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CONFIGURATION AND PERFORMANCE OF MODULATED FILTER BANKS

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Abstract

This paper examines the performance issues relating to the transmultiplexers synthesized in [1]. These transmultiplexers consist of modulated filter banks based on one or two lowpass prototypes. First, the limitations of the configured systems regarding intersymbol interference and crosstalk suppression arising from the use of practical filters are analyzed. Based on these observations, a new design technique for FIR prototypes that takes the practical degradations into account is formulated. The procedure involves the unconstrained optimization of an error function. The resulting performance is compared with minimax filters. For the one prototype systems, the new method is superior in that it leads to better intersymbol interference and crosstalk suppression with a smaller number of filter taps. In the case of the transmultiplexer with two prototypes, the main advantage of the new design method is the inclusion of crosstalk terms in the error function. Finally, we note that the five transmultiplexers can be converted into new subband systems and show how the design approach formulated for transmultiplexers carries over to the new subband systems.

1. Introduction

Multirate digital filter banks have been used in the realization of transmultiplexers (Fig. 1) and subband systems [2][3]. A class of these systems consist of modulated filter banks [1][4][5] in which the filters are modulated versions of lowpass prototypes. In [1], a number of bandwidth efficient transmultiplexers are synthesized. In this paper, we investigate the use of practical filters in the transmultiplexers. Filters are designed by minimizing a distortion measure incorporating both intersymbol interference and crosstalk terms. The resulting performance is evaluated and compared with minimax filters.

2. Input-Output Descriptions

An N channel transmultiplexer as depicted in Fig. 1 generates input-output relations given by

$$\hat{X}_i(z^N) = \frac{1}{N}\sum_{k=0}^{N-1} X_k(z^N)\sum_{l=0}^{N-1} A_k(zW^{-l})B_i(zW^{-l}) \quad \text{for} \quad 0 \le i \le N-1 \quad (1)$$

where $W = e^{-j\frac{2\pi}{N}}$. The output signal $\hat{X}_i(z^N)$ is related to the input signal $X_k(z^N)$ via a transfer function $T_{ki}(z^N) = \sum_{l=0}^{N-1} A_k(zW^{-l})B_i(zW^{-l})$. Crosstalk is eliminated when each output signal $\hat{X}_i(z)$ only depends on its corresponding input $X_i(z)$ and is not influenced by other input signals. In this case, the crosstalk function $T_{ki}(z^N)$ is zero for $k \neq i$. Furthermore, intersymbol interference (ISI) is eliminated if and only if $T_{kk}(z^N)$ is of the form cz^{-pN}. Then, perfect reconstruction to within a gain and delay is achieved.

Crosstalk-free transmultiplexers that have the same input-output transfer function $T_{kk}(z^N) = T(z^N)$ for every pair of terminals can be converted into alias-free subband systems [3]. These subband systems have an input-output relationship $\hat{X}(z) = \frac{1}{N}T(z^N)X(z)$.

3. Synthesized Transmultiplexers and Subband Systems

The combining filters $A_i(z)$ and the separation filters $B_i(z)$ of the transmultiplexers synthesized in [1] are given below. The transmultiplexers consist of modulated filter banks in which each filter is a frequency shifted version of a lowpass prototype $h(n)$ or $g(n)$. Under the assumption that both prototypes are strictly bandlimited to no more than $2\pi/N$ (systems T1, T2 and T3) or π/N (systems T4 and T5), the transmultiplexers are crosstalk-free and have the same input-output transfer function for every pair of terminals. Systems T1, T2 and T3 accomplish Orthogonal Quadrature Amplitude Modulation (OQAM). Systems T4 and T5 implement Vestigial Sideband Modulation (VSB). Transmultiplexers T1 through T5 are converted into alias-free subband systems S1 to S5 respectively.

