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SYNTHESIS OF BANDWIDTH EFFICIENT OQAM AND VSB TRANSMULTIPLEXERS

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Abstract

This paper develops a set of conditions which allow bandwidth efficient transmultiplexers to be synthesized. The synthesis procedure is based upon a generalized impulse response for the combining (modulating) and separation (demodulating) filters. In particular, the combining and separation filters are bandpass versions of one or two lowpass prototypes and are configured to cancel crosstalk by exploiting relationships between the center frequencies, delays and phases in their impulse response. Based on the derived conditions, five different transmultiplexers are synthesized. Three of them implement Orthogonal Quadrature Amplitude Modulation (OQAM) with repeated center frequencies. The other two accomplish Vestigial Sideband Modulation (VSB) with distinct frequencies. The transmultiplexers can be converted into new complementary subband systems. Intersymbol interference is eliminated in both the transmultiplexers and the subband systems by appropriately designing the prototypes.

1. Introduction

Multirate digital filter banks have been used in the realization of transmultiplexers and subband systems [1][2]. For systems with two bands, quadrature mirror filters (QMF) [3][4] are often used. For an arbitrary number of bands, approaches to specify the filter banks are based on a matrix formalism [1][2], lossless structures [5], and modulated banks [6][7].

A multi-input, multi-output transmultiplexer structure as shown in Fig. 1 is well suited for simultaneous transmission of many data signals across a single channel. This paper synthesizes five different transmultiplexers based on a set of combining and separation filters that are formed from frequency shifted versions of lowpass prototypes. A bandwidth efficiency of N signals/2π radians is achieved by configuring the filters such that spectral overlap is present. This introduces crosstalk in transmultiplexers, that is, a particular output is influenced by many inputs. The separation filters are designed to cancel the crosstalk and hence, allow for the reconstruction of the input(s).

2. Transmultiplexers: Input-Output Descriptions

An N channel transmultiplexer as depicted in Fig. 1 generates input-output relations given by

$$\hat{X}_i(z^N) = \frac{1}{N}\sum_{k=0}^{N-1} X_k(z^N)\sum_{l=0}^{N-1} A_k(zW^{-l})B_i(zW^{-l}) \quad \text{for} \quad 0 \le i \le N-1 \tag{1}$$

where $W = e^{-j\frac{2\pi}{N}}$. The output signal $\hat{X}_i(z^N)$ is related to the input signal $X_k(z^N)$ via a transfer function $T_{ki}(z^N) = \sum_{l=0}^{N-1} A_k(zW^{-l})B_i(zW^{-l})$. Crosstalk is eliminated when each output signal $\hat{X}_i(z)$ only depends on its corresponding input $X_i(z)$ and is not influenced by other input signals. In this case, the crosstalk function $T_{ki}(z^N)$ is zero for $k \ne i$. Furthermore, intersymbol interference is eliminated if and only if $T_{kk}(z^N)$ is of the form cz^{-pN}. Then, perfect reconstruction to within a gain and delay is achieved.

3. Assumptions and Input-Output Transfer Function

The basic assumptions made for the synthesis procedure are as follows. The filters are modulated and delayed versions of one lowpass prototype $h(n)$. This condition will be relaxed later to allow two prototypes. The impulse responses of the combining filters $A_k(z)$ and the separation filters $B_k(z)$ are parameterized by a center frequency, phase shift and delay. Their impulse responses are given by

$$\begin{aligned} a_k(n) &= h(n-n_k)\cos\left[\omega_k(n-n_k)+\alpha_k\right] \\ b_k(n) &= h(n+p_k)\cos\left[\omega_k(n+p_k)+\beta_k\right]. \end{aligned} \tag{2}$$

The center frequencies ω_k of each input-output pair are equally spaced and lie between 0 and π (inclusive). In addition, the center frequencies can either be distinct or can repeat. The bandwidth of the strictly bandlimited lowpass prototype $h(n)$ (stopband response is zero) is selected such that spectral overlap exists only between bandpass filters centered at adjacent frequencies and the desired bandwidth efficiency is achieved. The lowpass prototype $h(n)$ is bandlimited to no more than π/N for systems with distinct frequencies and $2\pi/N$ for those with repeated frequencies.

