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MINIMAX DESIGN OF FACTORABLE NYQUIST FILTERS

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Abstract

The use of Nyquist filters in data transmission systems is important in avoiding intersymbol interference. Moreover, the Nyquist filters should be factorable into lowpass transmitter/receiver filter pairs. Here, the design problem is formulated so as to generate zero-phase FIR lowpass Nyquist filters that can be split into minimum and maximum phase parts. Two factorable minimax design methods are given. These methods use the McClellan-Parks algorithm as a first step to control the stopband behaviour. The time domain constraints, imposed by solving a linear system of equations, determine the passband response. The final filter exhibits equiripple stopband behaviour. The advantages of these methods are that the minimum and maximum phase parts are obtained without direct factorization and that arbitrary frequency weighting can be easily incorporated to allow for a nonequiripple behaviour. The new design approach is compared with other methods. Finally, a practical design that conforms to a CCITT voice band modem specification is shown.

1. Introduction

Intersymbol interference occurs when a received data symbol is influenced by a combination of several transmitted data symbols. Intersymbol interference is avoided through the use of Nyquist filters. Nyquist filters have an impulse response with regular zero crossings. Furthermore, the filters for bandwidth efficient data transmission systems are based on lowpass prototypes. The design problem incorporates both time and frequency domain constraints.

This paper two design methods to generate zero-phase lowpass FIR Nyquist filters that can be split into minimum and maximum phase parts. The McClellan-Parks algorithm is used as a first step to control the stopband response. The subsequent step incorporates the time domain constraints and automatically generates the passband response. A few iterations of the above steps produces a factorable Nyquist filter with Chebyshev stopband response. Furthermore, the polynomial factorization problem for the determination of the minimum phase part is considerably eased in that a partial factorization of the transfer function of the Nyquist filter is obtained as a byproduct of the design procedure. We refer to the proposed approaches as factorable minimax design methods. The filters are fundamentally equiripple. Nonequiripple filters can be obtained by applying an additional frequency weighting factor.

Nyquist filters with Chebyshev stopband behaviour have been designed in [1] using a multistage structure. The focus in [1] is on a computationally efficient multistage implementation. However, the resulting filters are not necessarily factorable. Other methods of designing Nyquist filters include the use of linear programming techniques [2][3], the eigenfilter approach [4][5] and the use of the McClellan-Parks algorithm [6] as an intermediate step [1][7][8][9]. The methods in [3],[5] and [8] allow for the splitting of the filter into its minimum and maximum phase parts.

2. Factorable Nyquist Filters

Zero-phase FIR Nyquist filters $H(z)$ have the following impulse response characteristic:

$$h(iN) = \begin{cases} \dfrac{1}{N} & \text{for } i = 0 \\ 0 & \text{for } i \neq 0 \end{cases} \quad (1)$$

For a lowpass design, the minimum bandwidth solution is an ideal lowpass filter bandlimited to π/N. For practical filters, we allow an excess bandwidth $\beta\pi/N$ to bring the overall bandwidth to $(1+\beta)\pi/N$. The parameter β is the roll-off factor of $|H(e^{j\omega})|$. Furthermore, bandwidth efficient systems impose $\beta \leq 1$[†]. This sets the upper edge of the passband to be $\omega_p = (1-\beta)\pi/N$ and the lower edge of the stopband to be $\omega_s = (1+\beta)\pi/N$. The ideal frequency characteristic is

$$|H(e^{j\omega})| = \begin{cases} 1 & \text{for } 0 \leq |\omega| \leq \omega_p & \text{Passband} \\ 0 & \text{for } \omega_s \leq |\omega| \leq \pi & \text{Stopband} \end{cases} \quad (2)$$

The factorable minimax design methods generate filters that approximate this ideal magnitude characteristic. In addition, exact zero crossings in the impulse response are achieved and double zeros on the unit circle are imposed to assure factorization into minimum and maximum phase parts.

