6 Digital transmission of analog signals

6.1 Sampling of signals

Sampling: A continuous time signal $x(t)$ is converted into a corresponding sequences of samples such that the continuous time signal $x(t)$ can be recovered from its sequences of samples.

$$\{x(t), T_1 \leq t \leq T_2\} \xrightarrow{\text{sampling}} \{x(t_0), x(t_1), \ldots\}$$

Usually the samples are spaced uniformly in time.

a) Ideal sampling

An ideal sampled signal can be represented as

$$\begin{aligned} x_\delta(t) &= x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \\ &= \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s) \\ &= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \end{aligned}$$

where $f_s = \frac{1}{T_s}$ is the **sampling rate** and T_s is the sampling period.

Exercise: Draw an example of a continuous time signal and its ideal sampled version.

b) Practical sampling

$$x_s(t) = \left[x(t) \sum_{n=-\infty}^{\infty} p(t - nT_s) \right] * h(t)$$

where $p(t)$ is the sampling pulse and $h(t)$ is the pulse stretcher (ex. sample and hold S/H).

$x_A(t) = p(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ (mathematical model)

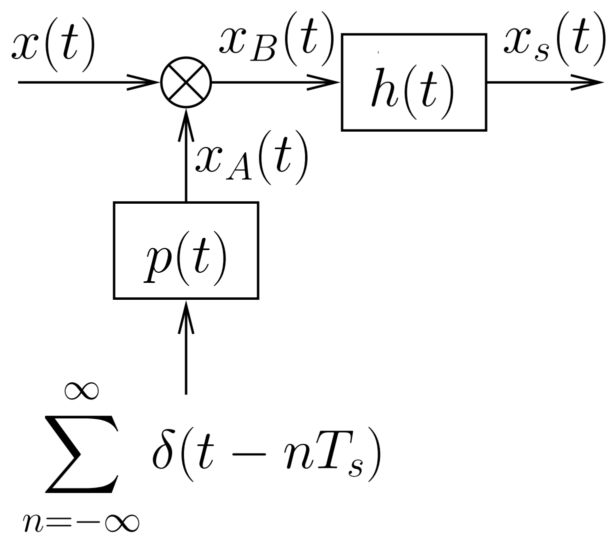

Figure 22: Model for practical sampling

$$= \sum_{n=-\infty}^{\infty} p(t - nT_s) \quad \text{(actual signal generated by practical sampler using a sampling pulse } p(t)\text{)}$$

$$x_B(t) = x(t) \sum_{n=-\infty}^{\infty} p(t - nT_s)$$

$$x(t) = x_B(t) * h(t) = \left[x(t) \sum_{n=-\infty}^{\infty} p(t - nT_s) \right] * h(t)$$

In order to find the expression of the Fourier transform of $x_s(t)$, we need the following result:

$$\mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right\} = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_s} \right)$$

Proof. $\sum_{n-\infty}^{\infty} \delta(t - nT_s)$ is periodic with period T_s so it has a Fourier series representation

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_s n t} \tag{20}$$

where the Fourier coefficient c_n is given by

$$c_n = \frac{1}{T_s} \int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \left[\sum_{p=-\infty}^{\infty} \delta(t - pT_s) \right] e^{j2\pi n f_s t} dt$$

$$= \frac{1}{T_s} \int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta(t) e^{-j2\pi n f_s t} dt \quad \text{since only } \delta(t) \text{ in } \sum_{p=-\infty}^{\infty} \delta(t - pT_s) \text{ is included in } \left[-\frac{T_s}{2}, \frac{T_s}{2}\right]$$
$$= \frac{1}{T_s}$$

Taking the Fourier transform of (20) completes the proof. **Q.E.D**

Let us investigate now the Fourier transform of $x_s(t)$.

$$\begin{aligned}
X_s(f) &\stackrel{\triangle}{=} \mathcal{F}\{x_s(t)\} = \mathcal{F}\left\{\left[x(t) \sum_{n=-\infty}^{\infty} p(t - nT_s)\right] * h(t)\right\} \\
&= \left[X(f) * \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} p(t - nT_s)\right\}\right] H(f) \\
&= \left[X(f) * \mathcal{F}\left\{p(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT_s)\right\}\right] H(f) \\
&= \left[X(f) * \left(P(f) \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_s}\right)\right)\right] H(f) \\
&= \left[X(f) * \left(\frac{1}{T_s} \sum_{n=-\infty}^{\infty} P\left(\frac{n}{T_s}\right) \delta\left(f - \frac{n}{T_s}\right)\right)\right] H(f) \\
&= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} P\left(\frac{n}{T_s}\right) X\left(f - \frac{n}{T_s}\right) H(f)
\end{aligned}$$

Exercise: Assuming that $X(f)$ is band-limited to W, draw a typical spectrum of its sampled version $x_s(t)$ and deduce the required condition of possible recovery of $X(f)$ from $X_s(f)$.