The aim of the synthesis procedure is to develop modulated filter banks that are crosstalk-free and have the same input-output transfer function between every pair of corresponding terminals. The kth input-output terminal pair has a transfer function given by

$$\begin{aligned} T_{kk}(z^N) = \frac{1}{4}z^{-(n_k-p_k)}\sum_{i=0}^{N-1} W^{i(n_k-p_k)} &\Big[e^{j(\alpha_k+\beta_k)}H^2(e^{-j\omega_k}zW^{-i}) \\ &+ e^{-j(\alpha_k+\beta_k)}H^2(e^{j\omega_k}zW^{-i}) \\ &+ 2\cos(\alpha_k-\beta_k)H(e^{-j\omega_k}zW^{-i})H(e^{j\omega_k}zW^{-i})\Big]. \end{aligned} \tag{3}$$

The strategy will be to try to make the transfer function $T_{kk}(z^N)$ independent of k. To this end, it is assumed that $n_k - p_k = s$ for every k. Also, note that the last term in Eq. (3) will be zero for center frequencies sufficiently away from 0 or π (the spectra in the $H(\cdot)$ terms do not overlap). Specifically, this will be true for $\omega_b \le \omega_k \le \pi - \omega_b$ where ω_b is the maximum bandwidth of the lowpass prototype (π/N for distinct center frequencies and $2\pi/N$ for repeated center frequencies). For the other end center frequencies near 0 or π, choosing $\alpha_k - \beta_k$ to be an odd multiple of $\pi/2$ will suffice to set the last term to zero. We now formulate two sets of conditions for identical input-output transfer functions.

3.1 Difference Criterion

For the difference criterion, the difference between any two center frequencies is constrained to be a multiple of $2\pi/N$ ($\omega_l - \omega_k = 2m\pi/N$ where m is an integer). The same transfer functions at terminals k and l are achieved by adhering to the following set of rules.

1. If a particular ω_k does not satisfy the inequality $\omega_b \le \omega_k \le \pi - \omega_b$, then $\alpha_k - \beta_k$ must be an odd multiple of $\pi/2$. The same restriction holds for terminal l.
2. The phases are such that $\alpha_k + \beta_k = \alpha_l + \beta_l$.
3. The delay factors are such that $n_k - p_k = n_l - p_l$ and moreover, are multiples of N.

3.2 Sum Criterion

It can be shown that if we restrict the sum of center frequencies to be of the form $\omega_k + \omega_l = 2m\pi/N$, another set of rules for which $T_{kk}(z^N) = T_{ll}(z^N)$ emerges as follows.

1. If a particular ω_k does not satisfy the inequality $\omega_b \le \omega_k \le \pi - \omega_b$, then $\alpha_k - \beta_k$ must be an odd multiple of $\pi/2$. The same restriction holds for terminal l.
2. The phases are such that $\alpha_k + \beta_k = -(\alpha_l + \beta_l)$.
3. The delay factors are such that $n_k - p_k = n_l - p_l$ and moreover, are multiples of N.

3.3 Equally Spaced Center Frequencies

Consider systems in which the center frequencies are equally spaced. For the case in which two signals may be transmitted at the same center frequency, we assume two cases for ease of exposition. The center frequencies of the two sets are

$$\text{Set 1}: \ 0,\ \frac{2\pi}{N},\ \frac{2\pi}{N},\ \frac{4\pi}{N},\ \frac{4\pi}{N},\ \cdots,\ \pi-\frac{2\pi}{N},\ \pi-\frac{2\pi}{N},\ \pi$$

or

$$\text{Set 2}: \ \frac{\pi}{N},\ \frac{\pi}{N},\ \frac{3\pi}{N},\ \frac{3\pi}{N},\ \cdots,\ \pi-\frac{\pi}{N},\ \pi-\frac{\pi}{N}\ .$$

Both Sets 1 and 2 ensure the desired bandwidth efficiency and a common input-output transfer function for every pair of terminals. Although these assumptions about the center frequencies are made at this time, we will later discuss why these are the only possible center frequencies for which two signals can be transmitted without crosstalk.

In the case of N distinct equally spaced center frequencies, a spacing of π/N ensures the desired bandwidth efficiency. The additional requirement of achieving the same input-output transfer function results in the unique choice

$$\text{Set 3}: \ \frac{\pi}{2N},\ \frac{3\pi}{2N},\ \frac{5\pi}{2N},\ \frac{7\pi}{2N},\ \cdots,\ \pi-\frac{\pi}{2N}\ .$$

The center frequencies of Set 3 alternately satisfy the sum or difference criterion and are the same as those in [7].

