Proc. IEEE Globecom Conf. (Orlando, FL), pp. 1769-1773, Dec. 1992

Trellis-Coded Phase/Frequency Modulation with Equal Usage of Signal Dimensions†

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Abstract: A trellis-coded modulation scheme, using a coded 2-FSK/2^m-PSK modulation format instead of a conventional coded 2^{m+1}-PSK format for m-bits/symbol data transmission, is considered. Previous work [1, 2] on coded 2-FSK/2^m-PSK modulation techniques choose two transmission frequencies equally often. In contrast, our method selects the signal points in such a manner that each of the signal dimensions is used equally. We compare the asymptotic coding gains of the proposed method to those of the previous schemes and calculate an upper bound to the bit-error probability. The spectral characteristics of the proposed coded modulation scheme are also studied *vis-a-vis* those of the coded 2^{m+1} PSK format and of the other [1, 2] coded 2-FSK/2^m-PSK formats. We show that the new scheme obtains higher coding gains, sometimes (but not always) at the expense of an increased bandwidth.

1 Introduction

Trellis-coded modulation (TCM), a combined coding and modulation technique [3], is generally used for digital transmission over band-limited channels. TCM schemes for data transmission over an AWGN channel are designed by optimizing not the free Hamming distance of the code, but the free Euclidean distance between the transmitted signal sequences. Remarkable coding gain, without compromising the data rate or requiring more bandwidth, has made this scheme attractive.

Coded modulation format defined over an expanded set of signals, but utilizing both frequency and phase, was suggested by Padovani and Wolf [1] to achieve further coding gain for the same number of trellis states. They have shown that the 2-FSK/2^m-PSK codes have better Euclidean distance and hence a higher coding gain than the Ungerboeck's 2^{m+1}-PSK codes for an uncoded 2^m-PSK signal set with the same decoding complexity. However, an increase in bandwidth has occurred in some cases as a price for the increase in noise immunity. These phase/frequency codes are considered useful for channels which can accommodate this nominal bandwidth expansion. By introducing a non-uniformity in the signal constellation points, further coding gain was obtained in [2] for phase/frequency coded systems. However, in practice, the non-uniformity in the phase/frequency signal constellation poses a serious problem in carrier synchronization. In this article, we consider uniform 2-FSK/2^m-PSK modulation signals as treated by Padovani and Wolf. But here the transmission frequencies are not used equally; instead the signal constellation points are placed in such a manner so as to use the signal dimensions equally. The equal usage of the signal dimensions rather than that of the transmission frequencies is the key concept applied here to achieve additional coding gain.

The remainder of the article is organized as follows. In Section 2, we describe a trellis-coded phase/frequency modulation scheme utilizing the available signal dimensions equally. This scheme is discussed with two exemplary systems—coded 2-FSK/4-PSK and coded 2-FSK/8-PSK. In Section 3, the resulting codes are compared to (a) Ungerboeck's and (b) Padovani and Wolf's codes in terms of the asymptotic coding gains for several constraint lengths of the convolutional coder. An upper bound to the bit-error probability is also calculated for the coded 2-FSK/4-PSK for the constraint length three. In Section 4, the spectral properties of the proposed phase/frequency modulation scheme are investigated for different values of the modulation index.

2 Description of Coded Modulation Scheme

In an uncoded system, for transmitting m bits of information in an interval T, any one of the available 2^m signals are sent each T seconds. However, in the TCM scheme, 2^m signals of the 2^{m+1} coded signals are candidates for transmission in any keying interval T. The encoder introduces the interdependence among the resulting sequences of channel signals such that the free Euclidean distance d_{free} between any two possible signal sequences becomes greater than the minimum distance d_0 between any two points in the 2^m signal constellation. This synthesized memory is exploited by the maximum-likelihood (Viterbi) decoder resulting in an asymptotic coding gain of $10\log_{10}(d^2_{\text{free}}/d^2_0)$ dB.