3. Minimax Factorable Design Procedures

The Nyquist filter $H(z)$ must have an odd number of coefficients in order to be factorable into minimum and maximum phase parts. As in [5], we factor $H(z)$ as $H(z) = H_0(z)H_1^2(z)$ where $H_1^2(z)$ contains all the double zeros of $H(z)$ on the unit circle and $H_0(z)$ contains the other zeros of $H(z)$. The double zeros of $H_1^2(z)$ on the unit circle imply that it has an odd number of coefficients and that it is a zero-phase function. The zeros of $H_0(z)$ must occur in mirror image pairs reflected about the unit circle. Hence, $H_0(z)$ also has an odd number of coefficients and is a zero-phase function.

Let the lengths of $H_0(z)$ and $H_1^2(z)$ to be $2l_0+1$ and $2l_1+1$ respectively. The number of coefficients of the overall Nyquist filter $H(z)$ is $M = 2(l_0+l_1)+1$[‡]. The inverse z-transforms of $H(z)$, $H_0(z)$ and $H_1^2(z)$ are defined to be $h(n)$, $h_0(n)$ and $f(n)$, respectively.

3.1 First Method

The design procedure for the first method is as follows:

1. Initialization: Fix l_0, l_1, N, ω_p and ω_s. Set $H_0(z) = 1$. The weighting is given as $W(e^{j\omega})$.
2. Design $H_1(z)$ using the McClellan-Parks algorithm such that it has zeros only on the unit circle in the stopband region $[\omega_s, \pi]$.
3. Impose the time domain constraints by solving for the coefficients of $H_0(z)$ through a linear system of equations.
4. Form the Nyquist filter $H(z)$. If the design warrants improvement, go back to step 2.
5. Split $H(z)$ into its minimum and maximum phase parts.

Step 2: Frequency Domain Specifications

The McClellan-Parks algorithm is used to get the coefficients of $H_1(z)$. The specifications are that the frequency response must be one at $\omega = 0$ and must approximate zero in the stopband region $[\omega_s, \pi]$. The weighting function applies to $H_1^2(z)$. Initially, the weighting function is $W(e^{j\omega})$. For the design of $H_1(z)$, tabulated values of the square root of the weighting function are inputs to the algorithm. In the stopband, the frequency response of $H_1(z)$ exhibits a ripple-like behaviour with local minima and maxima occurring at the extremal frequencies. If l_1 is even, $H_1(z)$ has an odd number of coefficients (l_1+1). Two of the extremal frequencies are 0 and π [10]. However, the total number of zeros is a multiple of two, all occurring in complex conjugate pairs (no zero at $z=-1$). At $\omega = \pi$, either a local maximum or a local minimum occurs. If l_1 is odd, $H_1(z)$ has an even number of coefficients. In this case, a zero occurs at $z = -1$. However, π is not an extremal frequency. The other zeros occur in complex conjugate pairs bringing the total number of zeros to l_1.

Step 3: Time Domain Constraints

Given $H_1(z)$, we form $H_1^2(z)$ and solve for the coefficients of $H_0(z)$ such that $H(z)$ has the Nyquist property. Since $h(n)$ has samples for $n = -(l_0+l_1)$ to l_0+l_1, the number of zero-valued samples that occur as n goes from 1 to l_0+l_1 is $\lfloor (l_0+l_1)/N \rfloor$. The same holds true as n

[†] Note that for $N > 2$, $\beta \leq 1$ is not a requirement for the design

[‡] The case $l_0+l_1 = kN$ for any integer k renders a Nyquist filter with $h(-l_0-l_1) = h(l_0+l_1) = 0$ thereby reducing the effective length by two.