Complete the proof:

$$X_s(f) = \underbrace{\qquad\qquad\qquad}_{\substack{\text{low-pass signal} \\ \text{band-limited to } W}} + \underbrace{\qquad\qquad\qquad}_{\text{sum of high-pass signals}}$$

If   or equivalently   , the low-pass term   and the sum of high-pass terms   do not overlap and $X(f)$ can be reconstructed from $X_s(f)$ by   and inverse filtering. Note that $2W$ is called the **Nyquist rate**.

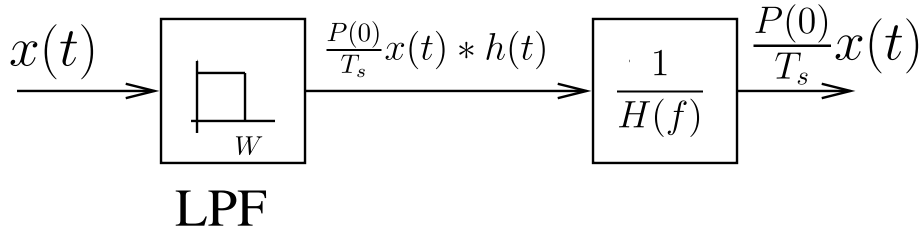

Figure 23: Reconstruction of $X(f)$ from $X_s(f)$ for practical sampling

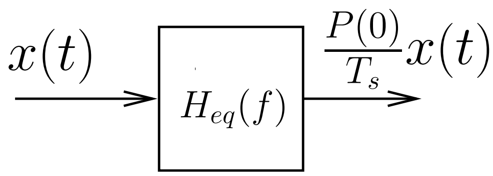

Figure 24: Reconstruction of $X(f)$ from $X_s(f)$ (equivalent to Fig. 23)

where $H_{eq}(f)$ is given by

$$H_{eq}(f) = \begin{cases} \dfrac{1}{H(f)}, & |f| \leq W \\ 0, & \text{else.} \end{cases}$$

Assume that $p(t) = h(t) = \delta(t)$ corresponding to ideal or instantaneous sampling, then the above result is summarized by the sampling theorem:

- A band-limited signal of finite energy, which has no frequency components higher then W Hz, is completely described by specifying the values of the signal at instants of time separated by $\frac{1}{2W}$ sec.
- A band-limited signal of finite energy, which has no frequency components higher then W Hz, may be completely recovered (by   filtering) from a knowledge of its samples taken at the rate of $2W$ samples per second.

c) Examples of sampling schemes:

Non-zero width sampling:

$$h(t) = \delta(t)$$
$$p(t): \quad \text{pulse of duration } T \text{ where } T < T_s$$

Exercise: Draw an example of a continuous signal and its non-zero width sampled version.

Natural sampling:

$$h(t) = \delta(t)$$
$$p(t) = u(t) - u(t-T) = \text{rect}\left(\frac{t - \frac{T}{2}}{T}\right)$$

$p(t)$ is a rectangular pulse spanning the interval $[0, T]$.

Exercise: Find the expression of $x_s(t)$ and draw a continuous time signal and its natural sampled version.

Zero-order hold sampling:

$$h(t) = u(t) - u(t-T_s) = \text{rect}\left(\frac{t - \frac{T_s}{2}}{T_s}\right)$$
$$p(t) = \delta(t)$$

$h(t)$ is a rectangular pulse spanning the interval $[0, T_s]$.

Exercise: Find the expression of $x_s(t)$ and draw a continuous time signal and its zero-order hold sampled version.

Flat top sampling:

$$h(t) = u(t) - u(t-T) = \text{rect}\left(\frac{t - \frac{T}{2}}{T}\right)$$
$$p(t) = \delta(t)$$

where $T < T_s$. $h(t)$ is a rectangular pulse spanning the interval $[0, T]$.

Exercise: Find the expression of $x_s(t)$ and draw a continuous time signal and its flat top sampled version.

Ideal or instantaneous sampling:

$$h(t) = p(t) = \delta(t)$$

6.2 Pulse Amplitude modulation

Pulse modulation is a mapping of discrete information a_i into sequences of pulses $p_i(t)$ such that a_i can be recovered from $p_i(t)$.

An example is **Pulse Amplitude Modulation (PAM)** defined as

$$x(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT)$$

Transmission of a message signal $s(t)$ using PAM:

$$x(t) = \sum_{n} s(nT_s) g(t - nT_s)$$

where $s(nT_s)$ are the samples of $s(t)$.
Flat top sampling PAM wave:

$$g(t) = u(t) - u(t - T) \quad T < T_s$$

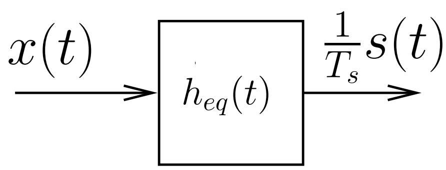

Figure 25: Reconstruction of $s(t)$ from it PAM modulated wave $x(t)$

where $H_{eq}(f) = \mathcal{F}\{h_{eq}(t)\}$ is given by

$$H_{eq}(f) = \begin{cases} \frac{1}{G(f)} = \frac{\exp\{j\pi fT\}}{T\text{sinc}(fT)}, & |f| \leq W \\ 0, & \text{else.} \end{cases}$$

