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SHAPING MULTIDIMENSIONAL SIGNAL CONSTELLATIONS

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Abstract: Consider an optimally shaped N-dimensional (N even) signal constellation on a lattice. Assuming continuous approximation, the boundary of the two-dimensional subconstellations is a circle and the boundary of the whole constellation is a hypersphere. We derive analytical expressions for the optimum tradeoff between the shape gain and the Constellation-Expansion-Ratio and also between the shape gain and the Peak-to-Average-power-Ratio.

We introduce a method for achieving a point on the optimum tradeoff curves. This is based on mapping the constellation to the Voronoi region of the lattice D_n^*, $n = N/2$. The addressing complexity is essentially that of decoding D_n^* which using $D_n^* = \{(2\mathbf{Z})^n\} \cup \{(2\mathbf{Z})^n + (1)^n\}$, where $\mathbf{Z}$ is the integer lattice, is simple. For dimensions up to 12 the point obtained is located on the knee of the curve. This coding method is less complex and has superior performance to that based on the Voronoi constellations.

In a second method, n-dimensional subconstellations are first shaped using previous method. The N/n-fold cartesian product of this subconstellations is further shaped by a lookup table. Analytical expressions show that even for small lookup tables this method obtains results near the optimum tradeoff curves.

Summary

We study two methods for shaping an N-dimensional constellation. In the first method $\mathbf{S}_2(R)$ [1] is the boundary of the two-dimensional subconstellations and $\mathbf{S}_N(\sqrt{\beta}R)$ is the boundary of the whole constellation. This method provides the optimum tradeoff between γ_s (shape gain) and CER$_s$ (shaping Constellation-Expansion-Ratio) and also between γ_s and PAR (Peak-to-Average-power-Ratio). The parameter β determines this tradeoff. Optimality is the result of the facts that, (i) CER$_s$ and PAR are measured on a two dimensional basis, (ii) the two-dimensional subconstellations are bounded within a circle, and (iii) for a fixed volume the signal constellation is the subset of the available signal space with the least second moment.

The energy shells of the $\mathbf{S}_2$'s can be mapped to the coordinates of an $n{=}N/2$ dimensional space. Half of each shell is mapped to the positive values and half to the negative values. The region $\mathbf{R}_N$ bounding the constellation will be mapped to $\mathbf{R}_n$. It is shown that,

$$\int_{\mathbf{R}_N} F(X_0^2 + \ldots + X_{N-1}^2) = (\pi R^2)^n \times \sum_{k=0}^{\lfloor\beta\rfloor} C_n^k \frac{(\beta-k)^n}{(n-1)!} \int_0^1 F\left\{R^2\left[(\beta-k)\tau + k\right]\right\} \tau^n \, d\tau. \tag{1}$$

Relation (1) is used to calculate the tradeoff between γ_s and CER$_s$ shown in Fig. 1, and also between γ_s and PAR. Using Eq. (1), it is shown that as $N \to \infty$, the distribution along each $\mathbf{S}_2$ approaches a truncated Gaussian distribution. This is used in calculating the curves of $N = \infty$ in Fig. (1). For $\beta{=}N/4$, the region $\mathbf{R}_n$ will be equal to the Voronoi region of lattice D_n^* and addressing is facilitated. This corresponds to the marked points in Fig. (1). For dimension up to 12 this point is located on the knee of the curve. The addressing complexity is essentially that of decoding D_n^* which using, $D_n^* = \{(2\mathbf{Z})^n\} \cup \{(2\mathbf{Z})^n + (1)^n\}$ is simple. Table 1 shows the results. This method has higher performance and is less complex than that using Voronoi constellations, [1].

In a second method, $\mathbf{S}_2(R)$ is the boundary of the two-dimensional subconstellations, $\mathbf{S}_n(\sqrt{\beta'}R)$ is the boundary of the n dimensional subconstellations and $\mathbf{S}_N(\sqrt{\beta'\beta''}R)$ is the boundary of the whole constellation. This method has two degrees of freedom, β' and β''. A relationship similar to (1) is derived. For $\beta'{=}n/4$, the region $\mathbf{R}_n$ bounding the n-dimensional subconstellations is mapped to the Voronoi region of $D_{n'}^*$, $n'{=}n/2$. The Voronoi region of $D_{n'}^*$ is partitioned into M shaping regions of equal volume and increasing average energy. A lookup table is used to select a region within each $\mathbf{R}_n$. Analytical expressions show that even for small lookup tables this method achieves values near the optimum tradeoff curve. Table 2 shows some results.

N	γ_s(dB)	CER$_s$	PAR
4	0.456	1.414	3.00
8	0.603	1.189	2.61
12	0.612	1.122	2.47
16	0.608	1.090	2.39

Table 1: Performance of the signal constellations obtained by using shell mapping (first method).

N	γ_s(dB)	CER$_s$	PAR
16	0.81	1.3	3.00
	0.88	1.4	3.26
	0.92	1.5	3.53
24	0.82	1.2	2.76
	0.93	1.3	3.00
	0.99	1.4	3.35
32	0.81	1.15	2.64
	0.90	1.2	2.85
	0.99	1.3	3.12

Table 2: Performance of the signal constellations obtained by using a two level shell mapping (second method).

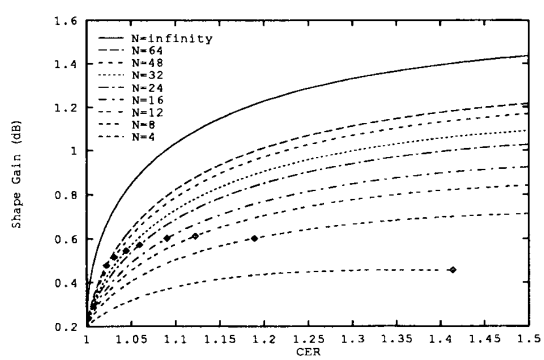

Figure 1: Optimum tradeoff between CER$_s$ and γ_s.

References

[1] G. D. Forney, Jr., "Multidimensional constellations –Part II: Voronoi constellations," *IEEE J. Select. Areas Commun.*, vol. SAC-7, pp. 941-958, 1989.

[1] $S_N(R)$ denotes an N-dimensional sphere of radius R.