goes from -1 to $-(l_0+l_1)$. Since the sample for $n = 0$ is also known the number of known coefficients of $H(z)$ is[1]

$$L = 2\left\lfloor \frac{l_0+l_1}{N} \right\rfloor + 1 \quad (3)$$

The coefficients of $H(z)$ are found by performing the convolution $h_0(n) * f(n)$. By expanding the convolution sum, one can uniquely determine $H_0(z)$ such that the time domain constraints are satisfied [5] if the number of unknown coefficients of $H_0(z)$ equals the number of known coefficients of $H(z)$. This results in a system of linear equations of dimension $2l_0+1$. By further exploiting the time domain symmetry of each filter, the problem is reduced to that of a system of dimension l_0+1. The system of equations can be expressed as $\mathbf{Fh} = \mathbf{c}$ where $\mathbf{h}^T = [h_0(0) \;\cdots\; h_0(l_0)]$, $\mathbf{c}^T = [1/N \;\; 0 \;\cdots\; 0]$ and

$$\mathbf{F} = \begin{bmatrix} f(0) & 2f(1) & \cdots & 2f(l_0) \\ f(N) & f(N-1)+f(N+1) & \cdots & f(N-l_0)+f(N+l_0) \\ \vdots & \vdots & & \vdots \\ f(Nl_0) & f(Nl_0-1)+f(Nl_0+1) & \cdots & f(Nl_0-l_0)+f(Nl_0+l_0) \end{bmatrix} \quad (4)$$

The constraint that $L = 2l_0+1$ is equivalent to $l_0 = \lfloor (l_0+l_1)/N \rfloor$ which in turn translates to constraints on l_0 and l_1 given by

$$l_0(N-1) \le l_1 < l_0(N-1) + N\ . \quad (5)$$

This leads to closed form expressions for l_0 and l_1 in terms of N and M,

$$l_0 = \left\lfloor \frac{M-1}{2N} \right\rfloor \quad \text{and} \quad l_1 = \frac{M-1}{2} - \left\lfloor \frac{M-1}{2N} \right\rfloor . \quad (6)$$

This method of satisfying the Nyquist property automatically takes care of the passband response of $H(z)$. Note that $H_0(z)$ is a highpass function that primarily controls the passband characteristic and hence has no zeros on the unit circle.

Step 4: Convergence

The coefficients of $H(z)$ are found from $H_0(z)$ and $H_1^2(z)$. Steps 2 and 3 are iterated if the design warrants improvement. For Step 2, the weighting function is updated to be $W(e^{j\omega})|H_0(e^{j\omega})|$ where $|H_0(e^{j\omega})|$ is calculated from the coefficients of $H_0(z)$ formed in Step 3 of the previous iteration. The application of this weighting factor significantly influences the stopband behaviour through the design of $H_1(z)$. In the weighting function, the factor $|H_0(e^{j\omega})|$ leads to a stopband behaviour of $H_1^2(z)$ that compensates for the highpass response of $H_0(z)$. The stopband behaviour is either equiripple or nonequiripple depending on the other factor $W(e^{j\omega})$ in the weighting function. The iterations are terminated when the extremal frequencies obtained by designing $H_1(z)$ do not change by more than a given threshold.

Step 5: Final Filter

This step factors $H(z)$ into minimum and maximum phase parts. Let the minimum phase part of $H(z)$ be $H^-(z) = H_0^-(z)H_1(z)$ where $H_0^-(z)$ is the minimum phase part of $H_0(z)$. The factor $H_1(z)$ is known as a byproduct of the design procedure. Only $H_0(z)$ needs to be factored in order to derive its minimum phase part.

3.2 Second Method

The difference between the second method and the previous approach lies in Step 2 in which a constrained form of the McClellan-Parks algorithm is used to directly compute the coefficients of $H_1^2(z)$ rather than to first design $H_1(z)$. The specifications are that the frequency response must be one at $\omega = 0$ and must approximate zero in the stopband region $[\omega_s, \pi]$. The initial weighting function is $W(e^{j\omega})$. Tabulated values of the weighting function are supplied as inputs. Since double zeros on the unit circle are required, we constrain the frequency response to be nonnegative in the stopband region. We implement the procedure in [11] to obtain a minimax approximation to a desired response that satisfies certain upper and lower constraints. For our specific application, a lower constraint of zero is imposed in the stopband region $[\omega_s, \pi]$.