Transmultiplexer T1 and Subband System S1

$$\begin{array}{ll}
a_0(n) = h(n)\cos\frac{\pi}{4} & b_0(n) = h(n)\cos\frac{\pi}{4} \\
a_1(n) = h(n-\frac{N}{2})\cos\frac{2\pi}{N}n & b_1(n) = h(n+\frac{N}{2})\cos\frac{2\pi}{N}n \\
a_2(n) = h(n)\sin\frac{2\pi}{N}n & b_2(n) = -h(n)\sin\frac{2\pi}{N}n \\
a_3(n) = h(n)\cos\frac{4\pi}{N}n & b_3(n) = h(n)\cos\frac{4\pi}{N}n \\
a_4(n) = h(n-\frac{N}{2})\sin\frac{4\pi}{N}n & b_4(n) = -h(n+\frac{N}{2})\sin\frac{4\pi}{N}n \\
\cdots\quad\cdots & \cdots\quad\cdots
\end{array} \quad (2)$$

Transmultiplexer T2 and Subband System S2

$$\begin{array}{ll}
a_0(n) = h(n)\cos(\frac{\pi}{N}n+\frac{\pi}{4}) & b_0(n) = h(n)\cos(\frac{\pi}{N}n-\frac{\pi}{4}) \\
a_1(n) = h(n-\frac{N}{2})\cos(\frac{\pi}{N}n-\frac{\pi}{4}) & b_1(n) = h(n+\frac{N}{2})\cos(\frac{\pi}{N}n+\frac{\pi}{4}) \\
a_2(n) = h(n)\cos(\frac{3\pi}{N}n-\frac{\pi}{4}) & b_2(n) = h(n)\cos(\frac{3\pi}{N}n+\frac{\pi}{4}) \\
a_3(n) = h(n-\frac{N}{2})\cos(\frac{3\pi}{N}n+\frac{\pi}{4}) & b_3(n) = h(n+\frac{N}{2})\cos(\frac{3\pi}{N}n-\frac{\pi}{4}) \\
\cdots\quad\cdots & \cdots\quad\cdots
\end{array} \quad (3)$$

Transmultiplexer T3 and Subband System S3

$$\begin{array}{ll}
a_0(n) = h(n)\cos\frac{\pi}{4} & b_0(n) = g(n)\cos\frac{\pi}{4} \\
a_1(n) = g(n-\frac{N}{2})\cos\frac{2\pi}{N}n & b_1(n) = h(n+\frac{N}{2})\cos\frac{2\pi}{N}n \\
a_2(n) = g(n)\sin\frac{2\pi}{N}n & b_2(n) = -h(n)\sin\frac{2\pi}{N}n \\
a_3(n) = h(n)\cos\frac{4\pi}{N}n & b_3(n) = g(n)\cos\frac{4\pi}{N}n \\
a_4(n) = h(n-\frac{N}{2})\sin\frac{4\pi}{N}n & b_4(n) = -g(n+\frac{N}{2})\sin\frac{4\pi}{N}n \\
\cdots\quad\cdots & \cdots\quad\cdots
\end{array} \quad (4)$$

Transmultiplexer T4 and Subband System S4

$$\begin{array}{ll}
a_0(n) = h(n)\cos(\frac{\pi}{2N}n-\frac{\pi}{4}) & b_0(n) = h(n)\cos(\frac{\pi}{2N}n+\frac{\pi}{4}) \\
a_1(n) = h(n)\cos(\frac{3\pi}{2N}n+\frac{\pi}{4}) & b_1(n) = h(n)\cos(\frac{3\pi}{2N}n-\frac{\pi}{4}) \\
a_2(n) = h(n)\cos(\frac{5\pi}{2N}n-\frac{\pi}{4}) & b_2(n) = h(n)\cos(\frac{5\pi}{2N}n+\frac{\pi}{4}) \\
\cdots\quad\cdots & \cdots\quad\cdots
\end{array} \quad (5)$$

Transmultiplexer T5 and Subband System S5

$$\begin{array}{ll}
a_0(n) = h(n)\cos(\frac{\pi}{2N}n-\frac{\pi}{4}) & b_0(n) = h(n)\cos(\frac{\pi}{2N}n+\frac{\pi}{4}) \\
a_1(n) = h(n-\frac{N}{2})\cos\frac{3\pi}{2N}n & b_1(n) = h(n+\frac{N}{2})\cos\frac{3\pi}{2N}n \\
a_2(n) = h(n)\cos(\frac{5\pi}{2N}n+\frac{\pi}{4}) & b_2(n) = h(n)\cos(\frac{5\pi}{2N}n-\frac{\pi}{4}) \\
a_3(n) = h(n-\frac{N}{2})\sin\frac{7\pi}{2N}n & b_3(n) = -h(n+\frac{N}{2})\sin\frac{7\pi}{2N}n \\
\cdots\quad\cdots & \cdots\quad\cdots
\end{array} \quad (6)$$

