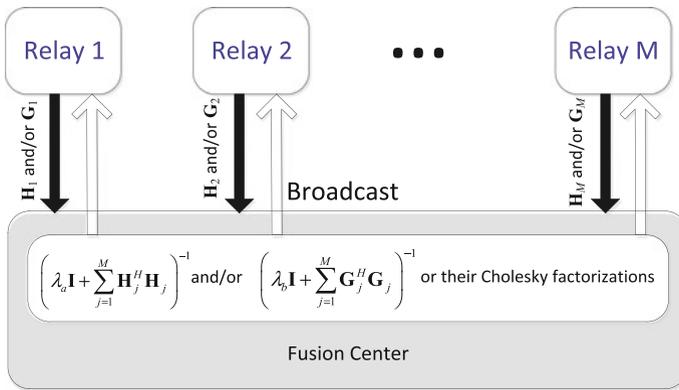


Fig. 2 CSI exchange for cooperative relaying strategies

where the superscript C stands for “cooperative”. Here, the equalizer (10a) is inspired by the works in [2, 8] and we extend it to the precoder side as well in (10b). The sums in (10a) and (10b) are the information needed to be shared between relays.

More generally, the hybrid relaying matrix \mathbf{F}_k in (6) can be formed by combining factors \mathbf{A}_k and \mathbf{B}_k selected from any of the above proposed non-cooperative and cooperative parametric families of matrices. For notational simplicity, we refer to these hybrid strategies as “A-B(λ_a, λ_b)” where for example, NC-C(0, 0) means that the \mathbf{A}_k factor of the relaying matrix \mathbf{F}_k is the non-cooperative substructure \mathbf{A}_k^{NC} and the \mathbf{B}_k factor is the cooperative substructure \mathbf{B}_k^{C} , with $\lambda_a = \lambda_b = 0$. In this sense, the proposed hybrid framework enables the formal classification of previously investigated methods as well as their generalization by supplementing them with a rich set of alternatives.

3.3 Implementation Issues

The relaying matrices are computed based on the knowledge of the wireless channels. For the NC-NC strategy, each relay only needs its own backward and forward channel matrices that can be obtained in the same way as in 1S-1R-1D systems [4, 7, 12, 15, 22, 26, 27]. For the cooperative hybrid relaying strategies, it is also essential to share the matrix sums $\sum_{k=1}^M \mathbf{H}_k \mathbf{H}_k^H$ and/or $\sum_{k=1}^M \mathbf{G}_k^H \mathbf{G}_k$ (but not all the channel matrices) among the relays. These sums can be computed at a fusion center, which may be one of the relays or the destination, and broadcasted to the relays, as shown in Fig. 2. In practice, the number of relays M will not be very large, e.g., between 2 and 4. Therefore, compared with non-cooperative NC-NC, the cooperative hybrid strategies can be implemented without much added difficulty, especially when the relays are not far from each other so that dedicated local wireless links or wireline connections are possible.

The procedures for computing \mathbf{A}_k , \mathbf{B}_k and η_k are simple and involve only a small number of matrix multiplications and inverses. The resulting complexity is very low, though the Cholesky/QR factorizations and backward-forward substitution can be used for further simplification [6].

4 Optimization of the Parameters

4.1 Motivations

λ_a and λ_b can be regarded as “regularization” or “diagonal loading” parameters for the substructures \mathbf{A}_k and \mathbf{B}_k : λ_a prevents over-amplification of the noise terms \mathbf{w}_k in (1) when equalizing ill-conditioned backward channels; λ_b prevents the transmit power of the relays from being wasted in pre-compensating ill-conditioned forward channels [13,30]. In the context of a point-to-point MIMO channel, the use of $\lambda_a = \rho_1^{-1}$ and $\lambda_b = \rho_2^{-1}$ in (9a) leads to the optimal linear MMSE equalizer and precoder, respectively. These are known to offer the best trade-off between noise and interference cancellation, outperforming both MF and ZF over the complete SNR range [9,29].