In the case of Set 1 and Set 2, note that the difference and sum criteria are the same. Either set of conditions should be satisfied. Both criteria are used for Set 3. At this stage, we confine $\alpha_k + \beta_k$ to be a multiple of π for Sets 1, 2 and 3. For the end center frequencies (those that do not satisfy the inequality $\omega_b \le \omega_k \le \pi - \omega_b$), the phase difference $\alpha_k - \beta_k$ is constrained to be an odd multiple of $\pi/2$. Combining this with the constraints on $\alpha_k + \beta_k$ gives the condition that the phases α_k and β_k are of the form $(2r+1)\pi/4$, where r is an integer, for the end frequencies.

4. Analysis of Crosstalk

This section analyzes the crosstalk functions between signals sent at adjacent center frequencies and the crosstalk functions between signals sent at the same center frequency. The crosstalk between signals whose allocated bandwidths do not overlap is zero. We will adhere to the restrictions generated for the input-output transfer function and formulate additional conditions for cancelling crosstalk. The cases when the center frequencies repeat and are distinct are considered separately. To start, we express the general crosstalk function for signals transmitted at two center frequencies ω_k and ω_l as

$$\begin{aligned} T_{kl}(z^N) = \frac{1}{4} z^{-(n_k - p_l)} \sum_{i=0}^{N-1} W^{i(n_k-p_l)} & \Big[e^{j(\alpha_k+\beta_l)} H(e^{-j\omega_k} z W^{-i}) H(e^{-j\omega_l} z W^{-i}) \\ & + e^{-j(\alpha_k+\beta_l)} H(e^{j\omega_k} z W^{-i}) H(e^{j\omega_l} z W^{-i}) \\ & + e^{j(\alpha_k-\beta_l)} H(e^{-j\omega_k} z W^{-i}) H(e^{j\omega_l} z W^{-i}) \\ & + e^{-j(\alpha_k-\beta_l)} H(e^{j\omega_k} z W^{-i}) H(e^{-j\omega_l} z W^{-i}) \Big] \ . \end{aligned} \tag{4}$$

4.1 Crosstalk: Different Center Frequencies of Sets 1 and 2

Consider the case of center frequencies belonging to Sets 1 and 2 which are multiples of π/N. For now, it is assumed that the different positive frequencies ω_k and ω_l are in the closed interval $[2\pi/N, \pi - 2\pi/N]$. Then, the last two terms of Eq. (4) are zero due to the bandlimitedness of $H(z)$. By substituting the relationship $\omega_l - \omega_k = 2\pi/N$ for the first two terms in Eq. (4), noting that $e^{j\omega_k} = W^p$ where p is a multiple of $1/2$ and performing algebraic manipulation to give identical crossterms in $H(\cdot)$, we develop a general rule relating the phases, delays, and p as given by

$$\alpha_k + \beta_l = \pi\Big[\frac{(\pm 1 - 2p)(n_k - p_l)}{N} + \frac{1}{2}\Big]\ . \tag{5}$$

4.1.1 Set 1

In Set 1, p is an even multiple of $1/2$ (center frequencies are even multiples of π/N). The two solutions to Eq. (5) are given below.

Solution One

1. The delays are such that $n_k - p_l$ is a multiple of N.
2. The phases are such that $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$.

Solution Two

1. The delays are such that $n_k - p_l$ is an odd multiple of $N/2$.
2. The phases are such that $\alpha_k + \beta_l$ is a multiple of π.

The only remaining crosstalk due to spectral overlap occurs between the end center frequency $\omega_k = 0$ and $\omega_l = 2\pi/N$. Retaining the restriction on α_k and β_k for the end center frequencies and the difference in the delay factors to be as above, two ways of eliminating crosstalk are as follows.

1. The delays are set such that $n_k - p_l$ and $n_l - p_k$ are multiples of N. The phases α_k and β_k are either $\pm\pi/4$ or $\pm 3\pi/4$. The phases α_l and β_l are odd multiples of $\pi/2$.
2. The delays are set such that $n_k - p_l$ and $n_l - p_k$ are odd multiples of $N/2$. The phases α_k and β_k are either $\pm\pi/4$ or $\pm 3\pi/4$. The phases α_l and β_l are multiples of π.