3.1 Nyquist Criterion for Zero ISI

To eliminate ISI in systems T1, S1, T2 and S2, $H^2(z)$ should be a Nyquist filter with an impulse response having regular zero crossings every Nth sample (except the zeroth sample). The same is true for $H(z)G(z)$ in systems T3 and S3. For transmultiplexers T4 and T5 and their subband complements, the Nyquist filter $H^2(z)$ must have an impulse response with zero crossings every $2N$th sample (except the zeroth sample).

4. Practical Considerations

Transmultiplexers T1 through T5 have been synthesized to cancel crosstalk and have the same input-output transfer function for every pair of terminals with strictly bandlimited filters [1]. In addition, satisfying the Nyquist criterion eliminates ISI and hence, achieves perfect reconstruction. Since practical filters are not strictly bandlimited, the input-output transfer function is not generally the same for all terminals and crosstalk is generally not cancelled. In addition, design procedures for practical filters may give filters such that the Nyquist criterion is not exactly satisfied. Therefore, the ISI is not generally eliminated at all terminals with practical filters.

Now, the question is to investigate the performance of the systems with practical filters. In this section, we consider two possible limitations, namely, the ISI is not exactly cancelled and the crosstalk is not exactly zero. The next section shows how the limitations are taken into account in an optimized design of the practical prototype.

4.1 The Input-Output Transfer Function

Consider the input-output transfer functions of the systems when practical filters are used. The bandlimitedness of the lowpass prototype was invoked in the analysis in [1] to cancel a term in the general expression for the input-output transfer function as a step in making it the same for all pairs of corresponding terminals. However, given the phase factors of the synthesized systems, this term is naturally cancelled (independently of $H(z)$ and $G(z)$) for systems T2, T4 and T5 and for the terminals in T1 and T3 operating at center frequencies of 0 and π. Therefore, the input-output transfer function is indeed the same for all terminals in each of the systems T2, T4 and T5 even with practical filters. The bandlimitedness assumption results in an error that shows up only in systems T1 and T3 and moreover, only at the terminals specified by center frequencies other than 0 and π.

The added consideration of satisfying the Nyquist criterion depends on the filter design procedure used (discussed later). If the Nyquist criterion is satisfied with practical filters, the ISI is cancelled at all terminals in T2, T4 and T5 and at the terminals specified by center frequencies of 0 and π in T1 and T3.

4.2 Crosstalk Suppression

From the synthesis procedure in [1], crosstalk cancellation occurs in two ways. First, crosstalk due to spectral overlap between adjacent sidebands is cancelled by choosing the center frequencies, delays and phases. These continue to be cancelled with practical filters. Second, crosstalk due to sidebands spaced further apart (no spectral overlap) is zero for strictly bandlimited prototypes. This crosstalk will appear with practical filters.

In further analyzing the crosstalk functions, we distinguish between exact cancellation in which the crosstalk function is inherently zero and the case when the bandlimitedness of the prototype must be used to achieve crosstalk cancellation. Exact crosstalk cancellation depends only on the center frequencies, phases and delay factors and occurs independently of the prototypes $H(z)$ and $G(z)$. In the other case, crosstalk cancellation occurs with strictly bandlimited prototypes but is admitted when practical filters are used. For a particular output terminal, there are $N-1$ crosstalk functions. For each of the transmultiplexers, a certain number of these $N-1$ functions may be exactly zero.

Of the transmultiplexers, T1 achieves the most number of exact crosstalk cancellations (about 3/4 of the total number of crosstalk functions). In systems T2 and T3, about half of the crosstalk functions are exactly zero. Transmultiplexers T4 does not achieve any exactly zero crosstalk cancellations. In system T5, one exactly zero crosstalk function exists for each output terminal only when N is not a multiple of 4.

