

## Homework 6

Due: December, 2, 2011

**Question 1.**

**Question 2.** Textbook 4.50

**Question 3.** Textbook 4.58

**Question 4.** Textbook 4.61

**Question 5.** Proakis and Mennolakis 11.26

**Question 6.** Proakis and Mennolakis 11.31

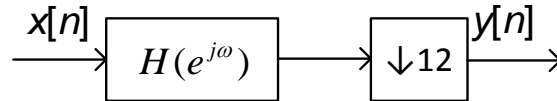
**Question 7.** Textbook 6.24

**Question 8.** Textbook 6.42

**Question 9.** Matlab Question

## Q1. Filter design

Suppose we want to design a LPF  $H(e^{j\omega})$  to be used in a filter bank with the following decimation structure:



The filter specification is as follows:

- Sampling frequency: 96 kHz.
- Passband:  $0 \leq f \leq 3900$  Hz.
- Transition band:  $3900 \text{ Hz} \leq f \leq 4000$  Hz.
- Stopband:  $4000 \text{ Hz} \leq f \leq 48$  kHz.
- Passband ripple:  $\delta_1 = 0.01$
- Stopband ripple:  $\delta_2 = 10^{-4}$ .

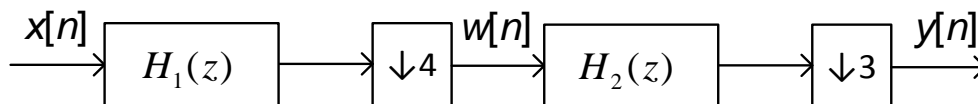
1)

- i. Sketch the frequency response  $H(e^{j\omega})$  (ignoring ripples), specifying all the passband and stopband edges.
- ii. Using Kaiser's formula, estimate the length of this filter. (remember length is  $M+1$ )

$$M = \frac{-10 \log_{10}(\delta_1 \delta_2) - 13}{2.324 \Delta \omega}$$

- iii. What type of filter is this?
- iv. If this filter is designed using the remez algorithm, how many alternations can it have? (There may be more than one answer).

2) Now we want to use a 2-stage design as follows to simplify the filters (so that the new filters have shorter length):



- i. What is the relation between  $H(z)$  and  $H_1(z), H_2(z)$ ?
- ii. What are the sampling frequencies at  $w[n]$  and  $y[n]$ ?
- iii. Now each of  $H_1(z)$  and  $H_2(z)$  is a lowpass filter. Sketch the frequency responses  $H_1(e^{j\omega})$  and  $H_2(e^{j\omega})$  such that the effective system is the same as the single-stage one with shortest filter length possible.
- iv. Let the passband ripples of  $H_1(e^{j\omega})$  and  $H_2(e^{j\omega})$  be  $\delta_1/2$  and the stopband ripples remain as  $\delta_2$ . Estimate the lengths of these 2 new filters.

4.50. Consider the QMF filterbank shown in Figure P4.50:

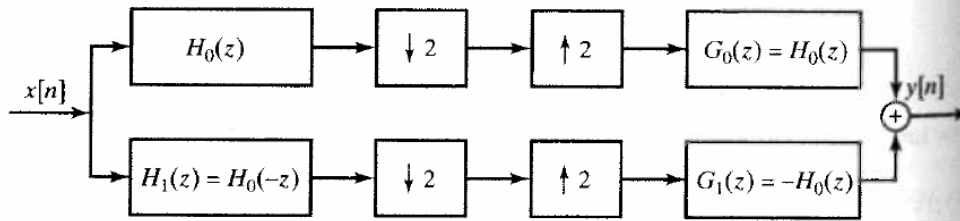


Figure P4.50

The input-output relationship is  $Y(z) = T(z)X(z)$ , where

$$T(z) = \frac{1}{2}(H_0^2(z) - H_0^2(-z)) = 2z^{-1}E_0(z^2)E_1(z^2)$$

and  $E_0(z^2)$ ,  $E_1(z^2)$  are the polyphase components of  $H_0(z)$ .

**Parts (a) and (b) are independent.**

(a) Explain whether the following two statements are correct:

- (a1) If  $H_0(z)$  is linear phase, then  $T(z)$  is linear phase.
- (a2) If  $E_0(z)$  and  $E_1(z)$  are linear phase, then  $T(z)$  is linear phase.