In the stopband, the frequency response of $H_1^2(z)$ exhibits a ripple-like behaviour with local minima and maxima occurring at the extremal frequencies. The local minima correspond to the frequencies at which the response touches zero. It is these frequencies which determine the double zeros of $H_1^2(z)$ on the unit circle. Given that $H_1^2(z)$ has $2l_1+1$ coefficients, a total of l_1+1 extremal frequencies result [10]. Two of the extremal frequencies are 0 and π regardless of the value of l_1. If l_1 is odd, the extremum at π is a local minimum thereby producing a double zero at $z=-1$. The other zeros occur in groups of four in the stopband region bringing the total number of zeros to $2l_1$. If l_1 is even, the extremum at π is a local maximum (no zero at $z = -1$). The total number of zeros is a multiple of four and occur in groups of four in the stopband region.

Steps 3 and 4 are identical to the first approach. In splitting $H(z)$ into its minimum and maximum phase parts, we take advantage of the fact that the frequencies corresponding to the double zeros of $H_1^2(z)$ are available as a byproduct of the modified McClellan-Parks algorithm. Given these frequencies and hence, the locations of the zeros on the unit circle, $H_1(z)$ can be formed without directly factoring $H_1^2(z)$. As before, only $H_0(z)$ must be factored to form $H^-(z) = H_0^-(z)H_1(z)$.

The next section discusses the merits of factoring only $H_0(z)$ as opposed to $H(z)$ in determining the minimum phase part. Also, observations concerning the relative orders of $H_0(z)$ and $H(z)$ are given.

4. The Factorization Problem

Polynomial factorization can be an ill-conditioned problem [12]. There is an advantage to substantially lowering the order of the polynomial to be factored. A general zero plot of $H(z)$ includes stopband zeros, passband zeros and extra real zeros [13]. The double stopband zeros on the unit circle contribute to the stopband ripples and the passband zeros that occur in mirror-image pairs contribute to the passband response. The extra real zeros adjust the band edges depending on the specification. If $H(z)$ were to be factored, polynomial deflation would be necessary as each zero is found. Note that finding the double zeros can be an ill-conditioned problem [12]. Furthermore, polynomial deflation has the danger of making the zeros of the quotient polynomial diverge from those of the original polynomial [12]. Our approach avoids polynomial deflation in that $H_0(z)$ is not determined by dividing $H(z)$ by $H_1^2(z)$ but rather, is determined by solving a linear system of equations. Hence, errors that would normally occur in locating the unit circle zeros are absent in our approach and do not affect the zeros of $H_0(z)$. Furthermore, the factorization of $H_0(z)$ does not involve multiple zeros since $H_0(z)$ has only the simple passband and extra zeros of $H(z)$.

Since only the zeros of $H_0(z)$ have to be determined, the extent to which the factorization problem is eased depends on the ratio l_1/l_0. The ratio l_1/l_0 is both a measure of the proportion of unit circle zeros to the other zeros of $H(z)$ and of the degrees of $H(z)$ and $H_0(z)$. The higher the value of l_1/l_0, the lower the relative orders of $H_0(z)$ and $H(z)$. It can be shown that l_1 is greater than l_0 by a factor of at least $N-1$. Therefore, the inherent advantage in terms of polynomial factorization increases as N increases. However, even for the lowest value, $N=2$, the degree of $H(z)$ is at least twice the degree of $H_0(z)$.

A typical designed Nyquist response $h(n)$ is depicted in Fig. 1. The time response consists of a main lobe between $n=-N$ and $n=N$ and a series of sidelobes each occurring between the zero crossings. The minimum value of l_1/l_0, namely, $N-1$, results when the end points of the impulse response are zero-valued. We discard this artificial case because the values of l_0 and M can be reduced by 1 and 2 respectively thereby giving a new value of l_1/l_0. As the number of coefficients is increased along a lobe (from $n = kN$ to $n = (k+1)N-1$), the value of l_0 is constant and l_1 continues to increase. Therefore, one can maximize l_1/l_0 within a particular lobe by increasing the number of taps. The largest disparity in the relative orders of $H_0(z)$ and $H(z)$ results by choosing the filter lengths to be of the form $M = 2kN-1$. These filter lengths are also useful since we get an integral number of sidelobes.

