Midterm Exam

4:00 PM – 6:00 PM, October 27, 2010

Duration: 120 minutes

This exam is closed-book. You can bring one single-sided sheet of notes. This sheet of notes must be entirely hand-written, no portions may be machine-produced or photocopied. Calculators are permitted, but no cell phones or laptops are allowed.

Problem 1. (20 points) Conceptual questions

Indicate whether the following statements are true or false. If true, give a brief explanation. If false, give a simple counter-example or a clear reason.

a. Linear phase filters must be FIR.

b. If the discrete-time Fourier transform doesn't converge, then the ROC of the z-transform is not defined.

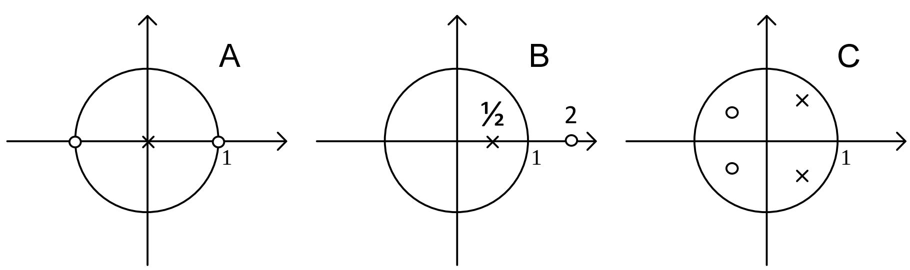
c. The group delay of a causal system is always non-negative.

d. Sum of two minimum-phase systems is another minimum-phase system.

e. A modulator $y[n] = (-1)^n x[n]$ is a linear and time-invariant system.

f. A low pass linear phase FIR filter must be type I. ($h[n] = h[M-n]$, M even)

g. Consider the following 3 pole-zero plots of 3 systems:



Which system is (put A, B, C or any combination or none)

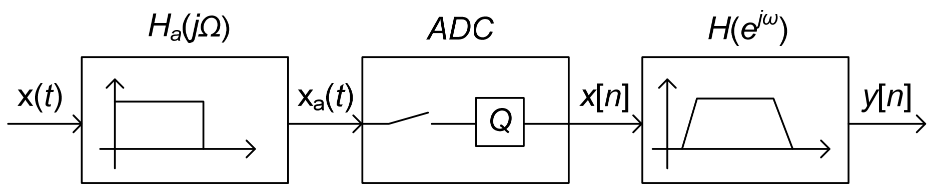
1) All pass?

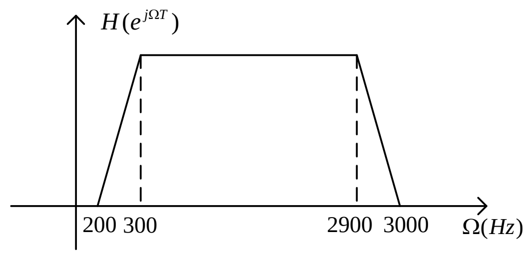
2) Minimum phase?

3) FIR linear phase of type III? ($h[n] = -h[M-n]$, M even)

4) FIR linear phase of type II? ($h[n] = h[M-n]$, M odd)

Problem 2. (40 points) Sampling and quantization





a) In designing a telephone system, a near-ideal analog anti-aliasing filter $H_a(j\Omega)$ of bandwidth 4 KHz is used before the sampling circuit (such that the output of $H_a(j\Omega)$ has no energy at frequencies above 4 KHz).

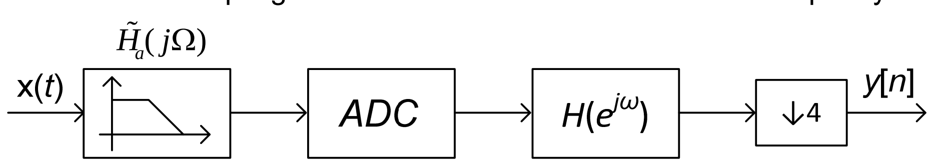
The digital bandpass filter is designed to have the passband from 300 Hz to 2900Hz and stopband edges at 200 Hz and 3000 Hz as shown in the above figure.