The same techniques result in cancelling crosstalk between signals sent at the other center frequencies of $\pi - 2\pi/N$ and π.

4.1.2 Set 2

For Set 2, p is an odd multiple of $1/2$ (center frequencies are odd multiples of π/N). A solution to Eq. (5) is given below.

Solution

1. The delays are such that $n_k - p_l$ is a multiple of $N/2$.
2. The phases are such that $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$.

For the end center frequency $\omega_k = \pi/N$, spectral overlap occurs with $\omega_l = 3\pi/N$. By substituting these frequencies in Eq. (4), it is found that the elimination of crosstalk is feasible if both of the conditions below are satisfied.

1. The delays are set such that $n_k - p_l$ and $n_l - p_k$ are multiples of $N/2$.
2. The phases are configured such that (α_k, β_l) and (β_k, α_l) are $(\pi/4, \pi/4 \pm m\pi)$, $(-\pi/4, -\pi/4 \pm m\pi)$, $(3\pi/4, 3\pi/4 \pm m\pi)$ or $(-3\pi/4, -3\pi/4 \pm m\pi)$ where m is an integer.

The same conditions result for cancelling the crosstalk between signals sent at a center frequency of $\pi - 3\pi/N$ and the other end frequency $\pi - \pi/N$.

4.2 Crosstalk: Repeated Center Frequencies

Here, we examine the crosstalk term associated with two signals transmitted with the same center frequency. We return to the original expression for the crosstalk function as in Eq. (4) and let ω_l be equal to ω_k $(2\pi/N \le \omega_k \le \pi - 2\pi/N)$. We have many degrees of freedom with which to force a zero crosstalk term. To maintain compatibility with the solutions formulated earlier, we restrict the differences in the delays to be multiples of $N/2$ and the sum of the phases to be multiples of $\pi/2$. Given the delays and phases as above, the analysis procedure investigates the question of which center frequencies can be utilized for transmitting more than one signal. The following results are obtained.

1. If $n_k - p_l$ is a multiple of N and $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$, the center frequency must be a multiple of π/N.
2. If $n_k - p_l$ is an odd multiple of $N/2$ and $\alpha_k + \beta_l$ is a multiple of π, the center frequency must be an odd multiple of π/N.
3. If $n_k - p_l$ is an odd multiple of $N/2$ and $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$, the center frequency must be an even multiple of π/N.

Although it was initially assumed that repeated frequencies are multiples of π/N, we see that with appropriate limitations on the delays and phases, only these frequencies ensure zero crosstalk.

The remaining case is to consider the end center frequency π/N in Set 2. Two signals can be transmitted over this frequency without crosstalk subject to both conditions given below.

1. The delays are set such that $n_k - p_l$ is an odd multiple of $N/2$.
2. The phases are configured such that $(\alpha_k, \beta_l) = (\pi/4, -\pi/4 \text{ or } 3\pi/4)$, $(-\pi/4, \pi/4 \text{ or } -3\pi/4)$, $(3\pi/4, -3\pi/4 \text{ or } \pi/4)$ or $(-3\pi/4, 3\pi/4 \text{ or } -\pi/4)$.

The same conditions hold for the other end frequency of $\pi - \pi/N$ in Set 2.

4.3 Distinct Center Frequencies of Set 3

Now, we consider the distinct center frequencies of Set 3. In Set 3, let two center frequencies be given by $\omega_k = (2p+1)\pi/2N$ and $\omega_l = (2p+3)\pi/2N$ for $p = 0, 1, \cdots, N-2$. By substituting these frequencies in Eq. (4), invoking the bandlimitedness assumption for $H(z)$ and performing algebraic manipulation gives two solutions that lead to two different transmultiplexers.

1. The delays are such that $n_k - p_l$ and $n_l - p_k$ are multiples of N. The phases are such that $\alpha_k + \beta_l$ and $\alpha_l + \beta_k$ are odd multiples of $\pi/2$.
2. The delays are such that $n_k - p_l$ and $n_l - p_k$ are odd multiples of $N/2$. If p is odd, $\alpha_k + \beta_l$ and $\alpha_l + \beta_k$ are odd multiples of $\pi/2$. If p is even, $\alpha_k + \beta_l$ and $\alpha_l + \beta_k$ are multiples of π.