Then, for MIMO relaying systems, it is legitimate to ask why it might be more appropriate to choose values other than 0, ρ_i^{-1} ($i = 1, 2$), or ∞ ? To begin with, two independent noise sources arise in the signal model. For the first hop, the outputs from the equalizer \mathbf{A}_k are not decoded immediately but need further processing, and thus setting λ_a to 0, ρ_1^{-1} or ∞ is not necessarily optimal. For the second hop, the input signals have already been impaired by noise and interferences before being processed by the precoders \mathbf{B}_k and retransmitted, and therefore choosing $\lambda_b = 0, \rho_2^{-1}$ or ∞ is not optimal, either. Furthermore, the combining of signals from multiple relays makes it more complicated to predict the joint effects of λ_a and λ_b on system performance.

Another important concern is that the presence of a linear MIMO equalizer at the destination makes it possible to *exploit* the inter-stream interferences. In contrast to the ZF-type substructures with λ_a or $\lambda_b = 0$, these interferences do not necessarily have to be small or completely eliminated at intermediate steps, such as in the output of the substructure \mathbf{A}_k or in the received signal vector \mathbf{r} . Provided that the interfering streams can be efficiently recombined at the destination, the power used by the relays to transmit them can actually contribute to performance improvement.

Therefore, in a relaying scenario, the parameter values 0, ρ_i^{-1} ($i = 1, 2$), or ∞ are not optimal in general. Our proposed parametric approach provides additional flexibility in balancing various factors that hinder system performance, and thereby can fully exploit the potential in these seemingly simple substructures.

4.2 Performance Measures and Power Constraints

Here, we introduce two classes of performance criteria that can be used to optimize the parameters λ_a and λ_b , as well as to compare the performance of different relaying strategies.

The most fundamental theoretical limit is the channel capacity. In a strict sense, it is the maximum asymptotically achievable rate over *all* possible transceiver schemes and relaying strategies. Here, we abuse this terminology slightly by viewing the AF relaying matrices as parts of the channel. Different relaying schemes result in different equivalent channels between the source and the destination, and we refer to the maximum mutual information between \mathbf{s} and \mathbf{r} as the channel capacity. For deterministic channels, it can be written as

$$C(\mathbf{F}) = \frac{1}{2} \log \det \left(\mathbf{I} + \mathbf{H}_{\text{eq}} \mathbf{R}_s \mathbf{H}_{\text{eq}}^H \right), \tag{11}$$

where $\mathbf{H}_{\text{eq}} = (\mathbf{GFR}_w \mathbf{F}^H \mathbf{G}^H + \mathbf{R}_n)^{-1/2} \mathbf{GFH}$ [27]. The factor of 1/2 in (11) is due to the half duplex mode of operation. Under *slow fading*, as assumed throughout this paper, the system

performance is characterized through the outage probability $p_{\text{out}}(R) = \Pr(C(\mathbf{H}_{\text{eq}}, \mathbf{F}) < R)$ and the corresponding outage capacity defined as the supremum

$$C_{\text{out}}(\epsilon) = \sup \{R | p_{\text{out}}(R) < \epsilon\}. \tag{12}$$

Practical systems compromise transmission rate for lower complexity, cost and latency [23]. In this sense, it is also of interest to examine other criteria such as the MSE, the signal to interference-plus-noise ratio (SINR) and the average BER. In this paper, we assume a V-BLAST (Vertical-Bell-Laboratories-Layered-Space-Time) scheme in which the source antennas transmit independent symbol streams with the same average power, and the destination applies a linear MMSE MIMO combiner followed by single-stream decoding [29, p. 333]. Within this framework, the MSE, SINR and theoretical BER of each substream are linked together through the normalized MSE matrix, as defined in [19] and [22] by

$$\mathbf{E} = \left(\mathbf{I} + \rho_1 \mathbf{H}^H \mathbf{F}^H \mathbf{G}^H \left(\mathbf{G} \mathbf{F} \mathbf{F}^H \mathbf{G}^H + \frac{\sigma_n^2}{\sigma_w^2} \mathbf{I} \right)^{-1} \mathbf{G} \mathbf{F} \mathbf{H} \right)^{-1}. \tag{13}$$