Given that the filter lengths are constrained to be of the form $M = 2kN-1$, the ratio l_1/l_0 is

$$\frac{l_1}{l_0} = \frac{k(N-1)}{k-1} . \quad (7)$$

This ratio is not only a maximum for $k=2$ but also a unique maximum for a general M. As k increases, a tradeoff results in that a higher stopband attenuation due to a longer filter is obtained at the expense of both a lower l_1/l_0 and a higher l_0. The subsequent examples show that a value of $k=5$ results in about an 80 dB stopband attenuation for a roll-off factor of 0.52. Smaller roll-off factors require a larger number of taps (larger value of k) and hence, a lower value of l_1/l_0 and a higher value of l_0 for an 80 dB stopband attenuation.

5. Discussion of the Design Techniques

Given that two factorable minimax design methods are proposed, the questions regarding their differences and how they compare with other approaches are now answered. Design examples are also given.

5.1 Factorable Minimax Design Methods

Comparison of the Two Proposed Methods

The two methods in this paper can be used to design factorable Nyquist filters with Chebyshev stopband behaviour. An equiripple stopband is obtained when $W(e^{j\omega}) = 1$. A nonequiripple design is achieved by specifying a nonconstant $W(e^{j\omega})$. The difference between the two design methods

[1] This formula is a corrected version of the formula given in [5].

given in this paper is that the first commences with an unconstrained design of $H_1(z)$ while the second starts with a constrained design of $H_1^2(z)$. Although the two methods should give the same Nyquist filter numerical differences do arise. The coefficients of $H_1^2(z)$ obtained by the two methods differ slightly. Even though these small differences lead to more pronounced differences in the coefficients of $H_0(z)$, the coefficients of the overall Nyquist filters formed by the two methods show only small differences. Slight differences in the frequency response occur and manifest themselves mostly in the stopband region. For an equiripple design, the heights of the stopband ripples as a result of using the second method differ slightly as compared to the first method in which the equiripple characteristic is more closely approached.

Examples using the first of our methods are presented to demonstrate both equiripple and nonequiripple designs. The design computations were done using double precision floating point arithmetic. Four iterations were necessary to resolve the coefficients.

Example 1

We generate an equiripple design with parameters $N = 6$, $l_0 = 4$, $l_1 = 25$, $\omega_p = 0.08\pi$ and $\omega_s = 0.254\pi$. This results in a filter with 59 coefficients having a roll-off factor $\beta = 0.52$ whose magnitude response is shown in Fig. 2. The passband response is flat to within 0.003 dB. The filter length is of the form $M = 2kN - 1$ with $k = 5$.

Example 2

The parameters used in this example are $N = 4$, $l_0 = 4$, $l_1 = 15$, $\omega_p = 0.12\pi$ and $\omega_s = 0.38\pi$. The weighting is

$$W(e^{j\omega}) = \begin{cases} 1 & \text{for } \omega = 0 \\ \dfrac{20}{2\pi}(\omega - \omega_s) + 1 & \text{for } \omega_s \le \omega \le \pi \end{cases} \qquad (8)$$

This gives a nonequiripple Nyquist filter with 39 coefficients and a roll-off factor $\beta = 0.52$. The filter length is of the form $M = 2kN - 1$ with $k = 5$. Figure 3 shows the magnitude response of the filter. The passband response is flat to within 0.002 dB.

Group Delay

An important question concerns the delay distortion of the minimum phase part. The group delay of the minimum phase part is only important in the passband and is primarily influenced by the passband zeros which are within the unit circle. For a given number of taps and a given N, the group delay tends to be more constant as the roll-off factor increases. Also, for a given roll-off factor and a given N, a larger number of taps produces a group delay with a greater deviation. The minimum phase filters generated in Examples 1 and 2 that achieve about a 40 dB stopband attenuation also have a relatively small passband group delay variation (approximately 0.15 zero crossing intervals).