We use a rate $R = m/(m+1)$ binary convolutional encoder which encodes m information bits $X_k^{(i)}, i = 1, \ldots, m$ into $(m+1)$-coded bits $Y_k^{(j)}, j = 1, \ldots, (m+1)$ for the k-th signal sequence. These coded bits are mapped to the channel signal vector S_k and then phase/frequency modulated (uniformly) to the channel signal waveform $S_k(t)$. Here, we consider signals which are combinations of binary FSK and 2^m-ary PSK. The two frequencies used for the signal transmission are $(\omega_c + \frac{h\pi}{T})$ and $(\omega_c - \frac{h\pi}{T})$ rad/sec with ω_c as the fundamental frequency and h as the modulation index. The resulting signal space is then four dimensional and the or-

† Part of this work was supported by the Natural Sciences and Engineering Research Council of Canada.

thonormal basis vectors for this space, using Gram-Schmidt orthonormalization procedure, is chosen to be

$$\begin{aligned}
\psi_1(t) &= \sqrt{\tfrac{2}{T}}\cos[(\omega_c+\omega_d)t] \\
\psi_2(t) &= \sqrt{\tfrac{2}{T}}\sin[(\omega_c+\omega_d)t] \\
\psi_3(t) &= \tfrac{1}{\sqrt{D}}\left\{\sqrt{\tfrac{2}{T}}\cos[(\omega_c-\omega_d)t] - C_1\psi_1(t) - C_2\psi_2(t)\right\} \\
\psi_4(t) &= \tfrac{1}{\sqrt{D}}\left\{\sqrt{\tfrac{2}{T}}\sin[(\omega_c-\omega_d)t] + C_2\psi_1(t) - C_1\psi_2(t)\right\},
\end{aligned}$$

where $\omega_d = h\pi/T$, $C_1 = \sin(2\pi h)/(2\pi h)$, $C_2 = [1-\cos(2\pi h)]/(2\pi h)$ and $D = 1 - C_1^2 - C_2^2$. We emphasize that the orthonormal basis vector set representation is not unique and any such basis vector set depends on the modulation index h. We illustrate the coded phase/frequency modulation scheme with the following two modulation formats.

A. Coded 2-FSK/4-PSK: A widely used scheme for 2 bits/symbol transmission is QPSK in which various combinations of the two bits select one of the four phase shifts to be applied to the carrier. Ungerboeck [3] used an expanded signal set, namely an 8-PSK constellation for this example, in which different combinations of the three coded bits at the output of the convolutional encoder (for each symbol interval) select one of the eight phase shifts. In Padovani and Wolf's scheme (referred to hereafter as the PW scheme) [1], for a coded 2-FSK/4-PSK format, one bit of the three coded bits selects one of the two carrier frequencies (the FSK part of the modulation) and the other two bits choose one of the four phase shifts to be applied to the corresponding carrier. Subsequently, Chalid *et al.* [2] have shown that by designing the signal constellations to be non-uniform, performance gains can be obtained over Padovani and Wolf's scheme in many instances. They have considered three mapping rules—two of them partition the original set into two subsets where each subset consists of binary FSK and binary PSK, and the other one partitions the original set into two subsets, each consisting of a 4-ary PSK signal.

In [1] and [2], the two transmission frequencies are used equally. However, a careful observation of the set of orthonormal basis vectors reveals that the existing coded phase/frequency modulation schemes do not effectively use all the signal dimensions. Relying on the principle of *Equal Utilization of the Signal Dimensions*, our scheme (referred to hereafter as the EUSD scheme) offers further coding gain.

In the present case, the expanded signal set consists of eight signals. These signals are placed on the eight non-adjacent vertices (out of sixteen vertices), as shown in Fig. 1, of a 4-D hypercube. They also lie on the surface of a 4-D hypersphere and thus all the signal points have the same energy. Normalization of the signal energy makes $4d^2 = 1$, where $2d$ is the length of each edge of the hypercube. The eight signal vectors $\{S_0, S_1, \ldots, S_7\}$ thus can be represented in terms of the ordered basis vectors as

$$\begin{aligned}
S_0 &= (+0.5,+0.5,+0.5,-0.5), & S_1 &= (+0.5,+0.5,-0.5,+0.5), \\
S_2 &= (+0.5,-0.5,-0.5,-0.5), & S_3 &= (+0.5,-0.5,+0.5,+0.5), \\
S_4 &= (-0.5,-0.5,-0.5,+0.5), & S_5 &= (-0.5,-0.5,+0.5,-0.5), \\
S_6 &= (-0.5,+0.5,+0.5,+0.5), & S_7 &= (-0.5,+0.5,-0.5,-0.5).
\end{aligned}$$