Specifically:

1. The normalized MSE of the k th substream is the k th diagonal entry

$$\text{MSE}_k = \mathbf{E}(k, k) \in (0, 1]. \tag{14}$$

2. The SINR of the k th substream is a function of its MSE:

$$\text{SINR}_k = \frac{1 - \text{MSE}_k}{\text{MSE}_k}. \tag{15}$$

3. If the interferences and noise terms are all Gaussian random variables, the symbol error-rate (SER) of the k th substream is upper bounded by a function of SINR_k :

$$p_s(k) = \alpha \text{Q} \left(\sqrt{\beta \text{SINR}_k} \right), \tag{16}$$

where α and β depend on the constellation, and $\text{Q}(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-y^2/2} dy$. If the source uses Gray codes in symbol-to-bit mapping, the BER of the k th substream is $\approx p_s(k)/n$, where 2^n is the constellation size.

Substituting (8) and (6) into (11) or (13), the above mentioned performance measures all become continuously differentiable functions of $\boldsymbol{\lambda} = [\lambda_a, \lambda_b]^T$.

4.3 Methodology

Assume that we are *minimizing* the function $f(\boldsymbol{\lambda})$ based on a single instance of the fading channels. In general, setting the gradient to zero, i.e. $\nabla_{\boldsymbol{\lambda}} f = \mathbf{0}$, does not lead to a closed-form optimal solution. Instead, we can resort to several numerical algorithms that start from an initial point, $\boldsymbol{\lambda}_0$, and search for the optimal $\boldsymbol{\lambda}_{\text{opt}} = [\lambda_a^o, \lambda_b^o]^T$ iteratively. In this process, due to the large dynamic range of the SNR parameters, it is more convenient to work with the logarithmic values of $\boldsymbol{\lambda}$. Furthermore, the initial point may be taken as $\boldsymbol{\lambda}_0 = [\rho_1^{-1}, \rho_2^{-1}]^T$.

Gradient-based methods such as gradient descent, Newton and quasi-Newton methods, update $\boldsymbol{\lambda}$ in the following way

$$\log \boldsymbol{\lambda}_{k+1} = \log \boldsymbol{\lambda}_k - \alpha_k \mathbf{B}_k \nabla_{\log \boldsymbol{\lambda}} f |_{\boldsymbol{\lambda}=\boldsymbol{\lambda}_k}, \tag{17}$$

where \mathbf{B}_k depends on the specific algorithm and α_k is the step size which satisfies the Wolfe conditions [31]. If the closed form of $\nabla_{\log \lambda} f$ is too complicated or unavailable, a finite difference can be used to approximate it [17, Section 8.1].

The above approach is applied on-line after each update of the channel matrices. Alternatively, we can optimize λ off-line based on a priori knowledge of system configurations, fading statistics and SNR values. The above gradient descent method still applies, provided that $f(\lambda)$ is replaced by its expectation $E_{\mathbf{H}, \mathbf{G}}\{f(\lambda)\}$.³ The latter is computed by averaging $f(\lambda)$ over channel realizations numerically, but this can be done *beforehand* for various possible fading statistics and SNRs. Following this approach, we have found through numerous experiments that the resulting optimal λ_{opt} can be well approximated by linear functions of $\log \rho_1$ and $\log \rho_2$, as in e.g.,

$$\log_{10} \lambda_a^o \approx \mathbf{c}_a^T \boldsymbol{\rho} + d_a, \tag{18}$$

where $\boldsymbol{\rho} = [\log_{10} \rho_1, \log_{10} \rho_2]^T$, and \mathbf{c}_a, d_a are model coefficients. Consequently, we have to minimize $E_{\mathbf{H}, \mathbf{G}}\{f(\lambda)\}$ for only a *small* set of representative SNRs, and then use total least-square fitting to get the model coefficients. Then, (18) is used in practical system implementation to update λ based on the instantaneous SNR measurements.