5. Optimized Design

For the optimized design of FIR prototypes, we establish an error function that takes the various distortions into account. The error function is a weighted linear combination of various factors, each of which is discussed below. Minimizing the error function should give a lowpass prototype that leads to low ISI and crosstalk distortions. The stopband edge frequency is $\omega_s = (1+\beta)\omega_{\min}$ where $\omega_{\min} = \pi/N$ for T1, T2 and T3, or $\omega_{\min} = \pi/2N$ for T4 and T5 and $0 \leq \beta \leq 1$. Note that the passband characteristic is not explicitly considered since an approximately zero stopband and a low ISI distortion ensure an approximately constant passband.

A linear phase prototype $h(n)$ is designed for systems T1, T2, T4 and T5 (for the linear phase case, $h(n)$ must have an odd number of taps). For notational convenience, we assume that $h(n)$ has $2L+1$ taps from $n=-L$ to L. A nonlinear phase $h(n)$ with $M+1$ taps from $n=0$ to M is designed for transmultiplexer T3 with $G(z) = H(z^{-1})$.

Stopband

The factor in the error function representing the stopband characteristic is the square of the stopband energy (denoted by $E_{\rm sb}$) where

$$\sqrt{E_{\rm sb}} = \frac{1}{2\pi}\int_S \left|H(e^{j\omega})\right|^2 d\omega \ , \tag{7}$$

$S = [-\pi, -\omega_s] \bigcup [\omega_s, \pi]$ and ω_s is the stopband edge. For a linear phase $H(z)$, $\sqrt{E_{\rm sb}} = \mathbf{b}^T\mathbf{P}\mathbf{b}$ where $\mathbf{b} = [b(0)\ b(1)\ \cdots\ b(L)]^T$, $b(0) = h(0)$, $b(n) = 2h(n)$ for $n \neq 0$ and $\mathbf{P}$ is a positive definite symmetric matrix independent of $\mathbf{b}$. Since $G(z) = H(z^{-1})$ in system T3, the stopband energies of both filters are the same. Again, $\sqrt{E_{\rm sb}} = \mathbf{h}^T\mathbf{R}\mathbf{h}$ where $\mathbf{h} = [h(0)\ h(1)\ \cdots\ h(M)]^T$

Intersymbol Interference Distortion

At output terminal l, the mean-square ISI distortion is given by $\frac{1}{N^2}\sum_{n\neq 0} t_{ll}^2(n)$ where $t_{ll}(n)$ is the inverse z-transform of the input-output transfer function $T_{ll}(z)$. The factor representing the mean-square ISI distortion, $E_{\rm isi}$, generally includes every input-output terminal. In systems T2, T4 and T5, $t_{ll}(n)$ is the same for every terminal l, even with practical filters. Hence, it is sufficient to consider only one terminal. Moreover, ISI cancellation depends solely on the satisfaction of the Nyquist criterion.

In systems T1 and T3, the bandlimitedness assumption must be invoked to ensure the same $t_{ll}(n)$ for each input-output terminal pair. However, ISI cancellation depends solely on the satisfaction of the Nyquist criterion at terminals specified by center frequencies of 0 and π. The is not the case with the other terminals since the input-output transfer functions are different with practical filters. Since the stopband characteristic is taken into account separately, we ignore these differences that arise with practical filters and only consider $t_{ll}(n)$ for the 0 center frequency when dealing with T1 and T3.

Given the preceding discussion, the factor $E_{\rm isi}$ is given by

$$E_{\rm isi} = \begin{cases} \sum\limits_{\substack{n=cN \\ c\neq 0}} \left[h(n)*h(n)\right]^2 & \text{for systems T1 and T2} \\ \sum\limits_{\substack{n=2cN \\ c\neq 0}} \left[h(n)*h(n)\right]^2 & \text{for systems T4 and T5} \\ \sum\limits_{\substack{n=cN \\ c\neq 0}} \left[h(n)*h(-n)\right]^2 & \text{for system T3} \end{cases} \tag{8}$$

Crosstalk Distortion

At output terminal l, the total crosstalk power due to the undesired input signals is $P_{\rm ctk}(l)$. In developing a mathematical formula for $P_{\rm ctk}(l)$, we assume that each of the input data signals are zero-mean, white, of equal power and uncorrelated with other inputs. The crosstalk power at output terminal l contributed by a signal at input terminal k is given by the input signal power $P_{\rm s}$ multiplied by $\frac{1}{N^2}\sum_n t_{kl}^2(n)$ where $t_{kl}(n)$ is the inverse z-transform of $T_{kl}(z)$. Also, the total crosstalk power at output terminal l is the sum of the crosstalk powers contributed by each of the undesired signals and is given by

$$P_{\rm ctk}(l) = \frac{P_{\rm s}}{N^2}\sum_{\substack{k=0 \\ k\neq l}}^{N-1}\sum_n t_{kl}^2(n) \ . \tag{9}$$