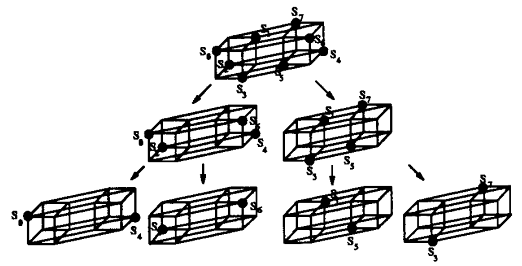

Fig. 1 2-FSK/4-PSK signal points and their partitions into sub-constellations

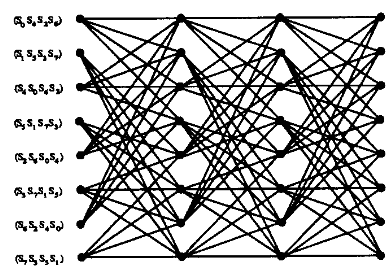

Fig. 2 Trellis diagram for 2-FSK/4-PSK scheme with $\nu = 3$

The signal constellation is partitioned into subconstellations, using a 'mapping by set partitioning' rule, as illustrated in Fig. 1. Following the notation introduced in [3], we use Δ_i to denote the minimum Euclidean distance in the i-th partitioned set. Here, $\Delta_0 = 2\sqrt{2}d = \sqrt{2}$ units, $\Delta_1 = 2\sqrt{2}d = \sqrt{2}$ units and $\Delta_2 = 4d = 2$ units.

In this work, we have considered the same set of systematic convolutional encoders with feedback for which the parity check polynomial coefficients and the number of information bits involved in parity check operations are provided in Tables II and III of [1]. Fig. 2 presents a trellis diagram for 2-FSK/4-PSK scheme with $\nu = 3$. While calculating the free distance of this trellis code, the symmetries of the signal set and the systematic nature of the code are exploited. This avoids finding the minimum distance among all the correct paths; we need to search the minimum-weight paths only against the all-zero possible transmitted paths [3]. We calculate d_{free}^2 to be 6 units for $\nu = 3$. Therefore, G_∞, the asymptotic coding gain over uncoded 4-PSK $= 10\log_{10}(\frac{6}{2})$ dB $= 4.77$ dB. For $\nu = 4$, d_{free}^2 and hence the coding gain over uncoded 4-PSK have been found to remain the same as those for $\nu = 3$; but $N(d_{\text{free}})$, the average number of codewords at Euclidean distance d_{free} from any given codeword, becomes relatively small compared to that for the $\nu = 3$ case. For $\nu = 5$, d_{free}^2 has been calculated to be 8 units and thus $G_\infty = 10\log_{10}(\frac{8}{2})$ dB $= 6.00$ dB.

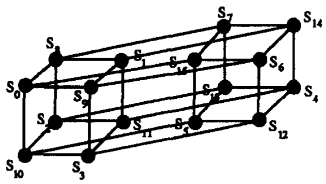

Fig. 3 2-FSK/8-PSK signal points placed on vertices of a 4-D hypercube

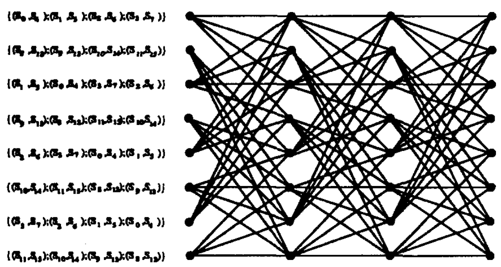

Fig. 4 Trellis diagram for 2-FSK/8-PSK scheme with $\nu = 3$ (any drawn line between two subsequent states actually represents two parallel transitions)

B. Coded 2-FSK/8-PSK: For data transmission of 3 bits/symbol, we consider sixteen equal-energy signal waveforms $\{S_0, S_1, \ldots, S_{15}\}$. These signal points are placed on the sixteen vertices, as shown in Fig. 3, of a 4-D hypercube with each edge having one unit length. The subconstellation partitions of the original constellation are (i) $\{(S_0,S_4);(S_1,S_5);(S_2,S_6);(S_3,S_7)\}$ and (ii) $\{(S_8,S_{12});(S_9,S_{13});(S_{10},S_{14});(S_{11},S_{15})\}$