Fig.26 represents an equivalent reconstration scheme where $H_{eq}(f)$ is separated into a Low Pass filter and equalizer. The amplitude response of the equalizer (magnitude of the Fourier transform

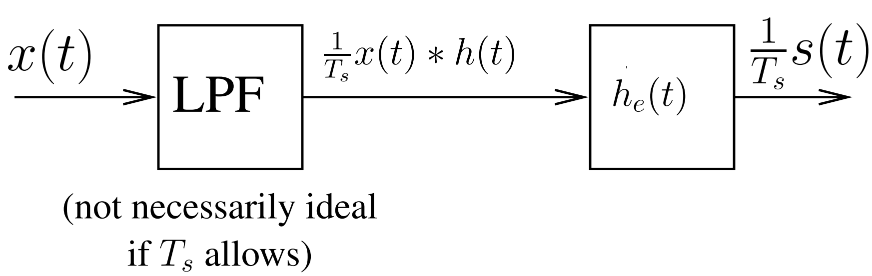

Figure 26: Reconstruction of $s(t)$ from its PAM modulated wave $x(t)$

of $h_e(t)$) of the equalizer is

$$|H_e(f)| = \frac{1}{T\text{sinc}(fT)} = \frac{1}{|G(f)|}$$

6.3 Time-Division Multiplexing (TDM)

Let $s_1(t), \ldots, s_N(t)$ be N signals band-limited to W. let $\frac{1}{T_s} > 2W$. Let $s_{n\delta}(t)$ be the ideal sampled version of $s_n(t)$ sampled at rate $\frac{1}{T_s}$ defined as

$$s_{n\delta}(t) = \sum_{k=-\infty}^{\infty} s_n\left[(n-1)\frac{T_s}{N} + kT_s\right]\delta\left(t - \left[(n-1)\frac{T_s}{N} + kT_s\right]\right)$$

If $n \neq m$ then $s_{n\delta}(t)$ and $s_{m\delta}(t)$ do not overlap in time and the time division multiplexed signal is defined as

$$s_\delta(t) = \sum_{n=1}^{N} s_{n\delta}(t) = \sum_{n=1}^{N}\sum_{k=-\infty}^{\infty} s_n\left[(n-1)\frac{T_s}{N} + kT_s\right]\delta\left(t - \left[(n-1)\frac{T_s}{N} + kT_s\right]\right)$$

Therefore, the ideally sampled TDM signal is composed of delta functions spaced at intervals of $\frac{T_s}{N}$.

Exercise: Draw a TDM signal obtained by multiplexing two signals $s_1(t)$ and $s_2(t)$

Exercise: Draw the block diagram of a TDM system including the TDM multiplexer and the TDM de-multiplexer. Assume that the input signals $s_i^o(t)$ are not all band-limited to W.

Figure 27: Block diagram of a TDM system

- The input signals $s_i(t)$, all band-limited to W, are sequentially sampled at the transmitter by a rotary switch or commutator. The switch makes f_s revolutions per second and extracts one sample from each input during each revolution. If $s_i(t)$ are not band-limited to W, they can be passed through a bank of low-pass filters as illustrated in Fig. 27 with the non-band-limited input signals $s_i^o(t)$.
- The TDM signal is applied to a pulse modulator. This can be a PAM modulator or the samples values may be quantized and transmitted using Pulse Code modulation (PCM) defined in Section 6.5.
- At the receiver, the received signal is applied to a pulse demodulator. The samples from individual channels are separated and distributed by another rotary switch called a distributor or decommutator which must be synchronized with the rotary switch of the multiplexer. The samples from each channel are filtered to reproduce the original message signal.

Note that for PAM, a separate pulse demodulator is not really needed since demodulation can be included in the reconstruction filter.

Bandwidth of TDM signals:

Exercise: Regarding $s_\delta(t)$ as a sampled version of a continuous time signal $s(t)$, find the minimum bandwidth of a TDM signal for perfect reconstruction.

Exercise: a) Design a TDM system for 4 signals $s_1(t), s_2(t), s_3(t), s_4(t)$, each band-limited to $W \leq 4\text{kHz}$, including a frame synchronization pulse. Compute the sample rate of the multiplexed signal.

b) Suppose now that the bandwidth of $s_1(t)$ is 8kHz and the bandwidth of $s_2(t), s_3(t), s_4(t)$ is 4kHz. Assume that we want to keep the switch rotating at 8000 revolutions per second. Compute the sample rate of the multiplexed signal.

6.4 Quantization

The results of sampling a continuous time signal is a sequence $\{A_1, A_2, \ldots\}$ where A_i assume continuous values.

Quantization: Transformation of the samples A_i into the samples A_{iq} which can assume a discrete finite number of values.