(b) The prototype filter is known,  $h_0[n] = \delta[n] + \delta[n-1] + \frac{1}{4}\delta[n-2]$ :

- (b1) What are  $h_1[n]$ ,  $g_0[n]$  and  $g_1[n]$ ?
- (b2) What are  $e_0[n]$  and  $e_1[n]$ ?
- (b3) What are  $T(z)$  and  $t[n]$ ?

4.58. We are interested in upsampling a sequence by a factor of 2, using a system of the form of Figure 4.23. However, the lowpass filter in that figure is to be approximated by a five-point filter with impulse response  $h[n]$  indicated in Figure P4.58-1. In this system, the output  $y_1[n]$  is obtained by direct convolution of  $h[n]$  with  $w[n]$ .

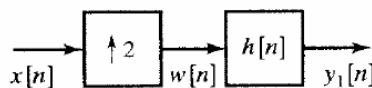
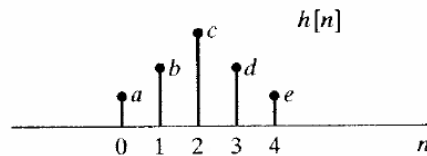


Figure P4.58-1

(a) A proposed implementation of the system with the preceding choice of  $h[n]$  is shown in Figure P4.58-2. The three impulse responses  $h_1[n]$ ,  $h_2[n]$ , and  $h_3[n]$  are all restricted to be zero outside the range  $0 \leq n \leq 2$ . Determine and clearly justify a choice for  $h_1[n]$ ,  $h_2[n]$ , and  $h_3[n]$  so that  $y_1[n] = y_2[n]$  for any  $x[n]$ , i.e., so that the two systems are identical.

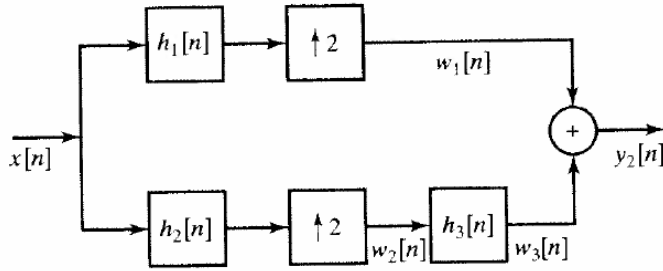


Figure P4.58-2

- (b) Determine the number of multiplications per output point required in the system of Figure P4.58-1 and in the system of Figure P4.58-2. You should find that the system of Figure P4.58-2 is more efficient.

4.61. Communication systems often require conversion from time-division multiplexing (TDM) to frequency-division multiplexing (FDM). In this problem, we examine a simple example of such a system. The block diagram of the system to be studied is shown in Figure P4.61-1. The TDM input is assumed to be the sequence of interleaved samples

$$w[n] = \begin{cases} x_1[n/2] & \text{for } n \text{ an even integer,} \\ x_2[(n-1)/2] & \text{for } n \text{ an odd integer.} \end{cases}$$

Assume that the sequences  $x_1[n] = x_{c1}(nT)$  and  $x_2[n] = x_{c2}(nT)$  have been obtained by sampling the continuous-time signals  $x_{c1}(t)$  and  $x_{c2}(t)$ , respectively, without aliasing.

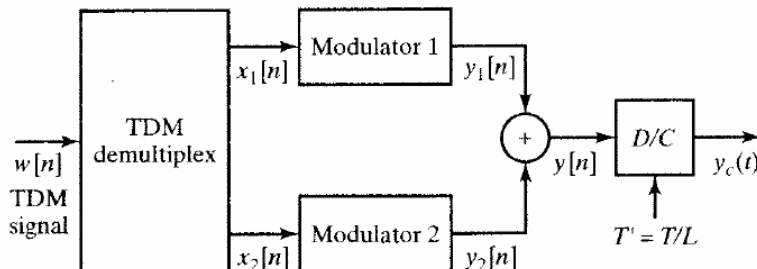


Figure P4.61-1

Assume also that these two signals have the same highest frequency,  $\Omega_N$ , and that the sampling period is  $T = \pi/\Omega_N$ .

- (a) Draw a block diagram of a system that produces  $x_1[n]$  and  $x_2[n]$  as outputs; i.e., obtain a system for demultiplexing a TDM signal using simple operations. State whether or not your system is linear, time invariant, causal, and stable.