Now, let us consider any particular signal point (say, S_0) and calculate Δ_0, Δ_1 and Δ_2. Here, $\Delta_0 = 1$ unit, $\Delta_1 = \min\{S_0S_4,\ S_0S_1,\ S_0S_5,\ S_0S_2,\ S_0S_6,\ S_0S_3,\ S_0S_7\} = \sqrt{2}$ units and $\Delta_2 = S_0S_4 = 2$ units. From the trellis diagram of the coded 2-FSK/8-PSK scheme with $\nu = 3$, shown in Fig. 4, d^2_{free} can easily be determined to be 4 units. Furthermore, the minimum squared distance d_0^2 between any two points in the 8-PSK signal constellation is calculated to be $4\sin^2 22.5° = 0.5857$ unit. Hence, G_∞ of $10\log_{10}(\frac{4}{0.5857})$ dB = 8.34 dB is obtained for coded 2-FSK/8-PSK scheme over uncoded 8-PSK with $\nu = 3$.

3 Coding Gain and Probability-of-Error

In this section, we study the performances, namely the asymptotic coding gain and the bounds on the probabilities of event and bit errors, of the proposed coded scheme.

3.1 Asymptotic Coding Gain

Asymptotic coding gains of the coded phase/frequency modulation scheme for 2 and 3 bits/ transmission are compared in Tables I and II, respectively.

	PW scheme (dB)				Cod. 8-PSK	EUSD
ν	$h = 0.25$	$h = 0.50$	$h = 0.75$	$h = 1.0$	(dB)	(dB)
3	3.7	4.1	4.5	4.8	3.6	4.8
4	4.4	4.8	4.8	4.8	4.1	4.8
5	4.8	4.9	5.5	6.0	4.6	6.0

TABLE I: Comparison of Asymptotic Coding Gains for 2 bits/symbol Transmission

	PW scheme (dB)			Cod. 16-PSK	EUSD
N	$h = 0.50$	$h = 0.75$	$h = 1.0$	(dB)	(dB)
3	6.8	7.9	8.3	5.3	8.3
4	7.5	8.3	8.3	6.1	8.3

TABLE II: Comparison of Asymptotic Coding Gains for 3 bits/symbol Transmission

In the PW scheme, the coding gains are dependent on the modulation index h. This is attributed to the fact that the equal usage of the transmission frequencies necessitates a reorientation of the signal points with h in reference to the basis vectors. As the value of h is lowered, information flow along some of the dimensions reduces. This results in relatively lower coding gain for small values of h. In our code, although the transmitted signal waveforms depend on h, the allowable signal vectors are fixed with respect to any basis vector set. This makes the coding gain independent of the modulation index. We also notice that the coding gains obtained in the EUSD scheme are the same as those of the PW scheme for $h = 1.0$. With $h = 1.0$, we calculate $C_1 = C_2 = 0$ and $D = 1$. These parameter values imply that the idea of the equal usage of the transmission frequencies $(\omega_c + \frac{\pi t}{T})$ and $(\omega_c - \frac{\pi t}{T})$ is synonymous with the notion of the equal utilization of the available signal dimensions.

3.2 Probability-of-Error Performance

The error performance of the EUSD scheme is studied when the codes are used on the AWGN channel. Perfect phase and timing synchronization are assumed. To calculate the event-error and the bit-error probability bounds, conventional generating function is defined over the expanded set of signals. For high signal-to-noise ratios, the event-error probability $P(e)$ approaches from above the value [3]

$$P(e) \gtrsim N(d_{\text{free}})\frac{1}{2}\text{erfc}\left(\frac{d_{\text{free}}}{2}\sqrt{\frac{E_s}{N_0}}\right), \tag{1}$$

where E_s is the (2^m-ary) symbol energy, $N_0/2$ is the double-sided noise power spectral density and

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt. \tag{2}$$

To find the average bit error probability performance of the Viterbi decoder, the pairwise error probability between the coded sequence and the estimated sequence is evaluated using the Bhattacharyya bound. A tight upper bound to the bit-error-probability is provided in [4] as

$$P_b \leq \frac{1}{2m}\text{erfc}\left(\frac{d_{\text{free}}}{2}\sqrt{\frac{mE_b}{N_0}}\right) D^{-d^2_{\text{free}}}\frac{d}{dz}T(D,z)\,|_{z=1}\,, \tag{3}$$

where the Bhattacharyya parameter D is related to the system bit energy-to-noise ratio E_b/N_0 as $D =$