Let Q be a quantizer with N amplitude levels. Let A be a continuous variable. Its quantized value is $A_q = Q(A)$ defined as follows:

$$\forall\, i = 1, \ldots, N \quad \text{if } a_{i-1} < A \leq a_i \quad A_q = Q(A) = \hat{a}_i \tag{21}$$

with the convention $a_0 = -\infty$ and $a_N = \infty$. Therefore $Q(A)$ is a staircase function of A. a_i are called the **decision levels** of the quantizer and $\hat{a}_i$ are called the **representation levels or quantization levels** of the quantizer. The step size [3] is

$$\Delta_i = a_i - a_{i-1}$$

Performance measure of a quantizer:

The performance measure of a quantizer is given by the **Quantizer Output-to-noise ratio** usually called **Signal-to-Quantization-noise-ratio (SQNR)** defined as

$$\text{SQNR} = \frac{P_A}{P_\epsilon}$$

where ϵ is the **Quantization error** given by

$$\epsilon = A - Q(A)$$

If $A(t)$ is a random process quantized to $Q\big(A(t)\big)$, the powers P_A and P_ϵ are given by

$$P_A = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} E\left[A^2(t)\right] dt$$
$$P_\epsilon = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} E\left[\big(A(t) - Q\big(A(t)\big)\big)^2\right] dt$$

If $A(t)$ is a WSS random process quantized to $Q\big(A(t)\big)$ and $\epsilon(t) = A(t) - Q(A(t))$ is also a WSS random process, the powers P_A and P_ϵ are given by

$$P_A = R_A(0) = E\left[A^2\right] \qquad P_\epsilon = E\left[\big(A - Q(A)\big)^2\right]$$

[3] In [1], the step size is defined as $\Delta_i = \hat{a}_i - \hat{a}_{i-1}$

If A is a random variable quantized to $Q(A)$, the powers P_A and and P_ϵ are given by

$$P_A = E\left[A^2\right] \qquad P_\epsilon = E\left[\big(A - Q(A)\big)^2\right]$$

Assume that $A(t)$ is a WSS random process quantized to $Q\big(A(t)\big)$ and $\epsilon(t) = A(t) - Q(A(t))$ is also a WSS random process. It can be shown that the optimum quantizer (in the sense that it maximizes the SQNR or equivalently minimizes the distortion P_ϵ) must satisfy:[4]

1) The decision levels of the quantizer are the midpoints of the quantization levels, i.e.

$$a_i = \frac{1}{2}\left(\hat{a}_i + \hat{a}_{i+1}\right)$$

2) The representation levels are the centroids of the quantization regions

$$\hat{a}_i = E\left[A(t) | a_{i-1} < A(t) \leq a_i\right]$$

a) Uniform quantization

First definition [2]:

A quantizer is said to be **uniform** if it has uniformly spaced decision levels excluding the step size Δ_1 and Δ_N, i.e.

$$\forall\, i = 2, \ldots, N-1 \qquad \Delta_i = a_i - a_{i-1} = \Delta$$

Alternative definition [1]:

A quantizer is said to be **uniform** if it has uniformly spaced representation levels, i.e.

$$\forall\, i = 2, \ldots, N \qquad \hat{a}_i - \hat{a}_{i-1} = \Delta$$

Simpler definition that agrees with both previously defined definitions:

Rationale: Assume that A is an uniformly distributed random variable between $A_{\min}$ and $A_{\max}$, then the optimum quantizer is uniform and satisfies

$$a_i = \frac{1}{2}\left(\hat{a}_i + \hat{a}_{i+1}\right)$$

yielding with $a_i - a_{i-1} = \Delta$

$$\hat{a}_{i+1} - \hat{a}_{i-1} = 2\Delta$$

Therefore for sake of simplicity in this course we will consider the following definition of a uniform

[4]These relationships need not to be known, but will be used to justify the forms of the representation and quantization levels of a uniform quantizer.

quantizer:

$$a_i - a_{i-1} = \hat{a}_i - \hat{a}_{i-1} = \Delta \quad i = 2, \ldots, N-1$$

Therefore the decision levels and representation levels of N levels uniform quantizer with an infinite dynamic range are given by

$$\begin{aligned}
a_0 &= -\infty \\
\hat{a}_1 &= a_1 - \frac{\Delta}{2} \\
\hat{a}_i &= \frac{a_{i-1} + a_i}{2} \quad i = 2, \ldots, N-1 \\
\hat{a}_N &= a_{N-1} + \frac{\Delta}{2} \\
a_N &= +\infty \\
\Delta &= a_i - a_{i-1} \quad i = 2, \ldots, N-1
\end{aligned}$$

Exercise: Draw $Q(A)$ versus A for a **midtread** (i.e. N odd) uniform quantizer and a **midrise** (i.e. N even) uniform quantizer.