5. Synthesized Transmultiplexers

Given these solutions, we establish values for the free parameters and synthesize five different transmultiplexers. The first three use repeated center frequencies (Set 1 or 2). The other two use the distinct frequencies of Set 3. In four of the five systems, it is necessary to implement delays which are odd multiples of $N/2$ (N is even).

System T1

$$
\begin{aligned}
a_0(n) &= h(n)\cos\frac{\pi}{4} & b_0(n) &= h(n)\cos\frac{\pi}{4} \\
a_1(n) &= h(n-\frac{N}{2})\cos\frac{2\pi}{N}n & b_1(n) &= h(n+\frac{N}{2})\cos\frac{2\pi}{N}n \\
a_2(n) &= h(n)\sin\frac{2\pi}{N}n & b_2(n) &= -h(n)\sin\frac{2\pi}{N}n \\
a_3(n) &= h(n)\cos\frac{4\pi}{N}n & b_3(n) &= h(n)\cos\frac{4\pi}{N}n \\
a_4(n) &= h(n-\frac{N}{2})\sin\frac{4\pi}{N}n & b_4(n) &= -h(n+\frac{N}{2})\sin\frac{4\pi}{N}n \\
\cdots & \quad \cdots & \cdots & \quad \cdots
\end{aligned}
\tag{6}
$$

A delay element is associated with a center frequency of π only if $N = 2, 6, 10, \cdots$.

System T2

$$
\begin{aligned}
a_0(n) &= h(n)\cos(\frac{\pi}{N}n+\frac{\pi}{4}) & b_0(n) &= h(n)\cos(\frac{\pi}{N}n-\frac{\pi}{4}) \\
a_1(n) &= h(n-\frac{N}{2})\cos(\frac{\pi}{N}n-\frac{\pi}{4}) & b_1(n) &= h(n+\frac{N}{2})\cos(\frac{\pi}{N}n+\frac{\pi}{4}) \\
a_2(n) &= h(n)\cos(\frac{3\pi}{N}n-\frac{\pi}{4}) & b_2(n) &= h(n)\cos(\frac{3\pi}{N}n+\frac{\pi}{4}) \\
a_3(n) &= h(n-\frac{N}{2})\cos(\frac{3\pi}{N}n+\frac{\pi}{4}) & b_3(n) &= h(n+\frac{N}{2})\cos(\frac{3\pi}{N}n-\frac{\pi}{4}) \\
\cdots & \quad \cdots & \cdots & \quad \cdots
\end{aligned}
\tag{7}
$$

For the last center frequency $\pi - \pi/N$, the delay at the transmitter is associated with a resultant phase of $\pi/4$ only if $N = 4, 8, 12, \cdots$. Otherwise, it is associated with a resultant phase of $-\pi/4$.

System T3

System T3 is a modification of T1 that uses two lowpass prototypes $h(n)$ and $g(n)$ which are each bandlimited to $2\pi/N$. By alternating the positions of $h(n)$ and $g(n)$ between the combining and separation filters for each center frequency, crosstalk due to spectral overlap is eliminated as in T1. For the case when two signals are transmitted with the same center frequency ω where $2\pi/N \le \omega \le \pi - 2\pi/N$, it can be shown that crosstalk is eliminated.

$$
\begin{aligned}
a_0(n) &= h(n)\cos\frac{\pi}{4} & b_0(n) &= g(n)\cos\frac{\pi}{4} \\
a_1(n) &= g(n-\frac{N}{2})\cos\frac{2\pi}{N}n & b_1(n) &= h(n+\frac{N}{2})\cos\frac{2\pi}{N}n \\
a_2(n) &= g(n)\sin\frac{2\pi}{N}n & b_2(n) &= -h(n)\sin\frac{2\pi}{N}n \\
a_3(n) &= h(n)\cos\frac{4\pi}{N}n & b_3(n) &= g(n)\cos\frac{4\pi}{N}n \\
a_4(n) &= h(n-\frac{N}{2})\sin\frac{4\pi}{N}n & b_4(n) &= -g(n+\frac{N}{2})\sin\frac{4\pi}{N}n \\
\cdots & \quad \cdots & \cdots & \quad \cdots
\end{aligned}
\tag{8}
$$

In system T3, crosstalk cancellation between equal center frequencies in the closed interval $[2\pi/N, \pi - 2\pi/N]$ relies on the bandlimitedness of the prototypes. In general, two prototypes cannot be utilized for crosstalk cancellation between equal frequencies outside this interval since the bandlimitedness cannot be invoked. Hence, T2 which has center frequencies of π/N and $(N-1)\pi/N$ cannot be modified to use two prototypes.