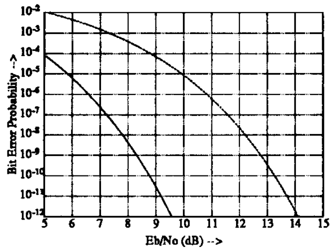

Fig. 5 Bit-error probability for uncoded **QPSK** (shown by - - -) and a tight upper bound on bit-error probability for coded 2-FSK/4-PSK EUSD scheme (shown by —-) vs. bit-energy to noise ratio

$\exp(-mE_b/4N_0)$ and m is the number of bits transmitted in a keying interval T. The generalized transfer function $T(D,z)$ of the 2-FSK/4-PSK, eight-state trellis code is computed by drawing a signal flow graph (in a similar fashion as given in [5]) and applying Mason's gain formula. Fig. 5 plots the bit-error probability of (uncoded) QPSK signal and also the tight upper bound of (3) to the bit-error probability of the EUSD 2-FSK/4-PSK coded signal.

4 Power Spectral Density

The bandwidth occupancy of a signal is an aspect as vital as the error performance. By providing a random phase-shift to the transmitted signal (for converting it to a wide-sense stationary process), we compute the power spectral density (PSD) in the long-term average sense [6]. The EUSD coded scheme has a different PSD than that of the uncoded, Ungerboeck-type coded or even the other phase/frequency coded schemes discussed earlier. The PSD for the EUSD coded scheme is computed assuming the following:

1. For the k-th signal sequence, the data $X_k^{(i)}, i = 1,\cdots,m$, are considered as a sequence of *i.i.d.* binary random variables which take on the values 0 and 1 with equal probability.
2. The output of the encoder $Y_k^{(j)}, j = 1,\cdots,(m+1)$, is mapped as described in Section 2.
3. The baseband pulse $p(t)$ is a full-duty cycle, unit amplitude rectangular pulse of duration T seconds.

Now, we write the n-th possible transmitted bandpass signal $S_n^{bp}(t)$ as

$$S_n^{bp}(t) = S_n^I(t)\cos\omega_c t - S_n^Q(t)\sin\omega_c t, \qquad n = 0,\cdots,2^{m+1}-1 \tag{4}$$

where $S_n^I(t)$ and $S_n^Q(t)$ are termed the in-phase and the quadrature components, respectively. The bandpass signal $S_n^{bp}(t)$ can alternatively be written as

$$\begin{aligned} S_n^{bp}(t) &= A_n\sqrt{\frac{2}{T}}\cos(\omega_c+\omega_d)t + B_n\sqrt{\frac{2}{T}}\sin(\omega_c+\omega_d)t \\ &+ C_n\sqrt{\frac{2}{T}}\cos(\omega_c-\omega_d)t + D_n\sqrt{\frac{2}{T}}\sin(\omega_c-\omega_d)t \end{aligned} \tag{5}$$

The signal amplitude levels A_n, B_n, C_n, D_n of the four sine-cosine frequency components for the eight signal points in the 2-FSK/4-PSK code format is provided in Table III. From (4) and (5), the followings can be derived:

$$S_n^I(t) = \sqrt{\frac{2}{T}}[(A_n+C_n)\cos\omega_d t + (B_n-D_n)\sin\omega_d t] \tag{6}$$

$$S_n^Q(t) = \sqrt{\frac{2}{T}}[(A_n-C_n)\sin\omega_d t - (B_n+D_n)\cos\omega_d t] \tag{7}$$

The complex low-pass equivalent signal $S_n^{lp}(t)$ can be defined as

$$S_n^{lp}(t) = \frac{1}{\sqrt{2}}[S_n^I(t) + jS_n^Q(t)], \tag{8}$$

where

$$S_n^{bp}(t) = \sqrt{2}\,\mathrm{Re}\{S_n^{lp}(t)e^{j\omega_c t}\}. \tag{9}$$

By calculating the autocorrelation function and taking its Fourier transform, the average PSD $G_{lp}(f)$, equally averaged over all the signal waveforms, becomes

$$\begin{aligned} G_{lp}(f) &= \frac{1}{T}\cdot\frac{1}{2^{m+1}}\sum_{n=0}^{2^{m+1}-1}\Big[\{A_nP(f-f_d)+C_nP(f+f_d)\}^2 \\ &+ \{B_nP(f-f_d)+D_nP(f+f_d)\}^2\Big], \end{aligned} \tag{10}$$