The complexity of optimizing λ depends on which of the above two approaches is taken. If λ is optimized for each channel realization, the complexity is relatively high, but still lower than the methods in [11]. The major complexity comes from computing the gradients. For instance, in order to obtain the gradient of the MSE (using finite difference), NC-NC needs approximately $8M$ matrix multiplications and 6 matrix inverses (of size $N_S \times N_S$) per iteration. In contrast, the method in [11] requires $13M$ multiplications and $2M$ inverses (matrix sizes between $N_S \times N_S$ and $N_R \times N_R$) per iteration. In our simulations using finite difference, it usually takes the gradient descent method fewer iterations to converge than the method in [11].

More importantly, if the optimal parameters are designed off-line, the complexity of obtaining the parameters from a table lookup or the linear formula in (18) is almost negligible. This simplicity is one of the most attractive aspects of the proposed hybrid framework, especially for systems with fixed relay infrastructures whose channels remain relatively stationary.

5 Numerical Results and Discussion

In this section, the performance criteria introduced in Sect. 4.2 are studied numerically to gain a better understanding of the proposed hybrid relaying framework. First, the behaviors of the capacity and sum MSE provide new insights into how the parameters λ_a and λ_b affect system performance, which complements the interpretations in Sect. 4.1. Then, numerical comparisons with existing designs illustrate that the hybrid framework achieves a good balance between performance and complexity. Lastly, the linear formula in (18) brings further simplifications with minor performance loss.

The following system configurations and parameters are used throughout this section. Unless otherwise stated, the system of interest is a 1S-3R-1D system with $N_S = N_R = N_D = 4$ and $\rho_1 = \rho_2 = 15$ dB. The noises induced at the relay and destination have the same power: $\sigma_w^2 = \sigma_n^2$. The M relay stations transmit the same amount of power: \mathcal{P}/M , which means that

³ For the outage capacity, $f(\lambda)$ is replaced by a corresponding implicit function, cf. (12), instead of the expectation.