Assume that we want to represent a continuous variable A, $A_{\min} \le A \le A_{\max}$ by a word of b bits using uniform quantization. Assume that we want to match the dynamic range of A with the dynamic range of the quantizer.
There are 2^b possible words of b bits, which can represent $N = 2^b$ representation levels. The decision levels and representation levels of this $N = 2^b$ levels uniform quantizer with a finite range $[A_{\min}, A_{\max}]$ are given by

$$\begin{aligned}
\Delta &= \frac{A_{\max} - A_{\min}}{N} = \frac{A_{\max} - A_{\min}}{2^b} \\
a_i &= A_{\min} + i\Delta \quad \text{i=0, \ldots, N} \\
\hat{a}_i &= A_{\min} + i\Delta - \frac{\Delta}{2} \quad \text{i=1, \ldots, N}
\end{aligned}$$

For a uniform quantizer,

$$-\frac{\Delta}{2} \le \epsilon \le \frac{\Delta}{2}$$

since $a_{i-1} - \hat{a}_i = \frac{\Delta}{2}$ and $\hat{a}_i - a_i = -\frac{\Delta}{2}$. If the step size Δ is sufficiently small(i.e. the number of representation level is sufficiently large) then it can be assumed that ϵ is uniformly distributed in $\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$ and P_ϵ is given by

$$P_\epsilon = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{\epsilon^2}{\Delta} d\epsilon = \frac{\Delta^2}{12} = \frac{(A_{\max} - A_{\min})^2}{12 \cdot 4^b}$$

Therefore the SQNR is given by

$$\begin{aligned}
\text{SQNR} &= \frac{4^b 12 \cdot P_A}{(A_{\max} - A_{\min})^2} \\
\text{SQNR}_{dB} &= 10log\,(\text{SQNR}) \\
&= 10log\left(4^b\right) + 10log\left(\frac{12P_A}{(A_{\max} - A_{\min})^2}\right) \\
&= 6b + K \quad \text{6dB law}
\end{aligned}$$

K depends on the signal statistics, $A_{\max} - A_{\min}$ is the dynamic range of the quantizer and b the number of bits/samples.

Exercise: a) Suppose that $A(t) = a\cos(2\pi ft + \theta)$ where a, f are fixed and known and θ is a RV uniformly distributed in $[-\pi, \pi]$. Find the SQNR in dB of a uniform quantizer whose dynamic range matches the dynamic range of $A(t)$.

b) Suppose the quantizer from a) is used to quantize the signal $B(t) = \frac{a}{2}\cos(2\pi ft + \theta)$, find the new SQNR.

c) Suppose the quantizer from a) is used to quantize the signal $C(t) = 2a\cos(2\pi ft + \theta)$. What is Q(C) ? What is the difficulty in that case ?

b) Quantizer with companding

If the signal to be quantized is not uniformly distributed, then the probability of certain values of the signal is higher than other values. In this case we may want to use a nonuniform quantizer such that high probability values are quantized more accurately $\Longrightarrow$ companding.

compand: COMpress and exPAND

Exercise: Draw the block diagram of non-uniform quantization system.

Standards:

- μ law compression

$$|y| = \frac{\log\left(1 + \mu |x/x_{\max}|\right)}{\log(1+\mu)}$$

  $\mu \geq 0$ (USA & CANADA).

- A law compression

$$|y| = \begin{cases} \dfrac{A|x/x_{\max}|}{1+\log(A)} & 0 \leq |x/x_{\max}| \leq \frac{1}{A} \\ \dfrac{1+\log\left(A|x/x_{\max}|\right)}{1+\log(A)} & \frac{1}{A} \leq |x/x_{\max}| \leq 1 \end{cases}$$

6.5 Pulse Code modulation (PCM)

Pulse Code Modulation (PCM): Analog-digital conversion of a where the information contained in the instantaneous samples of an analog signal is represented digitally.

- *Low-pass filter of the PCM modulator*: prevents aliasing due to sampling.
- *Sampler*: The low pass filtered continuous time signal $s(t)$ is sampled and represented by its samples.
- *Quantizer*: Each sample $s(nT_s)$ is mapped to a discrete quantity assuming a finite number of values $\hat{s}(nT_s) = Q\left(s(nT_s)\right)$.
- *Encoder*: $\hat{s}(nT_s)$ is represented by a digital word composed of several symbols. The symbols can be bits, i.e. 0 or 1 (binary codes) or other type of symbols (ex. taken from alphabet '1,2,3, ...'). The system designer specifies the exact code words that will represent a particular