The  $k^{\text{th}}$  modulator system ( $k = 1$  or  $2$ ) is defined by the block diagram in Figure P4.61-2. The lowpass filter  $H_i(e^{j\omega})$ , which is the same for both channels, has gain  $L$  and cutoff frequency  $\pi/L$ , and the highpass filters  $H_k(e^{j\omega})$  have unity gain and cutoff frequency  $\omega_k$ . The modulator frequencies are such that

$$\omega_2 = \omega_1 + \pi/L \quad \text{and} \quad \omega_2 + \pi/L \leq \pi \quad (\text{assume } \omega_1 > \pi/2).$$

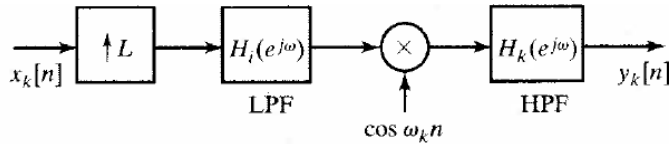


Figure P4.61-2

- (b) Assume that  $\Omega_N = 2\pi \times 5 \times 10^3$ . Find  $\omega_1$  and  $L$  so that, after ideal D/C conversion with sampling period  $T/L$ , the Fourier transform of  $y_c(t)$  is zero, except in the band of frequencies

$$2\pi \times 10^5 \leq |\omega| \leq 2\pi \times 10^5 + 2\Omega_N.$$

- (c) Assume that the continuous-time Fourier transforms of the two original input signals are as sketched in Figure P4.61-3. Sketch the Fourier transforms at each point in the system.

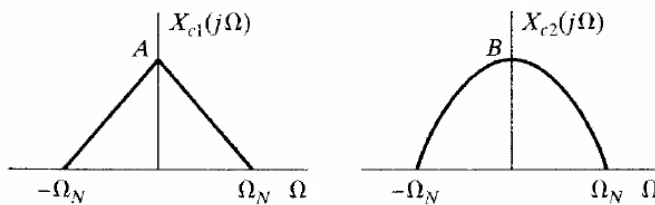


Figure P4.61-3

- (d) Based on your solution to parts (a)–(c), discuss how the system could be generalized to handle  $M$  equal-bandwidth channels.

- 11.26 In the polyphase filter structures for a uniform DFT filter bank described in Section 11.10.1, the polyphase filters in the analysis section were defined by equation (11.10.10). Instead of this definition, suppose we define the  $N$ -band polyphase filters for the lowpass prototype filter,  $H_0(z)$ , as

$$H_0(z) = \sum_{i=0}^{N-1} z^{-i} P_i(z^N)$$

where

$$P_i(z) = \sum_{n=0}^{\infty} h_0(nN + i)z^{-n}, \quad 0 \leq i \leq N - 1$$

Then,

$$H_k(z) = H_0(z e^{-j2\pi k/N}) = H_0(z W_N^k)$$

where  $W_N = e^{-j2\pi/N}$ .

- (a) Show that the filters  $H_k(z)$ ,  $1 \leq k \leq N - 1$ , can be expressed as

$$H_k(z) = \begin{bmatrix} 1 & W_N^{-k} & W_N^{-2k} & \dots & W_N^{-(N-1)k} \end{bmatrix} \begin{bmatrix} P_0(z^N) \\ z^{-1} P_1(z^N) \\ \vdots \\ z^{-(N-1)} P_{N-1}(z^N) \end{bmatrix}$$

(b) Show that

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{N-1}(z) \end{bmatrix} = N\mathbf{W}^{-1} \begin{bmatrix} P_0(z^N) \\ z^{-1}P_1(z^N) \\ \vdots \\ z^{-(N-1)}P_{N-1}(z^N) \end{bmatrix}$$

where  $\mathbf{W}$  is the DFT matrix

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & W_N^1 & W_N^2 & \dots & W_N^{(N-1)} \\ 1 & W_N^2 & W_N^4 & \dots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & W_N^{(N-1)} & W_N^{2(N-1)} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

(c) Sketch the analysis section of the uniform DFT filter bank.

(d) Determine and sketch the synthesis section of the uniform DFT filter bank.