System T4

$$
\begin{aligned}
a_0(n) &= h(n)\cos(\frac{\pi}{2N}n-\frac{\pi}{4}) & b_0(n) &= h(n)\cos(\frac{\pi}{2N}n+\frac{\pi}{4}) \\
a_1(n) &= h(n)\cos(\frac{3\pi}{2N}n+\frac{\pi}{4}) & b_1(n) &= h(n)\cos(\frac{3\pi}{2N}n-\frac{\pi}{4}) \\
a_2(n) &= h(n)\cos(\frac{5\pi}{2N}n-\frac{\pi}{4}) & b_2(n) &= h(n)\cos(\frac{5\pi}{2N}n+\frac{\pi}{4}) \\
\cdots & \quad \cdots & \cdots & \quad \cdots
\end{aligned}
\tag{9}
$$

In fact, system T4 is the same as the transmultiplexer formed as the complement of the subband system in [7] except for the phase factors. We cannot modify T4 to include two prototypes since the resulting input-output transfer functions will not be the same for all pairs of terminals.

System T5

$$
\begin{aligned}
a_0(n) &= h(n)\cos(\frac{\pi}{2N}n-\frac{\pi}{4}) & b_0(n) &= h(n)\cos(\frac{\pi}{2N}n+\frac{\pi}{4}) \\
a_1(n) &= h(n-\frac{N}{2})\cos\frac{3\pi}{2N}n & b_1(n) &= h(n+\frac{N}{2})\cos\frac{3\pi}{2N}n \\
a_2(n) &= h(n)\cos(\frac{5\pi}{2N}n+\frac{\pi}{4}) & b_2(n) &= h(n)\cos(\frac{5\pi}{2N}n-\frac{\pi}{4}) \\
a_3(n) &= h(n-\frac{N}{2})\sin\frac{7\pi}{2N}n & b_3(n) &= -h(n+\frac{N}{2})\sin\frac{7\pi}{2N}n \\
\cdots & \quad \cdots & \cdots & \quad \cdots
\end{aligned}
\tag{10}
$$

System T5 is a new alternative to T4 which has a delay element for every other center frequency. As in T4, two prototypes cannot be accommodated.

5.1 Elimination of Intersymbol Interference

The five preceding transmultiplexers have been synthesized to eliminate crosstalk. The elimination of intersymbol interference in T1 and T2 is accomplished by designing $H^2(z)$ to satisfy the discrete form of the Nyquist criterion in which every Nth sample of its impulse response (except the zeroth sample) is zero valued. In T3, the product $G(z)H(z)$ must satisfy the Nyquist criterion. The elimination of intersymbol interference in T4 and T5 is accomplished by designing $H^2(z)$ to satisfy the discrete form of the Nyquist criterion in which every $2N$th sample of its impulse response (except the zeroth sample) is zero valued. Perfect reconstruction is achieved if the prototypes are strictly bandlimited (as discussed earlier) and the Nyquist criterion is satisfied.

6. OQAM and VSB Systems

Each system can be interpreted in terms of modulation by examining the input signal spectrum and the filter responses as shown in Fig. 2. As shown in Fig. 2(a), modulation is implicitly accomplished in the interpolation step in that copies of the input signal spectrum appear at intervals of $2\pi/N$. The three systems T1, T2 and T3 accomplish Orthogonal Quadrature Amplitude Modulation (OQAM). For each unique center frequency (except 0 and π), two signals are sent in quadrature. Systems T1 and T3 explicitly accomplish OQAM in that a particular combining filter extracts one of the replicated copies of the input spectrum around carrier frequencies at multiples of $2\pi/N$. The same is not true of T2 in that the combining filters which are centered at odd multiples of π/N, extract a portion of two adjacent copies of the input spectrum. System T2 can be converted to a true OQAM as follows. If each of the input signals is multiplied by $(-1)^n$ prior to interpolation, the input spectrum shifts in such a way that the replicated copies are centered at implicit carriers equal to odd multiples of π/N. Now, the combining filters will extract a replicated copy centered at odd multiples of π/N. The original signals can be recovered by multiplying each of the outputs by $(-1)^n$. Orthogonal Quadrature Amplitude Modulation systems have been realized in continuous time [8] and in discrete time [9].