5.2 Other Approaches

Linear Programming and Eigenfilter Formulations

In [3], a linear programming approach that is also based on a minimax criterion is used to design a factorable Nyquist filter. Moreover, arbitrary weighting can be easily applied (see [2]). Our approach gives similar results to the linear programming formulation. The eigenfilter approach [5], based on a least-squares design, also simplifies the factorization problem and meets the time domain constraints by solving a linear system of equations. The incorporation of an arbitrary weighting factor into the eigenfilter formulation involves the use of numerical integration techniques. Our approach naturally generates an equiripple behaviour whereas the eigenfilter method naturally renders nonequiripple filters.

Direct Use of the McClellan-Parks Algorithm

Factorable Nyquist filters can also be designed by invoking the constrained form of the McClellan-Parks algorithm [11] to get a nonnegative response that approximates a raised cosine characteristic. Another approach is to design a linear phase filter to approximate the square root of a raised cosine response using the McClellan-Parks algorithm (no constraints required) and convolve it with itself to produce a Nyquist filter. The basic drawback of these two approaches is that exact zero crossings in the impulse response are not guaranteed as compared to other design methods. The direct approaches used to design approximations to raised cosine Nyquist filters can be used for modem design. The CCITT recommendation V.22 [14] includes the specification of a pair of transmitter/receiver filters which should approximate the square root of a raised cosine response. The specified roll-off factor is 0.75. In addition, upper and lower limits in the frequency response and an upper limit in the the group delay variation are specified over certain frequency ranges.

We design Nyquist filters with a roll-off factor of 0.75 and with $N = 4$ using the first factorable minimax method and the direct approaches that use the McClellan-Parks algorithm. The approaches are described in slightly more detail as follows:

1. Design a filter that approximates a raised cosine response by invoking the constrained form of the McClellan-Parks algorithm [11] such that the response is nonnegative and its minimum and maximum phase parts have a frequency response that satisfies the upper and lower bounds specified by V.22.
2. Design a linear phase filter that approximates a square root raised cosine characteristic by invoking the modified form of the McClellan-Parks algorithm such that its frequency response satisfies the upper and lower bounds specified by V.22.
3. Use the first factorable minimax method to design a Nyquist filter such that its minimum and maximum phase parts satisfy the V.22 specifications of the frequency response.

In all cases, the smallest number of taps that satisfy the constraint $M = 2kN - 1$ is used. This leads to 15 tap Nyquist filters for the three methods. Note that the factorable minimax method does not guarantee a filter that satisfies any prescribed specifications of the frequency response. However, filters that satisfy the V.22 specifications can be designed by choosing the number of taps, carrying out the design and finally verifying that the constraints are met. We find that the constraints are met with 15 taps. The weighting, $W(e^{j\omega}) = 1$, is used in the factorable minimax method.

In comparing the performance of the three methods, we consider the stopband attenuation of the Nyquist filter, the group delay of the factorized filter in the region considered in the V.22 specifications and quantitative measures of the intersymbol interference. The measures of the intersymbol interference are the peak distortion D_{P} and the RMS distortion D_{RMS} defined by

$$D_{\mathrm{P}} = \frac{\sum_{n \neq 0} |h(nN)|}{|h(0)|} \quad \text{and} \quad D_{\mathrm{RMS}} = \sqrt{\frac{\sum_{n \neq 0} h^2(nN)}{h^2(0)}}. \qquad (9)$$

The stopband attenuations produced for methods 1, 2 and 3 are about 45, 42 and 50 dB respectively. The allowable variation in group delay as specified by V.22 is 0.18 zero crossing intervals. Method 1 generates a minimum phase filter whose group delay variation is slightly under the prescribed 0.18 zero crossing intervals. Method 2 generates a filter with no group delay variation. Only Method 3 does not meet the group delay requirement in that the filter it produces has a variation of 0.24 zero crossing intervals[†]. In terms of peak and RMS distortion, Method 3 assures exact zero crossings and hence, produces no such distortion. Method 1 produces peak and RMS distortions of 0.0004 and 0.0003, respectively. Method 2 leads to much higher peak and RMS distortions of 0.186 and 0.132, respectively. This coupled with the lower stopband attenuation achieved by the second method leads us to choose Method 1 over Method 2. Method 3 gives a higher stopband attenuation than Method 1 and produces exact zero crossings in the impulse response. This enhanced stopband attenuation comes at the expense of a larger group delay variation.