**11.31 Zoom-frequency analysis** Consider the system in Fig. P11.31(a).

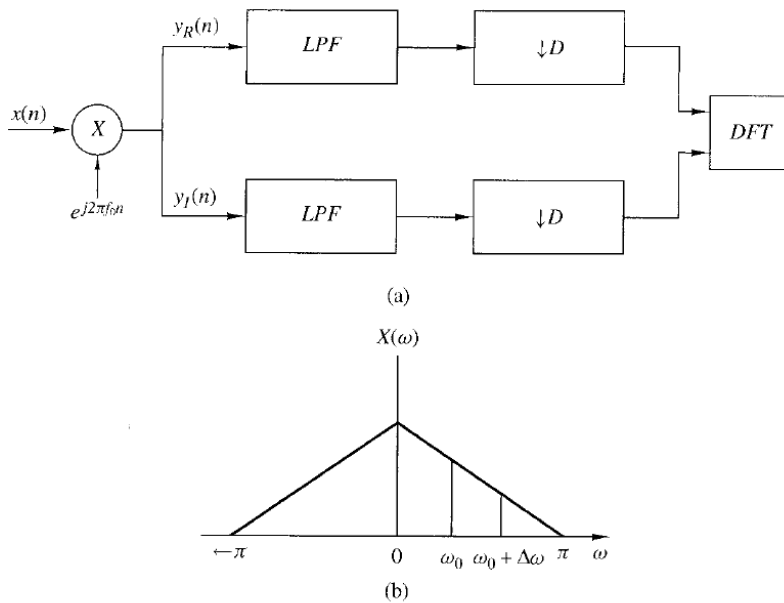


Figure P11.31

- (a) Sketch the spectrum of the signal  $y(n) = y_R(n) + jy_I(n)$  if the input signal  $x(n)$  has the spectrum shown in Fig. P11.31(b).
- (b) Suppose that we are interested in the analysis of the frequencies in the band  $f_0 \leq f \leq f_0 + \Delta f$ , where  $f_0 = \pi/6$  and  $\Delta f = \pi/3$ . Determine the cutoff of a lowpass filter and the decimation factor  $D$  required to retain the information contained in this band of frequencies.

(c) Assume that

$$x(n) = \sum_{k=0}^{p-1} \left(1 - \frac{k}{2p}\right) \cos 2\pi f_k n$$

where  $p = 40$  and  $f_k = k/p$ ,  $k = 0, 1, \dots, p - 1$ . Compute and plot the 1024-point DFT of  $x(n)$ .

(d) Repeat part (b) for the signal  $x(n)$  given in part (c) by using an appropriately designed lowpass linear phase FIR filter to determine the decimated signal  $s(n) = s_R(n) + js_I(n)$ .

(e) Compute the 1024-point DFT of  $s(n)$  and investigate to see if you have obtained the expected results.

6.24. Consider a causal LTI system whose system function is

$$H(z) = \frac{1 - \frac{3}{10}z^{-1} + \frac{1}{3}z^{-2}}{\left(1 - \frac{4}{5}z^{-1} + \frac{2}{3}z^{-2}\right)\left(1 + \frac{1}{5}z^{-1}\right)} = \frac{\frac{1}{2}}{1 - \frac{4}{5}z^{-1} + \frac{2}{3}z^{-2}} + \frac{\frac{1}{2}}{1 + \frac{1}{5}z^{-1}}$$

(a) Draw the signal flow graphs for implementations of the system in each of the following forms:

- (i) Direct form I
- (ii) Direct form II
- (iii) Cascade form using 1<sup>st</sup>- and 2<sup>nd</sup>-order direct form II sections
- (iv) Parallel form using 1<sup>st</sup>- and 2<sup>nd</sup>-order direct form I sections
- (v) Transposed direct form II.

(b) Write the difference equations for the flow graph of part (v) in (a) and show that this system has the correct system function.

6.42. Consider an all-pass system with system function

$$H(z) = -0.54 \frac{1 - (1/0.54)z^{-1}}{1 - 0.54z^{-1}}$$

A flow graph for an implementation of this system is shown in Figure P6.42.

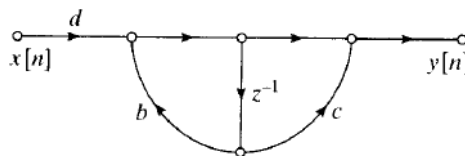


Figure P6.42

(a) Determine the coefficients  $b$ ,  $c$ , and  $d$  such that the flow graph in Figure P6.42 is a direct realization of  $H(z)$ .

(b) In a practical implementation of the network in Figure P6.42, the coefficients  $b$ ,  $c$ , and  $d$  might be quantized by rounding the exact value to the nearest tenth (e.g., 0.54 will be rounded to 0.5 and  $1/0.54 = 1.8518\dots$  will be rounded to 1.9). Would the resulting system still be an all-pass system?