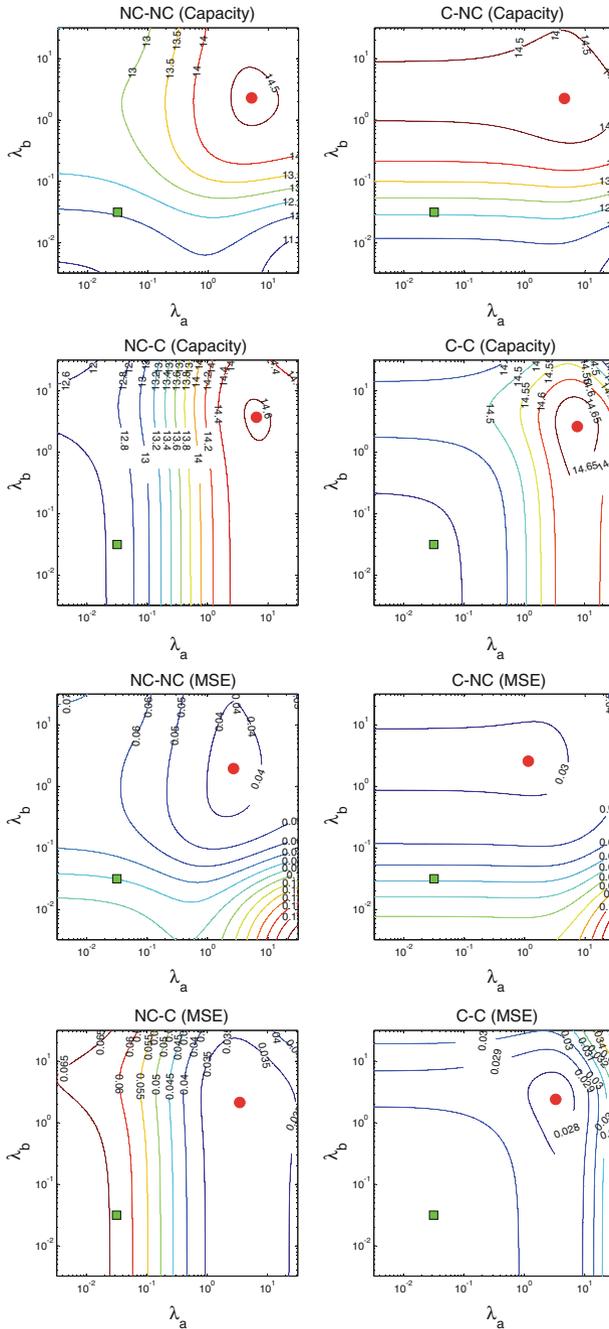


Fig. 3 Capacity and MSE contours versus (λ_a, λ_b) for a given realization of the backward and forward channels. The squares represent the parameter pair $(\rho_1^{-1}, \rho_2^{-2})$, and the circles represent the optimal pairs $(\lambda_1^0, \lambda_2^0)$. For capacity, the units are bits/s/Hz, while the MSE is normalized between 0 and N_S

the individual power constraints in (7) are uniquely specified by ρ_2 . The wireless channels undergo slow fading, and the channel matrices have statistically independent, circularly symmetric complex Gaussian entries with zero mean and unit variance.

5.1 Effects of the Parameters on Capacity and MSE Performance

We study the impact of λ on system performance by plotting the contours of the capacity and the sum of MSE's in Fig. 3. To obtain these contours, each of the backward and forward channel matrices, i.e., \mathbf{H}_k or \mathbf{G}_k for $k = 1, \dots, M$, is randomly generated but held constant. All the hybrid strategies, NC-NC, C-NC, NC-C and C-C, are considered and for simplicity, the same λ_a and λ_b are used for different relays.⁴ In each subplot, the circle represents the optimal operating point, while the square represents the parameter pair $\lambda_\rho \triangleq [\rho_1^{-1}, \rho_2^{-1}]^T$, which is associated to linear MMSE processing (cf. Table 1). Several observations and conclusions can be made from these performance contours (and those for other channel realizations not shown here):

1. Although optimizing λ is bound to improve performance, the performance gap can be quite remarkable. The optimal parameter pair $\lambda_{\text{opt}} = [\lambda_1^o, \lambda_2^o]^T$ is also notably larger than λ_ρ .
2. The capacity or MSE is not sensitive to small perturbations of λ . In addition, the system performance is less sensitive to the parameter of the cooperative substructure, than to that of the non-cooperative substructure. This can be explained by the fact that the term $\sum_{j=1}^M \mathbf{H}_j^H \mathbf{H}_j$, or $\sum_{j=1}^M \mathbf{G}_j \mathbf{G}_j^H$, is the sum of multiple statistically independent, positive semidefinite Wishart-distributed matrices, and therefore should be well-conditioned with high probability.
3. If either λ_a or λ_b is fixed and the other parameter increases from zero to infinity, the MSE (or capacity) using first decreases (increases) and then increases (decreases).

From the above observations, the proposed non-cooperative and cooperative relaying substructures, although simple, show strong potential and advantage which were not realized in previous works.