To include the crosstalk power for every terminal l, we formulate an overall crosstalk factor $E_{\rm ctk}$ given by

$$E_{\rm ctk} = \frac{1}{P_{\rm s}}\sum_{l=0}^{N-1} P_{ctk}(l) = \sum_{l=0}^{N-1}\sum_{\substack{k=0 \\ k\neq l}}^{N-1}\sum_{n=cN}\left[a_k(n)*b_l(n)\right]^2 \tag{10}$$

Note that the crosstalk functions which are known to be exactly zero can be excluded from $E_{\rm ctk}$.

Overall Error Function

The overall error function to be minimized is the weighted sum of the above factors. At this point, note that the zero solution ($\mathbf{b} = 0$ or $\mathbf{h} = 0$) is the global minimum. To avoid reaching this solution, we append a term $(\mathbf{b}^T\mathbf{b}-1)^2$ or $(\mathbf{h}^T\mathbf{h}-1)^2$ to the overall error function. Hence, the overall error function $E(\mathbf{b})$ and $E(\mathbf{h})$ are

$$\begin{aligned} E(\mathbf{b}) &= \gamma_1 E_{\rm sb} + \gamma_2 E_{\rm isi} + \gamma_3 E_{\rm ctk} + \gamma_4(\mathbf{b}^T\mathbf{b}-1)^2 \\ E(\mathbf{h}) &= \gamma_1 E_{\rm sb} + \gamma_2 E_{\rm isi} + \gamma_3 E_{\rm ctk} + \gamma_4(\mathbf{h}^T\mathbf{h}-1)^2 \end{aligned} \tag{11}$$

where the γ_i represent nonnegative weighting factors.

6. Design Examples

We use a Quasi-Newton approach [6] to get a local minimum of E in which the gradient is expressed in closed form and the Hessian matrix is computed by the Broyden-Fletcher-Goldfarb-Shanno formula [6]. For the one prototype systems, the initial condition we use is an equiripple minimax approximation [7] of the square root of a raised cosine spectrum with unity gain at zero frequency. For transmultiplexer T3, the initial condition we use is an equiripple minimum phase filter with unity gain at zero frequency that is designed by the approach in [8]. Examples of magnitude response plots of the designed filters are depicted in Figure 2 (system T3) and Figure 3 (system T4) for the case $N=6$ and $\beta = 0.52$. Figure 2 shows a 30 tap filter designed with weighting factors $(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (100, 1, 1, 0.01)$. Figure 3 shows a 59 tap filter designed with weighting factors $(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (100, 1, 1, 0.01)$.

The fact that some crosstalk terms which form the crosstalk function $T_{kl}(z^N)$ are exactly zero is reflected in the frequency response of the lowpass prototype. Consider the optimized filter for system T4 (Fig. 3). When a crosstalk weight $\gamma_3 = 1$ is used, the stopband is shaped so as to suppress the nonzero crosstalk terms. An analysis of system T4 revealed that none of the crosstalk functions $T_{kl}(z^N)$ is exactly zero. However, some of the terms in the crosstalk functions $T_{kl}(z^N)$ are zero. Among the crosstalk functions in T4 for the case $N = 6$, the terms involving sidebands whose center frequencies are separated by $\pi/3$, $2\pi/3$ and π are never zero. The other terms involving sidebands whose center frequencies are separated by $\pi/6$, $\pi/2$ and $5\pi/6$ are consistently zero. This manifests itself in that the magnitude response in the stopband around the frequencies of $\pi/3$, $2\pi/3$ and π exhibit a higher attenuation than neighbouring regions. It is the higher attenuation in these regions that suppress the nonzero crosstalk terms. Transmultiplexer T3 has nonzero crosstalk terms only involving sidebands separated by $2\pi/3$ when $N = 6$. When the crosstalk weight $\gamma_3 = 1$, the stopband response of the resulting filter is better than for a design with $\gamma_3 = 0$ about the frequency $2\pi/3$.