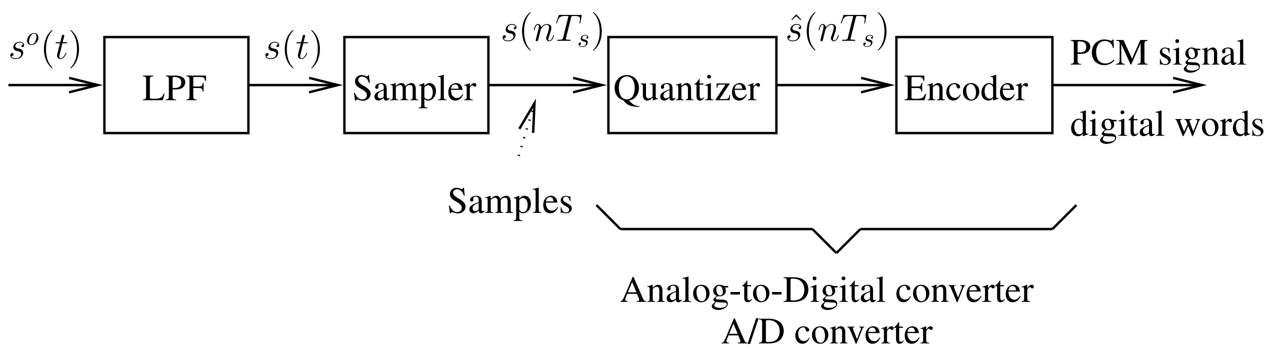

Figure 28: PCM modulator (transmitter)

quantized level. Note that in a complete communication system the digital words are then represented by continuous time waveforms used for their electronic transmission using either binary or M-ary modulation.

Ex: Let us assume binary coding using a Gray code for the quantized sample voltage $\{+5, +3, +1, 0\}$, and a binary modulation.

Quantized voltage	PCM digital words	Transmitted waveform
$+5$	00	$s_1(t) + s_1(t-T)$
$+3$	01	$s_1(t) + s_2(t-T)$
$+1$	11	$s_2(t) + s_2(t-T)$
0	10	$s_2(t) + s_1(t-T)$

A Gray code is a code such that two adjacent symbols (in the context of PCM two adjacent quantized voltages) differ by only one bit. Used as mapping of symbols of a M-ary modulation into bits words, it yields a minimum probability of bit error. In context of PCM, it yields to the minimum error between the transmitted PCM digital word and the recovered PCM word after reception as explained in the following. If a receiver decides that the digital word 01 was transmitted while in fact the digital word 00 was actually transmitted. After decoding of the PCM signal, the recovered quantized value will be set to $+3$ instead of $+5$. If $+5$ was coded as 00 and $+1$ as 01 , the recovered quantized value will be set to $+1$ instead of $+5$ which corresponds to a bigger error.

Ex: Let us assume quaternary coding for the quantized sample voltage $\{+5, +3, +1, 0\}$ and a quaternary (4-ary) modulation.

Quantized voltage	PCM digital words	Transmitted waveform
$+5$	1	$s_1(t)$
$+3$	2	$s_2(t)$
$+1$	3	$s_3(t)$
0	4	$s_4(t)$

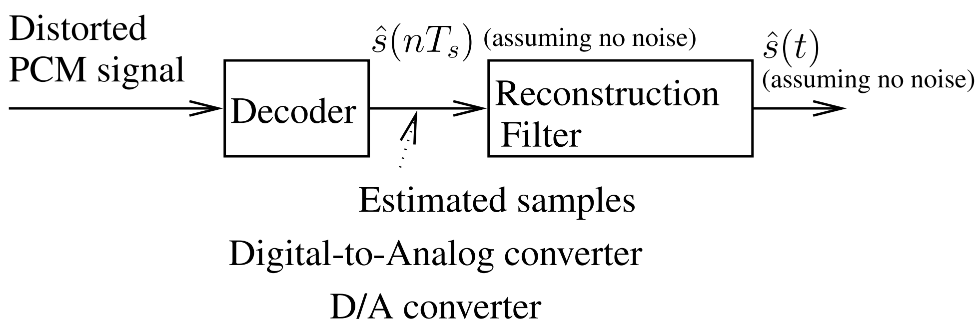

Figure 29: PCM demodulator (receiver)

- After transmission through the channel, the received signal is demodulated to regenerate the symbols (for example bits if binary modulation).
- *Decoder*: The symbols are regrouped into code words and decoded (i.e. mapped back) into analog samples.

Bandwidth requirement of binary PCM: Let $s(t)$ be a signal of bandwidth W sampled at $f_s \geq 2W$Hz. Assume that b bits are used to represent each sample $s(nT_s)$. Therefore, the bit rate is $R_b = bf_s \geq 2bW$, and with binary transmission the minimum required bandwidth to accomodate this rate is $R_b/2 \geq bW$. It is seen that the minimum PCM bandwidth varies linearly with the number of bits/samples.

Limitations of PCM: It does not exploit the correlation that may exist between adjacent samples of a continuous time signal. In other words the resulted encoded signal contains redundant information. If the signal to be quantized has a large dynamic range, to ensure low quantization error, a large number of representation level is needed or equivalently a large number of bits/ samples which translates into a large bandwidth requirement. In other words PCM is characterized by a waste of bandwidth and dynamic range. The remedies is to use Differential Pulse Code Modulation (DPCM).