5.2 Performance Comparison

We next compare the proposed hybrid relaying strategies with other existing approaches in terms of 10%-outage capacity and average BER, based on Monte-Carlo simulations. The non-regenerative MIMO relaying strategies under comparison are listed in Table 2. In Sect. 5.1, the design parameters $\lambda = [\lambda_a, \lambda_b]^T$ were optimized for a fading channel instance. Here for simplicity, they are optimized using (18) based on the a priori knowledge of the channel statistics. In the BER simulations, the source antennas transmit independent uncoded 16-QAM modulated streams, and the destination user employs the linear MMSE MIMO combiner described in Sect. 4.2 to decode the information bits. The theoretical SER for each substream is upper bounded by (16), in which $\alpha = 3$ and $\beta = 10$ for 16-QAM. The theoretical BER is approximately equal to $P_e/4$, which is used to search for the BER-optimal $\lambda_{\text{opt}} = [\lambda_a^o, \lambda_b^o]^T$. In the simulations, we set the first SNR $\rho_1 = 15$ dB and increase the second SNR ρ_2 from 5 to 25 dB.

The 10%-outage capacity of the 1S-3R-1D link for the different relaying strategies is plotted in Fig. 4. As predicted by previous analysis, SAF and SVD perform unsatisfacto-

⁴ In fact, we have verified numerically that choosing different values for each relay brings only marginal performance improvement, but leads to higher complexity because of the multi-dimensional search for the optimal parameters.

Table 2 The relaying strategies under comparison

Strategy	Comments
Simplistic AF (SAF)	$\mathbf{F}_k = \eta_k \mathbf{I}$
SVD with uniform power allocation	$\mathbf{F}_k = \eta_k \mathbf{V}_{2k} \mathbf{U}_{1k}^H$ [22]
MF [18]	NC-NC(∞, ∞)
ZF [18]	NC-NC(0, 0)
Linear MMSE [18]	NC-NC(ρ_1^{-1}, ρ_2^{-2})
Two-step MMSE [2]	C-NC($\rho_1^{-1}, 0$)
NC-NC, C-NC, NC-C, C-C	Proposed hybrid methods
Upper bound	Iterative algorithms [11]

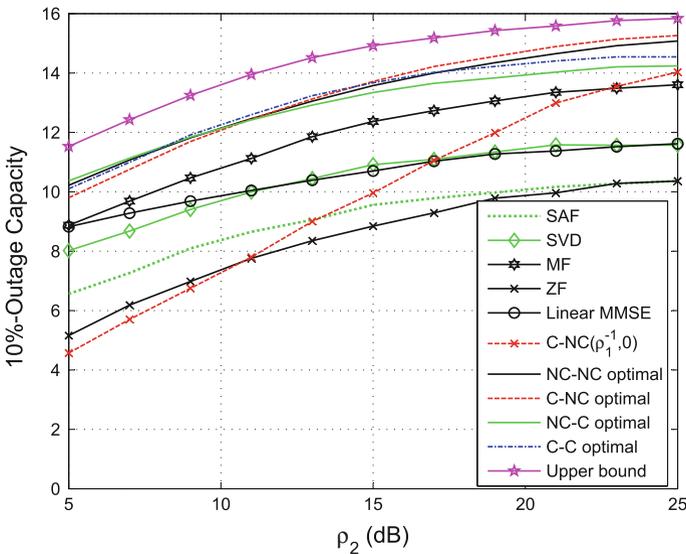


Fig. 4 10 %-outage capacity for 1S-3R-1D system with $\rho_1 = 15$ dB

rily due to their inability to achieve sufficient distributed array gain. Among the four hybrid relaying strategies with special parameter values, i.e., MF, ZF, linear MMSE and two-step MMSE, only MF can outperform SVD over a broad range of SNR values. This is because the good performance of a hybrid relaying strategy is guaranteed not only by coherent superposition of parallel transmissions, but also by efficient exploitation of the interferences without noise over-amplification. As expected, the performance of MF remains inferior to that of the proposed hybrid relaying strategies with optimal parameters. Indeed, the latter can result in significant improvement over MF in spectral efficiency by 1 to 1.5 bits/s/Hz. They come within less than one bit of the upper bound set by the iterative algorithm in [11], but with much lower complexity. Of the four methods, NC-C, C-C and C-NC are, respectively, the best strategy for high, intermediate and low ρ_2 values. The non-cooperative strategy NC-NC remains close to the best performance achieved by the other three cooperative strategies over the range of SNR values considered.