where $f_d = \omega_d/2\pi$ and $P(f)$, the Fourier transform of the signal $p(t)$, is $T\mathrm{sinc}(fT)$. Fig. 6 plots the power spectral density $G_{lp}(f)/T$ vs. the normalized frequency fT for the coded 2^{m+1}-PSK (h=0) and the coded 2-FSK/2^m-PSK modulation with h=0.25, 0.50, 0.75 and 1.00. Table IV summarizes the spectral efficiencies by tabulating the 99%, the 95%, the 90% and the half-power (3 dB) bandwidth. Here, the bandwidths are mentioned as the ratio of the bandwidth required for the PW scheme and the EUSD scheme to the corresponding bandwidth required for the coded 2^{m+1}-PSK scheme. In [4], the conditions for Ungerboeck's TCM codes to retain the same spectral characteristics as that for the uncoded case are provided. We assume these conditions while comparing the spectral efficiencies.

From Table IV, it may be noted that the EUSD scheme requires more bandwidth for low h values. However, from Tables I and II, we observe that the additional coding gains accrued with the low h values are also significantly more compared to those with the high h values. For example, with $h = 0.25$, the bandwidth expansion factor (relative to the PW scheme) is 1.225 (99% BW), 1.314 (95% BW), 1.434 (90% BW) and 1.114 (3 dB BW). However, at the same time, we obtain an additional asymptotic coding gain of 1.1 dB (with ν=3), 0.4 dB (with ν=4) and 1.2 dB (with ν=5) for the coded 2-FSK/4-PSK format. With h=0.5, the bandwidth requirement for the EUSD scheme is almost the same as the PW scheme, yet we obtain an additional asymptotic coding gain of 0.7 dB (with ν=3), 0.0 dB (with

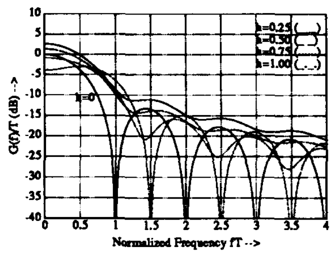

Fig. 6 Power spectral density $G_{lp}(f)/T$ vs. fT for coded 2^{m+1}-PSK modulation (h=0) and coded 2-FSK/2^m-PSK modulation with h=0.25,0.50,0.75 and 1.00

ν=4), 1.1 dB (with ν=5) for the coded 2-FSK/4-PSK format; and 1.5 dB (with ν=3), 0.8 dB (with ν=4) for the coded 2-FSK/8-PSK format. Surprisingly enough, we find that the bandwidth (in any of the above defined notions) requirement for the EUSD scheme is less than that for the PW scheme for h=0.75 while attaining the same or higher coding gain. Furthermore, with h=0.75 and the same decoder complexity, the EUSD scheme requires the same 95% bandwidth as that for the coded 2^{m+1}-PSK scheme while giving rise to substantial asymptotic coding gain as mentioned in Tables I and II. Finally, we note that the 3-dB bandwidth expansion factor is least with h=0.25 and highest with h=1.00 (among the four modulation index values considered). On the other hand, the 99% bandwidth expansion factor is least with h=1.00 and most with h=0.25.

5 Summary and Conclusions

In this article, we have considered uniform 2-FSK/2^m-PSK codes for transmitting information at m-bits/symbol. A scheme has been suggested where the signal dimensions are used equally rather than the transmission frequencies as proposed earlier in [1, 2]. In particular, codes for the data transmission at 2 and 3 bits per signaling period have been derived.

Assuming soft maximum-likelihood (Viterbi) decoding, the asymptotic coding gains for the proposed codes have been evaluated. We emphasize that in the proposed coded phase/frequency modulation (EUSD) scheme, unlike the previous schemes, the asymptotic coding gain is independent of the value of the modulation index. A tight upper bound to the bit-error-probability for transmission over the AWGN channel has been computed. The spectral characteristics of the EUSD scheme have been compared with those of the uncoded two-dimensional scheme and the PW scheme proposed in [1], studying the trade-off between the coding gain and the bandwidth expansion. Depending on the definition of the bandwidth (99%, 95%, 90% or 3 dB), the modulation index may be chosen to minimize the bandwidth expansion.