7. Transmultiplexer Performance

The performance of the transmultiplexers is evaluated and compared for minimax filters and for filters designed by the method in this paper. The transmultiplexers have six bands ($N = 6$) and use filters with $\beta = 0.52$. For systems T1, T2 and T3, the aim is to achieve a minimum stopband attenuation of 40 dB. A 35 dB stopband is used for systems T4 and T5.

For the one prototype systems, a minimax linear phase $H(z)$ is designed using the McClellan-Parks algorithm [7] such that its frequency response approximates the square root of a raised cosine spectrum. The factorable minimax method in [8] is used for T3. The resulting filters are not linear phase. For T1 and T2, the prototype has 77 taps. For T3, a 30 tap filter results. For T4 and T5, a 99 tap prototype is used. Equiripple designs are obtained by using a weighting function equal to unity. We also design nonequiripple filters. For the one prototype systems, the weighting function $W(\omega)$ is unity in the passband and the transition band. In the stopband, an increasing weighting function is used, $W(\omega) = 200/2\pi(\omega - \omega_s) + 1$ for $\omega_s \leq \omega \leq \pi$. In the case of T3, the method in [8] is exclusively based on stopband control. The weighting function $W(\omega)$ is as before for $\omega_s \leq \omega \leq \pi$. Using the new method for optimized filters, we design a 33 tap filter for T1 and T2, a 30 tap filter for T3 and a 59 tap filter T4 and T5. The weighting factors used are $(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (100, 1, 0, 0.01)$ and $(100, 1, 1, 0.01)$.

In measuring the performance of the transmultiplexers, we consider the normalized mean-square distortion D_{ISI} for the ISI and the normalized crosstalk power D_{CRP}. For the lth output terminal, $D_{\mathrm{ISI}}(l)$ is

$$D_{\mathrm{ISI}}(l) = \frac{\displaystyle\sum_{\substack{n \\ n \neq 0}} t_{ll}^2(n)}{t_{ll}^2(0)} . \qquad (12)$$

At output terminal l, $D_{\mathrm{CRP}}(l)$ is

$$D_{\mathrm{CRP}}(l) = \frac{P_{ctk}(l)}{\dfrac{P_s}{N^2}\displaystyle\sum_n t_{ll}^2(n)} = \frac{\displaystyle\sum_{\substack{k=0 \\ k \neq l}}^{N-1} \sum_n t_{kl}^2(n)}{\displaystyle\sum_n t_{ll}^2(n)} . \qquad (13)$$

The quantity $D_{\mathrm{CRP}}(l)$ is a crosstalk to signal ratio or the total crosstalk power divided by the power of the desired component at l. Tables 1 and 2 show the values of $D_{\mathrm{ISI}}(l)$ and $D_{\mathrm{CRP}}(l)$ for the transmultiplexers when $N = 6$. The values for the first three output terminals are sufficient since symmetry gives the same results for the other three terminals.

The two sources of ISI distortion, namely, that the design procedure does not give filters such that the Nyquist criterion is exactly satisfied and that the prototypes are not strictly bandlimited are reflected in Table 1. First, consider the minimax designs. The source of ISI distortion due to the limitation of the design procedure applies in every case except T3. In fact, this is the only cause of ISI distortion in systems T2, T4 and T5 and at terminal 0 of T1. There is no ISI distortion at terminal 0 of T3 since the method in [8] assures the satisfaction of the Nyquist criterion. The other source of distortion due to the prototype not being strictly bandlimited only affects terminals 1 and 2 of transmultiplexers T1 and T3. However, the small variation in the values of D_{ISI} for T1 and the low ISI distortion for terminals 1 and 2 of T3 show that this contribution is not severe. In fact, T3 outperforms the other systems indicating that the limitation of the design procedure is the major source of distortion. Applying an increasing weighting function for the stopband does not affect the ISI distortion significantly except for system T3. In T3, the only practical limitation is that the prototypes are not strictly bandlimited. Therefore, an enhanced stopband diminishes the effect of this only limitation and leads to a lower ISI distortion.