6.6 Differential Pulse Code Modulation (DPCM)

The principle of DPCM is: not to send the part of the signal that can be predicted from past samples. The DPCM modulator (transmitter) is illustrated in Fig. 30. The predicted signal $\hat{s}(nT_s)$ is predicted from $\{\hat{s}(kT_s) + \epsilon_q(kT_s), n - L \le k \le n - 1\}$, where L is the prediction order.

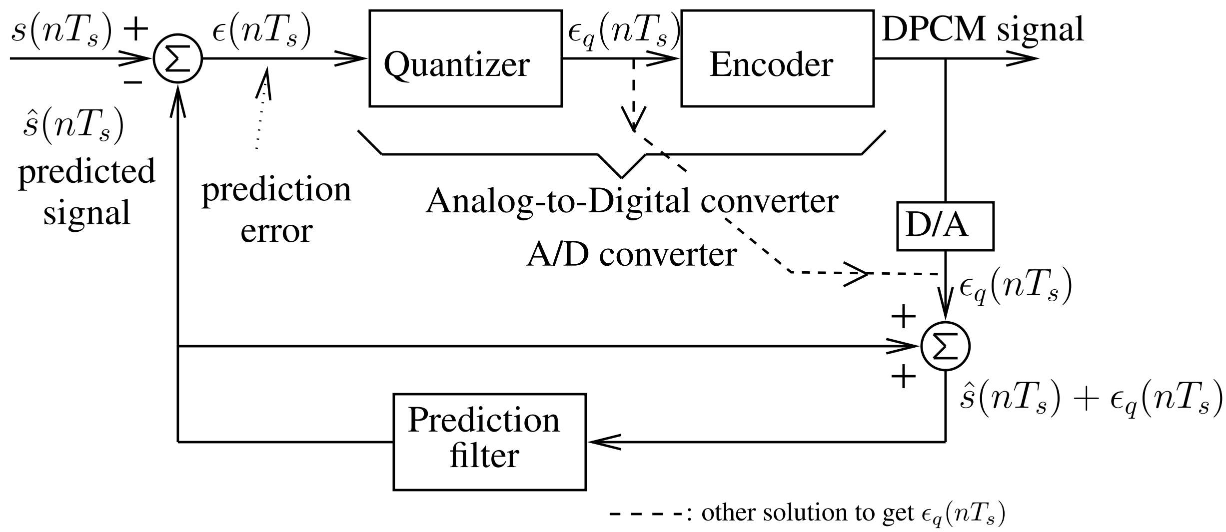

Figure 30: DPCM modulator (transmitter)

Assuming a L order linear prediction filter (tapped-delay line filter), we have

$$\hat{s}(nT_s) = \sum_{i=1}^{L} h(i) \left[\hat{s}\big((n-i)T_s\big) + \epsilon_q\big((n-i)T_s\big)\right]$$

The DPCM demodulator (receiver) is illustrated in Fig. 31.

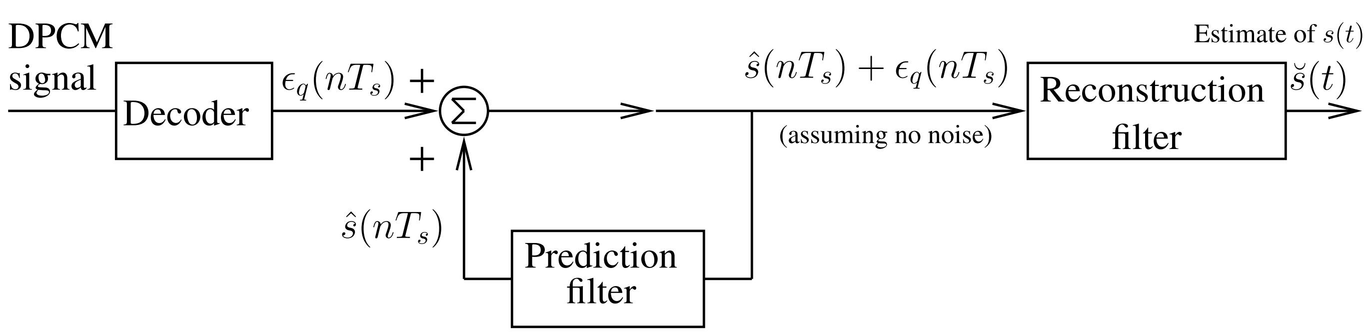

Figure 31: DPCM demodulator (receiver)

In DPCM, the quantization/encoding is done on the error signal $\epsilon(nT_s)$ which has a lower dynamic range than $s(nT_s)$, if the samples $s(nT_s)$ are correlated.

The Signal-to-quantization noise ratio of DPCM is given by

$$\mathrm{SQNR_{dB}} = 6b - 7.27 + \mathrm{SNI}_{dB}$$

where the signal-to-noise improvement ratio is given by

$$\begin{array}{ll} \mathrm{SNI_{dB}} \approx 13 - 25\mathrm{dB} & \text{speech} \\ \mathrm{SNI_{dB}} \approx 25 - 35\mathrm{dB} & \text{music} \end{array}$$

Therefore to achieve the same SQNR_{dB} with DPCM compared to PCM, $2 - 4$ less bits/samples are needed.