6. Summary and Conclusions

This paper describes two factorable minimax methods to design zero-phase FIR lowpass factorable Nyquist filters. The main advantages of the design techniques are that the polynomial factorization complexity in finding the minimum phase part is considerably eased and that arbitrary frequency weighting can be applied without additional computational overhead. The new methods can design both equiripple and nonequiripple filters.

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[†] A simple second order allpass equalizer brings the group delay within specifications. However, the use of such equalizers sacrifices the exact zero crossing property of the original design.

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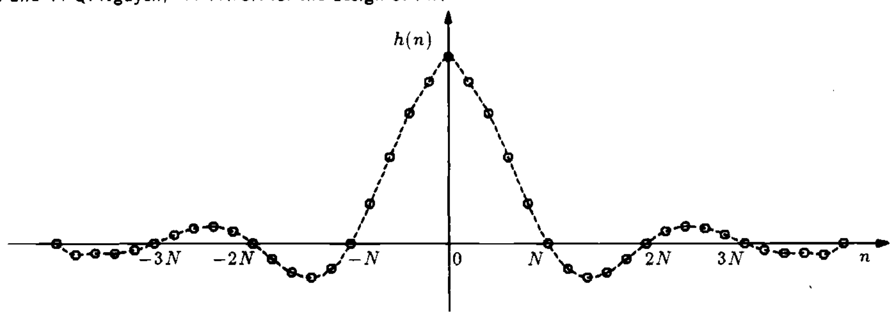

Fig. 1 Typical Nyquist response $h(n)$ (shown for $N = 5$, $M = 39$ and $\beta = 0.2$)

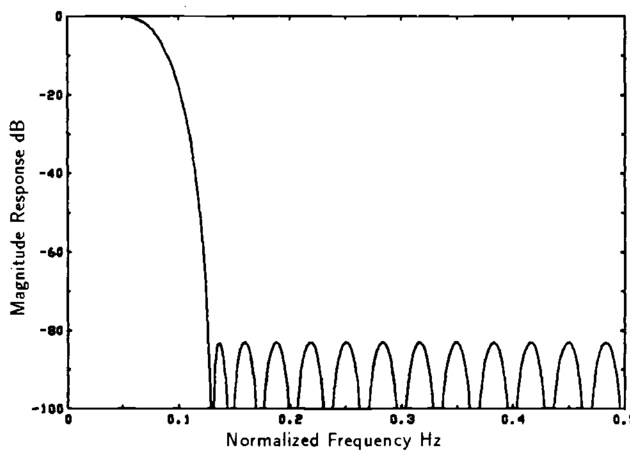

Fig. 2 Magnitude response of the Nyquist filter: Example 1

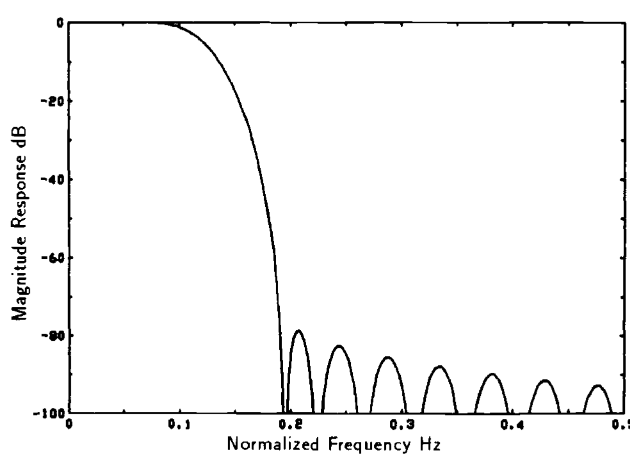

Fig. 3 Magnitude response of the Nyquist filter: Example 2