Now, consider the optimized design. In contrast to the minimax approach, the limitations of the design procedure in not satisfying the Nyquist criterion affects all the systems. The limitation of the prototypes in not being strictly bandlimited only affects terminals 1 and 2 of systems T1 and T3. In the case of the one prototypes systems (T1,T2, T4 and T5), the dominant (or only) source of distortion is due to the limitation of the design procedure. The optimized design leads to a much lower ISI distortion than the minimax approach. Furthermore, a much smaller number of taps is required in the optimized design. A positive crosstalk weight does not affect the ISI distortion significantly. In the case of T3, the major source of distortion is due to the prototypes not being strictly bandlimited. There is a large difference in the ISI distortion for terminals 1 and 2 compared with terminal 0. This contrast with the one prototype systems is due to the fact that the initial condition used is the minimum phase part of a Nyquist filter. Although the Nyquist property is sacrificed in obtaining the optimized filter, the distortion at terminal 0 remains very low. Increasing the crosstalk weight leads to more distortion at terminal 0 and less distortion at terminals 1 and 2. For terminals 1 and 2 of T3, the bandlimitedness property is used to cancel terms in the transfer function involving sidebands whose center frequencies are separated by $2\pi/3$. The enhanced stopband attenuation about $2\pi/3$ that results from the use of a positive crosstalk weight diminishes the effect of the degradation due to the prototypes not being strictly bandlimited. This results in a lower ISI distortion at terminals 1 and 2.

The OQAM systems (T1, T2 and T3) generally achieve a much lower normalized crosstalk power than the VSB transmultiplexers (T4 and T5) primarily because OQAM systems exhibit many more exactly zero crosstalk functions. An exception arises for the optimized design with $\gamma_3 = 0$. In this case, T4 and T5 achieve a lower normalized crosstalk power than T3. However, this occurs by using a filter in T4 and T5 that has more taps and a better overall stopband response than the filter used in T3. Notice that the crosstalk is exactly zero for terminal 2 of T1. Among the OQAM systems, T1 and T2 outperform T3 but at the expense of more filter coefficients (the disparity in the number of coefficients is much more for the minimax designs). Applying an increasing weighting function in the stopband for a minimax design diminishes the crosstalk power. Also, using a positive crosstalk weight γ_3 in the optimized design results in a substantially lower crosstalk power.

The optimized design approach is beneficial especially for the one prototype systems. The advantage of achieving a much lower ISI and crosstalk distortion (even with a crosstalk weight of zero) with many fewer filter taps as compared to a minimax design is clearly evident for systems T1, T2, T4 and T5. In addition, the optimized design allows for the flexibility of taking crosstalk into account by setting $\gamma_3 > 0$. In system T3, the number of filter coefficients for the minimax and optimized designs are the same. Moreover, the minimax filters serve as initial conditions for the optimized design. The advantage of the optimized design over the minimax design primarily lies in using a crosstalk weight to substantially diminish the crosstalk power. Moreover, the use of a positive crosstalk weight is responsible for a lower ISI distortion at terminals 1 and 2 of T3. However, in contrast to the minimax design for T3, the optimized filters do not ensure that the Nyquist criterion is satisfied. Without a crosstalk weight, there is no clear advantage of the optimized design.

8. Design for the Complementary Subband Systems

The complementary subband systems achieve perfect reconstruction if the prototypes are strictly bandlimited and the Nyquist criterion is satisfied. In a practical design, the stopband edge frequency is restricted as in the case of transmultiplexers. The error function is a weighted linear combination of E_{sb}, E_{isi} and the factor that avoids a zero solution. The filters that were previously designed with $\gamma_3 = 0$ can be used in the complementary subband systems.

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Transmultiplexer	$D_{ISI}(l)$ dB minimax design constant $W(\omega)$	$D_{ISI}(l)$ dB minimax design increasing $W(\omega)$	$D_{ISI}(l)$ dB new design $\gamma_3 = 0$	$D_{ISI}(l)$ dB new design $\gamma_3 = 1$
T1	−36 − 34 − 37	−34 − 34 − 34	−60 − 60 − 60	−60 − 60 − 60
T2	−36 − 36 − 36	−34 − 34 − 34	−60 − 60 − 60	−60 − 60 − 60
T3	−∞ − 45 − 45	−∞ − 54 − 54	−105 − 57 − 57	−96 − 83 − 88
T4	−31 − 31 − 31	−31 − 31 − 31	−62 − 62 − 62	−63 − 63 − 63
T5	−31 − 31 − 31	−31 − 31 − 31	−62 − 62 − 62	−62 − 62 − 62

Table 1 Mean-square ISI distortion for transmultiplexers T1 to T5. Entries along a row refer to output terminals $l = 0$, 1 and 2 respectively. The new designs are done with $(\gamma_1, \gamma_2, \gamma_4) = (100, 1, 0.01)$.