- (c) Show that the difference equation relating the input and output of the all-pass system with system function  $H(z)$  can be expressed as

$$y[n] = 0.54(y[n-1] - x[n]) + x[n-1].$$

Draw the flow graph of a network that implements this difference equation with two delay elements, but only one multiplication by a constant other than  $\pm 1$ .

- (d) With quantized coefficients, would the flow graph of part (c) be an all-pass system?

The primary disadvantage of the implementation in part (c) compared with the implementation in part (a) is that it requires two delay elements. However, for higher-order systems, it is necessary to implement a cascade of all-pass systems. For  $N$  all-pass sections in cascade, it is possible to use all-pass sections in the form determined in part (c) while requiring only  $(N + 1)$  delay elements. This is accomplished by sharing a delay element between sections.

- (e) Consider the all-pass system with system function

$$H(z) = \left( \frac{z^{-1} - a}{1 - az^{-1}} \right) \left( \frac{z^{-1} - b}{1 - bz^{-1}} \right).$$

Draw the flow graph of a "cascade" realization composed of two sections of the form obtained in part (c) with one delay element shared between the sections. The resulting flow graph should have only three delay elements.

- (f) With quantized coefficients  $a$  and  $b$ , would the flow graph in part (e) be an all-pass system?



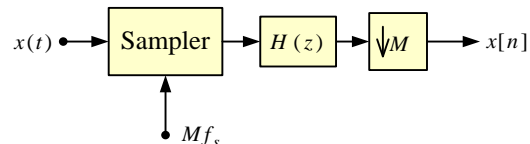
## Matlab Problem: Sampling and Multirate Processing

Consider the following signal (a double-sided damped sinusoid),

$$x(t) = e^{-|t|/\tau} \cos(2\pi f_o t).$$

For this problem, the parameters  $\tau$  (500  $\mu$ s time constant) and  $f_o = 1000$ Hz will be used.

This is not a bandlimited signal. The damped sinusoid will be input to a system as follows,



The input is sampled at  $Mf_s$ , filtered and then downsampled to  $f_s$ . The purpose of this problem is to find the “best” oversampling ratio  $M$  for a fixed filter length.

- The sampler will use the Matlab function `decimate`. This function includes both the filter and the downsampler. The filter can be chosen to be a linear phase FIR filter. The function takes care of the delay in the filter, so that the output will be aligned with the input. We will use the function in the form `decimate(x, M, N-1, 'FIR')`. In this case  $M$  is the downsampling factor and  $N$  is the number of coefficients in the filter. We will use  $N=31$  and evaluate the performance for  $M = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$ .
- The sampled signal can be created with the following Matlab function (use  $T=0.125$  for the duration of the signal and call this function with sampling frequency  $Mf_s$ )

```
function x = Ddsine (T, Fs)
% T is the duration of the signal (-T/2 to T/2)
% Fs is the sampling rate

f0 = 1000;
tau = 0.0005;

% Generate samples of the signal
t = linspace (-T/2, T/2, round (T * Fs) + 1);
x = exp (- abs (t) ./ tau) .* cos (2 * pi * f0 * t);

return
```

- We will use a fixed output sampling frequency,  $f_s = 4000$  Hz. The `decimate` function filters the signal, and then subsamples the output. For an FIR filter, one need not calculate the intermediate points which do not appear in the output, so that the filter output need only be calculated only  $f_s$  times a second, no matter what  $M$  is. Thus in an efficient implementation, the filtering complexity is fixed.
- There are two places at which aliasing can occur: due to the initial sampling and due to the downsampling. The first is reduced as  $M$  is increased, the second is reduced if the filter has a sharper cutoff.

- A reference signal is available. This signal has been created with a large oversampling ratio and a high order filter. Download the signal and then use the `wavread` command to make it available to Matlab. The signal file is [RefSig.wav](#)
1. The filter used in `decimate` can be found as `h = fir1(N-1, 1/M)`. For each value of  $M$  listed above, plot the frequency response in dB (using `freqz`) for frequencies between 0 and  $f_s$ . These plots should be superimposed to be able to compare the responses. Comment on the results you see, i.e. how does the frequency response of the filter vary with  $M$ .
  2. Create the output signal for the value of 10 values of  $M$  listed above. For each of these, calculate the squared error with respect to the reference signal you downloaded. For which value of  $M$  is the error the least? Explain your results, i.e. why does the minimum error occur for that value of  $M$ .