DPCM with 1 bit/samples is called Delta demodulation.

6.7 Delta modulation (DM)

Since for Delta modulation, only one bit/samples is used, the signal has to be oversampled (i.e. sampled at a rate much higher than the Nyquist rate) to purposely increase the correlation between adjacent samples of the samples. The DM modulator (transmitter) is illustrated in Fig. 32. The

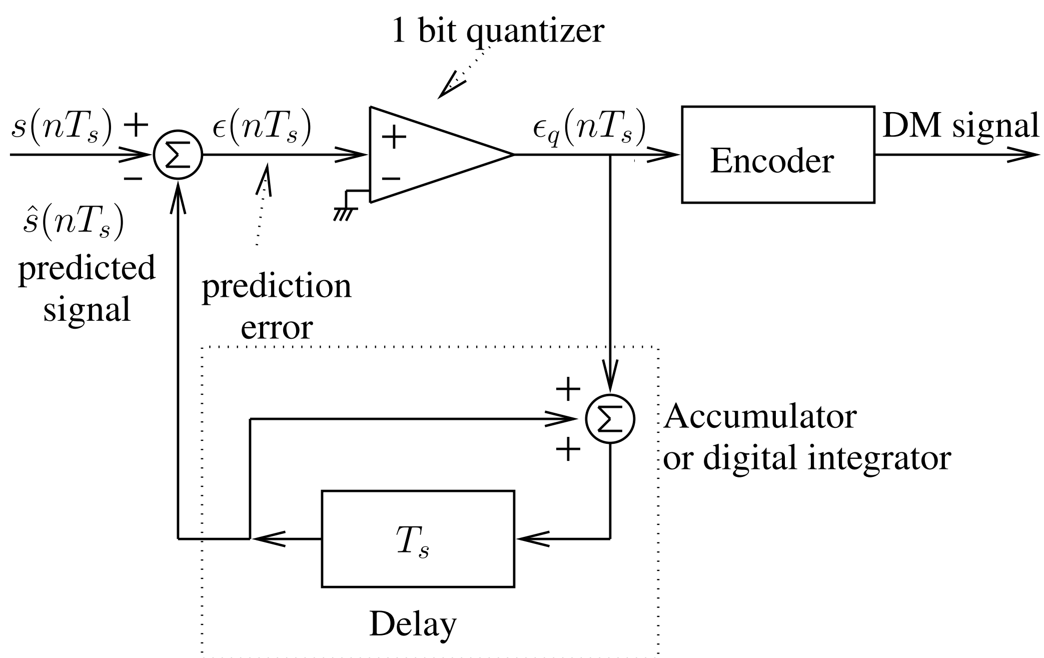

Figure 32: DM modulator (transmitter)

quantized value of the error between the true sample value and its predicted value is represented by 1 bit, hence is given by

$$\epsilon_q(nT_s) = \begin{cases} \Delta & \text{if } s(nT_s) - \hat{s}(nT_s) \geq 0 \\ -\Delta & \text{if } s(nT_s) - \hat{s}(nT_s) < 0 \end{cases} = \Delta \operatorname{sgn}\left(\epsilon(nT_s)\right)$$

The predicted value of a sample is expressed as

$$\hat{s}\big((n+1)T_s\big) = \hat{s}(nT_s) + \epsilon_q(nT_s) = \hat{s}(0) + \sum_{k=0}^{n} \epsilon_q(kT_s)$$

The DM demodulator (receiver) is illustrated in Fig. 33.

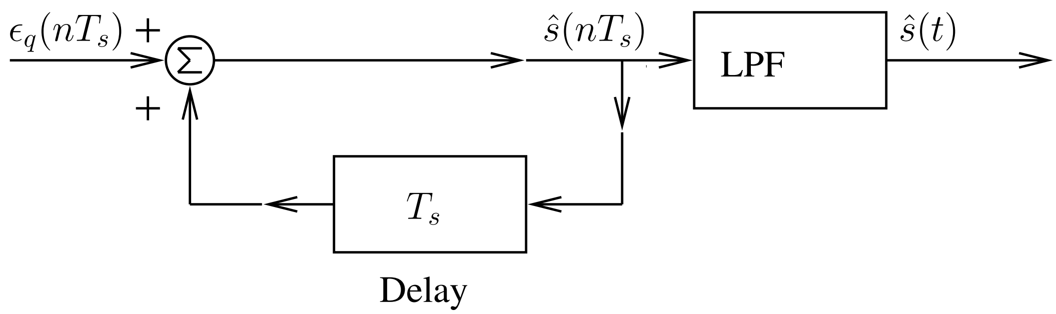

Figure 33: DM demodulator (receiver)

Delta modulator is characterized by the simplicity of its equipment. DM principle is illustrated by Fig. 34.

“staircase approximation of the signal”

Figure 34: Illustration of delta modulation