Rotated Multi-D Constellations in Rayleigh Fading: Mutual Information Improvement and a Pragmatic Approach for Near-Capacity Performance in High-Rate Regions

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Presentation outline

1. Motivation
2. System model
3. Rotation design
4. Code design
5. Conclusions
Motivation

Constellation rotation

- Improve the diversity order
  - without increasing bandwidth or power
- Convert the Rayleigh fading channel into an AWGN channel
  - At high SNR, high enough dimensions
  - Uncoded system [1]
  - Coded system (BICM-ID) [2]
- Improve the mutual information between channel input and output [3]
  - *Instantaneous* mutual information improvements by known rotations
- Fundamental questions
  - *Constraint capacity* - What is the mutual information improvement in Ergodic sense?
  - *Rotation design* - Which rotation maximizes the constraint capacity?
  - *Code design* - What kind of codes operate near the capacity limit?
System model (Multi-D Rotation)

Block Fading - Channel Gains

Deep fade

|h_1| |h_2| |h_3| |h_4| |h_5| ... 

Before rotation

S_1 S_2 S_3 S_4 ... ...

Σ_i g_1 s_i Σ_i g_2 s_i Σ_i g_3 s_i Σ_i g_4 s_i ... ...

x_1 x_2 x_3 x_4

After rotation

Multi-D rotation

\[
\begin{pmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 \\
\end{pmatrix} = G 
\begin{pmatrix}
  s_1 \\
  s_2 \\
  s_3 \\
  s_4 \\
\end{pmatrix}
\]

Received symbol \((r \in \mathbb{C}^{N \times 1})\)

\[
r = Hx + w = HGs + w
\]

\(G = \) Rotation matrix

\(H = \text{diag}(h_1, h_2, \ldots, h_N)\)

\(x = \) Rotated symbol

\(s = \) Unrotated symbol

\(w = \) Noise
Constraint capacity improvements are significant
**Design criterion**

Mutual information conditioned on channel fading

\[
I|H = \log_2(Q) - \left[ \frac{1}{Q} \sum_{x \in \Psi_r} \frac{1}{(\pi N_0)^N} \int_{r \in \mathbb{C}^N} \exp \left( -\frac{||r - H \cdot x||^2}{N_0} \right) \right]
\]

\[
\times \log_2 \left( \sum_{y \in \Psi_r} \exp \left( \frac{||r - H \cdot x||^2 - ||r - H \cdot y||^2}{N_0} \right) \right)
\]

\[
C \geq \log Q - \log \left( 1 + \frac{1}{Q} \sum_{s \in \Psi} \sum_{p \in \Psi, p \neq s} M(s, p, G)^{-1} \right) \tag{2}
\]

where \( M(s, p, G) = \prod_{k=1}^{N} \left( 1 + \frac{\gamma ||g_k(s - p)||^2}{4} \right) \)
Ergodic capacity and lower bound: 4-QAM

- Lower bound models the behavior of capacity (apx. 3 dB away)
- Dimensionality larger than 4 might not be necessary
  - A trade-off between performance & complexity
High SNR : Constraint capacity is saturated to its limit
Low SNR : Constraint capacity is close to the Shannon limit
Design should focus on medium SNR region
Rotation design

Full diversity rotation vs Capacity maximizing rotation

**Capacity maximizing rotation:**

\[
\text{minimize} \sum_{s \in \Psi} \sum_{p \in \Psi, p \neq s} \prod_{k=1}^{N} \left( 1 + \frac{\gamma \| g_k(s - p) \|^2}{4} \right)^{-1}
\]

(3)

**Full diversity rotation** (maximize the minimum product distance):

\[
\text{maximize} \left( \text{minimum} \left\{ \prod_{k=1}^{N} \| g_k(s - p) \|^2 \right\} \right) \quad \forall s, p \in \Psi
\]

(4)

- Capacity maximizing rotations
  - High SNR (\( \gamma \)) assumption is not valid
  - Need to consider every constellation point
- Unified analysis is difficult for
  - all the constellations
  - all dimensions (\( N \))
  - all SNR values (\( \gamma \))
Rotation design: 4-QAM, 4-Dimensional

**Lemma 1**
To maximize $M(s, p, G)$ for all single non-zero components, all the components of $G$ must be equal in magnitude.

**Lemma 2**
For 4-QAM, the unitary rotation $G$ maximizes $M(s, p, G)$ for $(s - p)$ having two complex non-zero components $\epsilon_1$ and $\epsilon_2$ with $\|\text{Re}(\epsilon_1^* \epsilon_2)\| = \|\text{Im}(\epsilon_1^* \epsilon_2)\|$, if and only if:

$$\theta_{i,1} = \theta_{i,2} + \pi/4 = \theta_{i,3} + \pi/2 = \theta_{i,4} + 3\pi/4, 1 \leq i \leq 4$$

**Rotation $G$ (Unitary matrix)**

$$G = \frac{1}{2} \begin{pmatrix} 1 & e^{-j \frac{\pi}{4}} & e^{-j \frac{\pi}{2}} & e^{-j \frac{3\pi}{4}} \\ 1 & -e^{-j \frac{\pi}{4}} & e^{-j \frac{\pi}{2}} & -e^{-j \frac{3\pi}{4}} \\ -1 & -e^{-j \frac{\pi}{4}} & e^{-j \frac{\pi}{2}} & e^{-j \frac{3\pi}{4}} \\ 1 & -e^{-j \frac{\pi}{4}} & -e^{-j \frac{\pi}{2}} & e^{-j \frac{3\pi}{4}} \end{pmatrix}$$

(5)
Capacity of proposed rotation: 4-QAM

Which codes can operate close to the capacity?
BICM-ID system with multi-D rotation

Iterative decoding

Best matching convolution codes through EXIT chart analysis
Error floor of about $10^{-6}$ is observed.

Interleaver length $3 \times 10^5$ and Matching convolution codes (CC) and Puncture patterns (PP): (1). Code rate=1/2, CC=[1, 0, 1; 1, 1], (2). Code rate=2/3, CC=[1, 0, 1; 1, 1, 1], PP=[1, 0; 1, 1], (3). Code rate=3/4, CC=[1, 0, 1; 1, 1, 1], PP=[1, 0; 1, 1, 0]
Remarkable improvements at higher code rates
Conclusion

- **Multi-D rotation design to maximize the mutual information (Ergodic)**
  - Over Rayleigh block fading channel
  - Remarkable improvements in high rate regions

- **Design of good Unitary rotation for 4-QAM in 4-Dimensional signal space**

- **Practical code design to operate near capacity**
  - BICM system with a short memory convolution codes, multi-D mapping and iterative decoding
  - Design through EXIT chart analysis

- **System which operates below the constraint capacity limit**
  - Close capacity operation at higher code rates
References


Thank you!

Questions?