Transmultiplexer	$D_{CRP}(l)$ in dB minimax design constant $W(\omega)$	$D_{CRP}(l)$ in dB minimax design increasing $W(\omega)$	$D_{CRP}(l)$ in dB new design $\gamma_3 = 0$	$D_{CRP}(l)$ in dB new design $\gamma_3 = 1$
T1	−47 − 47 − ∞	−65 − 65 − ∞	−70 − 70 − ∞	−87 − 87 − ∞
T2	−47 − 47 − 47	−65 − 65 − 65	−70 − 70 − 70	−87 − 87 − 87
T3	−39 − 40 − 41	−47 − 49 − 48	−46 − 48 − 45	−74 − 77 − 73
T4	−25 − 25 − 25	−40 − 40 − 40	−54 − 54 − 54	−65 − 65 − 65
T5	−26 − 26 − 26	−43 − 44 − 41	−49 − 50 − 52	−60 − 60 − 61

Table 2 Normalized crosstalk power for transmultiplexers T1 to T5. Entries along a row refer to output terminals $l = 0$, 1 and 2 respectively. The new designs are done with $(\gamma_1, \gamma_2, \gamma_4) = (100, 1, 0.01)$.

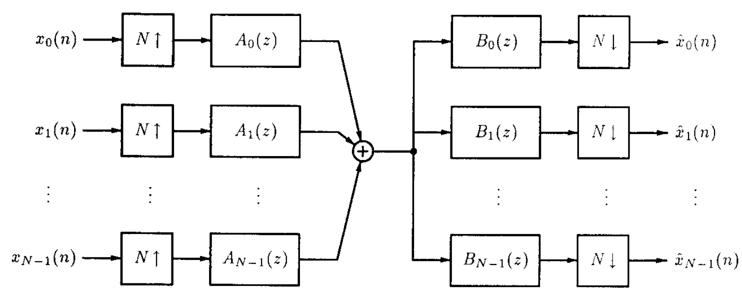

Fig. 1 A transmultiplexer system

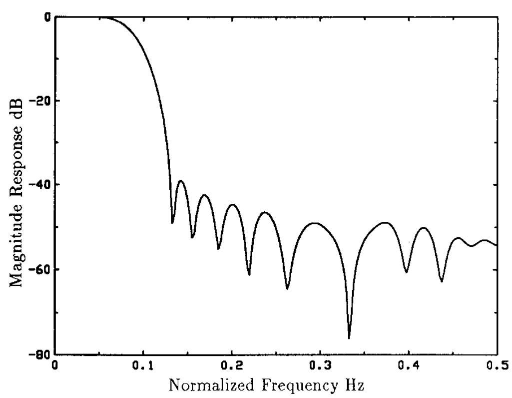

Fig. 2 Magnitude response of the lowpass filter for system T3. The weighting factors are $(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (100, 1, 1, 0.01)$.

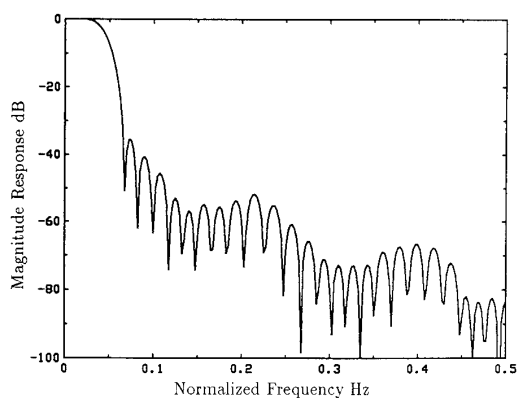

Fig. 3 Magnitude response of the lowpass filter for system T4. The weighting factors are $(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (100, 1, 1, 0.01)$.