For higher values of m, the proposed scheme, denoted as 2-FSK/2^m-PSK, suffers the same limitation as uncoded 2^m-ary PSK and offers almost no improvement as compared to the Ungerboeck codes. However, extending the signal space dimensionality beyond four dimensions may be considered. For instance, 4-FSK/2^m-PSK modulation formats, defined over an eight-dimensional signal space (8-D hypercube), is expected to yield additional coding gain over the Ungerboeck codes.

S_n	$2A_n$	$2B_n$	$2C_n$	$2D_n$
S_0	$(+1-\frac{C_1}{\sqrt{D}}-\frac{C_2}{\sqrt{D}})$	$(+1+\frac{C_1}{\sqrt{D}}-\frac{C_2}{\sqrt{D}})$	$(+\frac{1}{\sqrt{D}})$	$(-\frac{1}{\sqrt{D}})$
S_1	$(+1+\frac{C_1}{\sqrt{D}}+\frac{C_2}{\sqrt{D}})$	$(+1-\frac{C_1}{\sqrt{D}}+\frac{C_2}{\sqrt{D}})$	$(-\frac{1}{\sqrt{D}})$	$(+\frac{1}{\sqrt{D}})$
S_2	$(+1+\frac{C_1}{\sqrt{D}}-\frac{C_2}{\sqrt{D}})$	$(-1+\frac{C_1}{\sqrt{D}}+\frac{C_2}{\sqrt{D}})$	$(-\frac{1}{\sqrt{D}})$	$(-\frac{1}{\sqrt{D}})$
S_3	$(+1-\frac{C_1}{\sqrt{D}}+\frac{C_2}{\sqrt{D}})$	$(-1-\frac{C_1}{\sqrt{D}}-\frac{C_2}{\sqrt{D}})$	$(+\frac{1}{\sqrt{D}})$	$(+\frac{1}{\sqrt{D}})$
S_4	$(-1+\frac{C_1}{\sqrt{D}}+\frac{C_2}{\sqrt{D}})$	$(-1-\frac{C_1}{\sqrt{D}}+\frac{C_2}{\sqrt{D}})$	$(-\frac{1}{\sqrt{D}})$	$(+\frac{1}{\sqrt{D}})$
S_5	$(-1-\frac{C_1}{\sqrt{D}}-\frac{C_2}{\sqrt{D}})$	$(-1+\frac{C_1}{\sqrt{D}}-\frac{C_2}{\sqrt{D}})$	$(+\frac{1}{\sqrt{D}})$	$(-\frac{1}{\sqrt{D}})$
S_6	$(-1-\frac{C_1}{\sqrt{D}}+\frac{C_2}{\sqrt{D}})$	$(+1-\frac{C_1}{\sqrt{D}}-\frac{C_2}{\sqrt{D}})$	$(+\frac{1}{\sqrt{D}})$	$(+\frac{1}{\sqrt{D}})$
S_7	$(-1+\frac{C_1}{\sqrt{D}}-\frac{C_2}{\sqrt{D}})$	$(+1+\frac{C_1}{\sqrt{D}}+\frac{C_2}{\sqrt{D}})$	$(-\frac{1}{\sqrt{D}})$	$(-\frac{1}{\sqrt{D}})$

TABLE III: AMPLITUDES OF DIFFERENT FREQUENCY COMPONENTS FOR 2-FSK/4-PSK SIGNAL CONSTELLATION POINTS

Mod. Index	Code Type	Bandwidth			
h		99%	95%	90%	3 dB
0.00	Uncoded	1.000	1.000	1.000	1.000
0.00	Ungerboeck's	1.000	1.000	1.000	1.000
0.25	PW scheme	0.996	1.041	1.102	1.039
0.25	EUSD scheme	1.221	1.368	1.580	1.157
0.50	PW scheme	0.985	1.091	1.187	1.206
0.50	EUSD scheme	1.001	1.091	1.186	1.204
0.75	PW scheme	0.981	1.132	1.254	1.672
0.75	EUSD scheme	0.868	1.000	1.152	1.464
1.00	PW scheme	0.980	1.163	1.345	2.264
1.00	EUSD scheme	0.980	1.163	1.345	2.264

TABLE IV: COMPARISON OF THE SPECTRAL EFFICIENCIES

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