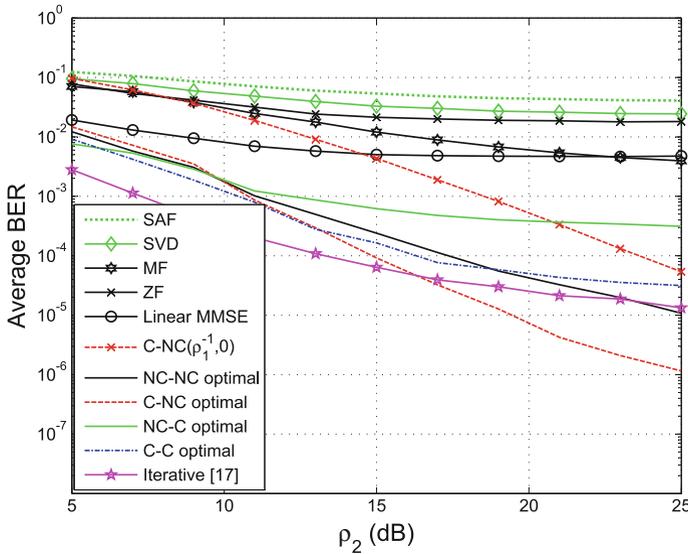


Fig. 5 BER performance for 1S-3R-1D system with $\rho_1 = 15$ dB

The BER results are plotted in Fig. 5 where again, the performance of SAF and SVD are unsatisfactory and ZF, MF and linear MMSE perform slightly better. The BER of C-NC($\rho_1^{-1}, 0$) is much lower for high ρ_2 , because this strategy can benefit from the cooperation between relays. However, with the optimal choice of parameters, 1S-MR-1D systems can fully exploit the potential of the proposed hybrid relaying framework: NC-NC, C-C and C-NC all lead to much lower BER values than previously investigated methods. In the mid-to-high SNR range, C-NC even performs better than the iterative MMSE method in [11]. In terms of BER, cooperation between relays brings in significant performance gain.

Finally, simulation results not shown demonstrate that small errors in channel estimation and SNR estimation only lead to small performance degradation. That is, the proposed hybrid strategies are not overly sensitive to such modeling errors.

5.3 Further Simplifications

In Sect. 4.3, we proposed that the logarithmic values of the optimal parameters, $\lambda_{\text{opt}} = [\lambda_a^o, \lambda_b^o]^T$, can be well approximated by linear functions of $\rho = [\log \rho_1, \log \rho_2]^T$. Considering for example the NC-NC method, the optimal parameters that maximize the outage capacity are plotted against ρ_1 and ρ_2 in Fig. 6, where the logarithmic scale is used. It is observed that the relationship between $\log \lambda_a^o$ (or $\log \lambda_b^o$) and ρ is very close to a plane, implying that the expression in (18) is sufficiently accurate. Similar relationships can be established for the criterion of average BER, and also for the other three hybrid methods: C-NC, NC-C and C-C.

The outage capacity under the parameters obtained from the empirical linear formulas is now compared with that when the parameters are optimized per channel instance. As seen in Fig. 7, the hybrid relaying strategies designed in this way cause negligible loss in performance, but the optimization of the parameters has much lower complexity.