What is the minimum sampling frequency such that there is no aliasing in *y*[*n*]? (Note that there may be aliasing in *x*[*n*] as long as $H(e^{j\omega})$ can remove it.) Assume the frequency response of $x_a(t)$ is the same as $H_a(j\Omega)$, sketch the frequency responses of *x*[*n*] and *y*[*n*] at the minimum sampling rate.

b) Let the sampling rate be 8 kHz. If the ADC uses 4-sigma signal scaling and quantizes to a total of 10 bits per sample, what is the quantization SNR (in dB) at *x*[*n*]?

For the same 10-bit ADC, if the SNR has to be greater than 55 dB, how many sigma (σ_x) will you need to fit in the maximum signal amplitude of the ADC? Is there any potential problem with this new signal scaling?

c) Now we want to use oversampling to relax the anti-aliasing filter as in the figure below. Suppose that the nominal sampling rate is 8 kHz but we choose to oversample by 4 times.



Note that $H(e^{j\omega})$ has different cut-off frequencies in terms of ω in this system. Let

$$\tilde{H}_a(j\Omega) = \begin{cases} 1, & |\Omega| \le \Omega_p \\ 0, & |\Omega| > \Omega_s \end{cases}$$

Find the smallest Ω_p and the largest Ω_s such that the above system is equivalent to the original system (at sampling rate 8 kHz), ignoring the effect on quantization noise.

d) With 4-sigma signal scaling and 10 bits per sample, what is the quantization SNR in the system with oversampling?

Problem 3. (30 points) LTI system analysis

Consider the following system function of an LTI system:

$$H(z)=\frac{\left(1-1.5z^{-1}\right)\left(1-\frac{1+j}{2}z^{-1}\right)\left(1-\frac{1-j}{2}z^{-1}\right)}{\left(1-0.8e^{j\frac{2\pi}{3}}z^{-1}\right)\left(1-0.8e^{-j\frac{2\pi}{3}}z^{-1}\right)\left(1-\frac{2}{3}z^{-1}\right)},\quad ROC:|z|\geq 0.8$$

a) Is this system stable? Why?

   Is it an FIR or IIR system? Why?

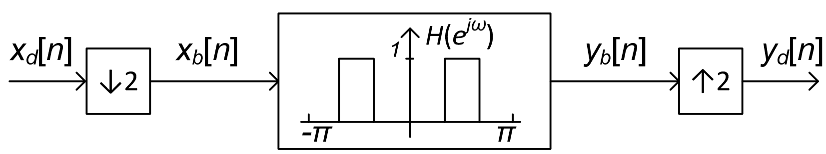
b) Plot the pole-zero diagram. Is this system a minimum-phase or all-pass system? Why?

c) Express $H(z)$ as a combination of an all-pass filter and a minimum-phase filter. Draw the pole-zero plots for each of these filters.

d) Express $H(z)$ as a combination of a minimum-phase filter and a linear-phase FIR filter. Draw the pole-zero plot for each of them. What type of FIR linear-phase filter is this one?

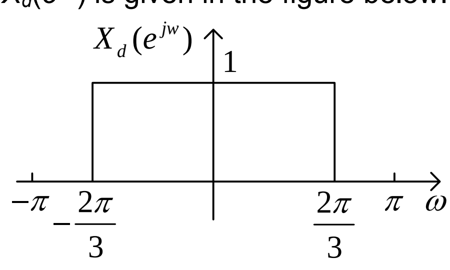
Problem 4. (10 points) Up/down sampling

Consider the following discrete-time system:



The ideal bandpass filter $H(e^{j\omega})$ has the passband as $\left[-\frac{2\pi}{3},-\frac{\pi}{3}\right]$ and $\left[\frac{\pi}{3},\frac{2\pi}{3}\right]$.

The input frequency response $X_d(e^{j\omega})$ is given in the figure below.



Plot the output frequency response $Y_d(e^{j\omega})$, showing all intermediate steps ($X_b(e^{j\omega})$ and $Y_b(e^{j\omega})$). Does $y_d[n]$ have aliasing?