In contrast, systems T4 and T5 can be thought of as being Vestigial Sideband (VSB) schemes. Given an implicit set of carriers at multiples of $2\pi/N$, there are both lower and upper sidebands at multiples of $2\pi/N$. A combining filter extracts either an upper or lower sideband of a particular copy of the input spectrum and a vestige of a suppressed sideband for transmission. Multiplication of the input signal by $(-1)^n$ prior to interpolation results in an implicit set of carriers at multiples of π/N. Again, one upper or lower sideband and a vestige of a suppressed sideband is extracted for transmission. In contrast to conventional frequency division multiplexing (FDM) schemes which avoid spectral overlap by using guard bands, the synthesized VSB systems allow overlap between the transmitted sidebands of different input signals.

7. Subband Complements

Transmultiplexers T1 through T5 can be converted into alias-free subband systems S1, S2, S3, S4 and S5 respectively [2]. Again, perfect reconstruction is achieved if the prototypes are strictly bandlimited and the Nyquist criterion is satisfied. For the special case of $N = 2$, S1 reduces to the classical QMF arrangement. If $G(z) = H(z^{-1})$, system S3 is lossless and reduces to the Smith-Barnwell structure [4] for the case $N = 2$. Hence, we have developed N band generalizations of the QMF bank and the Smith-Barnwell structure. The subband systems S4 and S5 have the same distinct center frequencies. System S4 resembles the one in [7] while S5 is an alternative employing delay factors.

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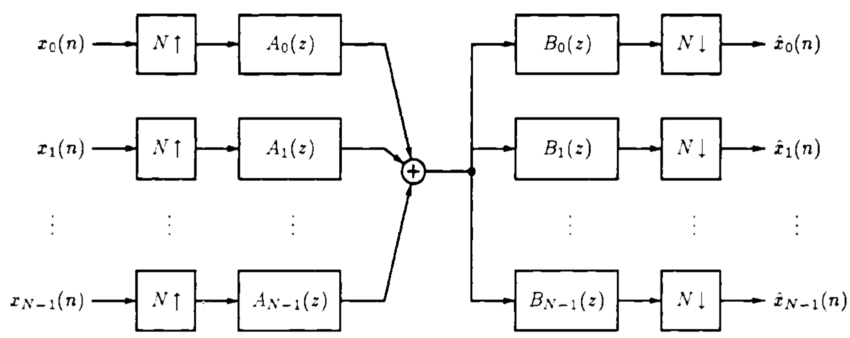

Fig. 1 A transmultiplexer system

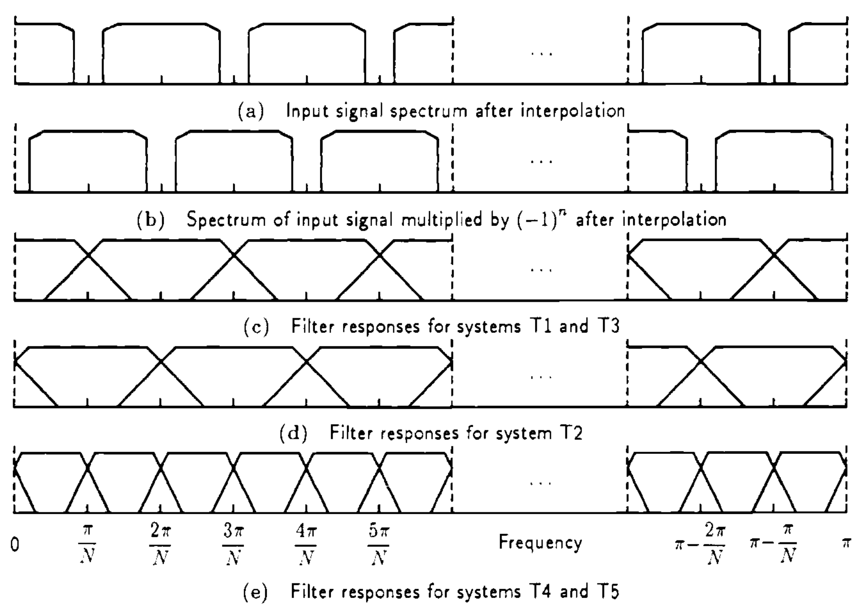

Fig. 2 Input signal spectrum and responses of the filters used in systems T1 to T5 (shown for N even)