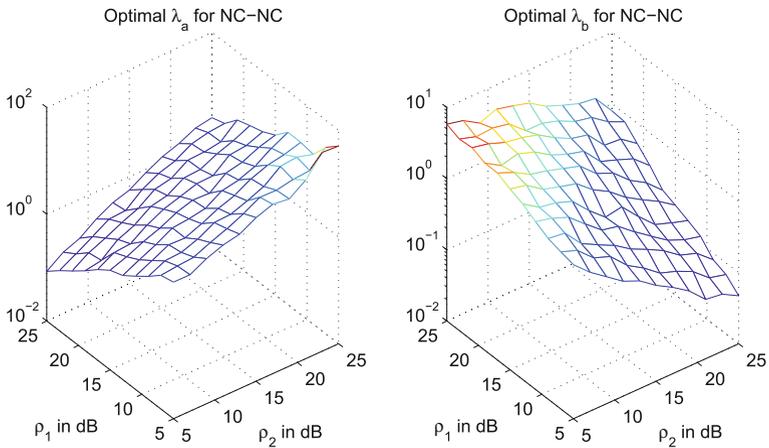


Fig. 6 The outage-capacity-optimal λ_a^o and λ_b^o values versus ρ_1 and ρ_2

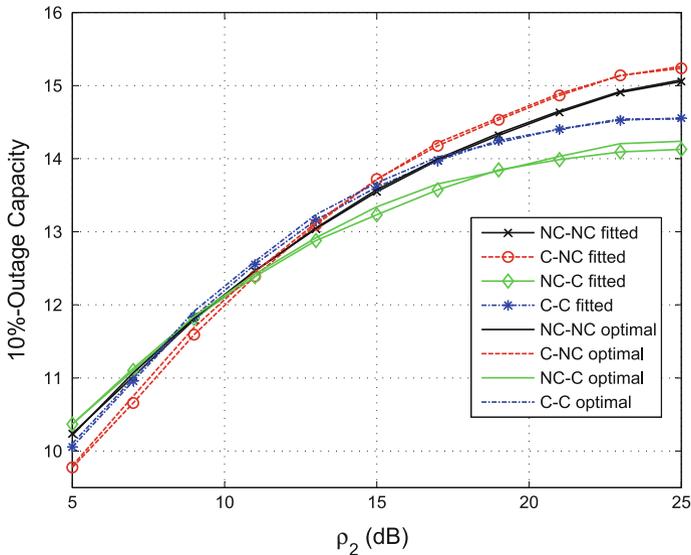


Fig. 7 10%-outage capacity: fitted parameters versus optimal parameters

6 Conclusion

In this work, to achieve a balance between performance and complexity for non-regenerative 1S-MR-1D relay systems, we proposed a unified hybrid framework in which the relaying matrices are generated by cascading two substructures. For each of these two substructures, we introduced both non-cooperative and cooperative, one-dimensional parametric families of candidate matrix transformations. This unified framework provides a generalization of several existing approaches and allows for the classification and comparison of all the possible combinations of the proposed substructures.

Within this hybrid framework, the design parameters λ can be further optimized, resulting in significant performance improvements. This can be done on-line based on individual channel estimates or off-line based on a priori knowledge of the channel statistics. In the latter case, the optimal parameters can be well approximated by linear functions of SNR $[\log \rho_1, \log \rho_2]^T$ with minor performance loss.

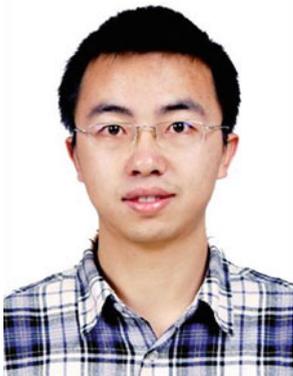
The optimal λ_{opt} differs significantly from those corresponding to the ZF, MF and linear MMSE relaying strategies. Through simulations, we showed that the capacity of selected hybrid schemes (with optimized parameters) comes within 1 bits/s/Hz of the upper bound achieved by the capacity-optimal iterative method in [11]. In the mid-to-high SNR range, the BER performance of C-NC even exceeds that of the MSE-optimal iterative method. The proposed hybrid methods therefore achieve a good balance between performance and complexity: they outperform existing low-complexity strategies by a large margin in terms of both capacity and BER, and at the same time, are significantly simpler than previous near-optimal iterative algorithms.

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