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Optimized PAM transmission over a fading multipath channel†

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Abstract: In a mobile communication system, the intersymbol interference (ISI) due to the multipath nature of the wave propagation has a serious effect on the performance. In this work, we study the structure of an optimized time-multiplexed Pulse Amplitude Modulation (PAM) system for the transmission over such a channel. The ISI is modeled by an additive Gaussian noise. To reduce the effect of the ISI, a number zeros are transmitted between successive time multiplexed impulses. By applying Quadrature Amplitude Modulation (QAM), we obtain two dimensions with identical statistics from each baseband time impulse. A coherent M-PSK signal constellation is employed over this two-dimensional space. The duration of the time impulses and also the number of zeros transmitted between them is selected to minimize the probability of the error between the constellation points. As the transmission of zeros reduces the bandwidth efficiency, this optimization procedure is more useful for lower bit rates. The performance of this scheme is compared with a PAM system without zero transmissions. The numerical results show substantial performance improvement without any increase in the complexity.

1 Introduction

Consider a channel with a statistical impulse response. The statistical nature of the channel impulse response results in :

1. a statistical phase shift which intervenes with the orthogonality of the two phases in a QAM transmission,
2. a variable transmission gain (fading) and,
3. a variable ISI (even for a fixed sequence of impulses at the channel input).

In this work, we assume that by using coherent demodulation, the first problem is solved.

In general, a channel impulse response of length greater than one results in ISI between successive transmissions. The ISI interferes with the orthogonality of the time multiplexed impulses. In this work, to reduce the effect of the ISI, an appropriate idle time is inserted between two successive transmissions. The duration of the idle time is optimized to minimize the average probability of error. By inserting the idle time, the bandwidth efficiency decreases. Consequently, the optimization procedure is more effective when the bit rate is low.

Our numerical examples are based on a channel model proposed in [1] for the propagation of the electromagnetic wave in an indoor mobile communication system.

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2 System model

The block diagram of the system under consideration is shown in Fig. (1). The channel **C** has a statistical impulse response. The additive noise is white Gaussian with zero mean and power σ^2. The transmission is achieved by the use of the time multiplexed impulses. The duration of each impulse is equal to t_0 seconds. By applying quadrature amplitude modulation, we obtain two dimensions with identical statistics from each baseband time impulse. A coherent M-PSK signal constellation is employed over this two dimensional space. The energy per channel use is normalized to unity. The rate to be transmitted per second is equal to R_t.

To reduce the effect of the ISI, L zeros are transmitted between two successive transmissions. This results in a total of $L+1$ channel uses per each block. The rate to be transmitted per block is equal to, $R_t t_0(L+1)$. The idle time is equal to Lt_0 seconds.

The objective is to minimize the probability of error between the constellation points for a given R_t. Our tools are the selection of the parameters L and t_0.

A value of $L>0$ decreases the bandwidth efficiency. In this case, the optimization procedure tries to use the available bandwidth in the best possible way. This can be also considered as an attempt to match the power spectrum of the modulator output to the channel frequency response. By increasing the bit rate, the optimum value of the idle time, Lt_0, decreases. This means that the improvement caused by the proposed scheme is higher for lower values of the bit rate.

3 Channel model

We assume that the channel is composed of an infinite number of independent parallel subchannels. In this case, due

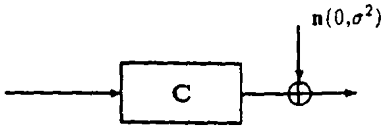

Fig. 1 System block diagram.

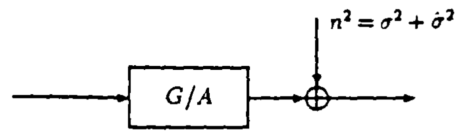

Fig. 2 The equivalent two-dimensional channel.

to the law of the large numbers, the samples of the channel impulse response have a Gaussian distribution. We also assume that the statistics of the channel is time-invariant. The channel transmission gain is denoted by C. Assuming complete phase recovery (coherent demodulation), the density function of C becomes Raleigh.

As an example, consider the channel corresponding to the propagation of the electromagnetic wave from an antenna to a receiver. We consider two kinds of reflections for the wave propagation. The reflections occurring in the immediate neighborhood of the receiver have an additive effect. Using the law of large numbers, this results in a Gaussian density for the voltage distribution.

The reflections occurring far from the receiver has a multiplicative effect on the power attenuation. We assume that the average number of such reflections is proportional to time. This results in an exponential time decay for the energy propagation. The corresponding time constant is denoted by τ.

Using these assumptions, the probability distribution of C is equal to,

$$P_C(\alpha) = \frac{2\alpha}{b} \exp\left(\frac{-\alpha^2}{b}\right), \qquad b = G/A, \tag{1}$$

where G depends on the gain of the receiver and transmitter antennas and also on the distance between the transmitter and receiver, [1], and the normalization factor A which is equal to,

$$A = \sum_{k=0}^{\infty} \exp(-kt_0/\tau) = 1/[1 - \exp(-t_0/\tau)], \tag{2}$$

is used to keep the total energy constant.

From (1), we obtain,

$$E[C] = \frac{\sqrt{\pi(G/A)}}{2}, \quad \& \quad E[C^2] = (G/A). \tag{3}$$

In our analysis, the ISI is modeled by an additive Gaussian noise of power $\hat{\sigma}^2$. The assumption of Gaussianity is justified by considering that the channel impulse response has a Gaussian distribution. Defining,

$$B = \frac{\exp[-(L+1)t_0/\tau]}{1 - \exp[-(L+1)t_0/\tau]}, \tag{4}$$

it is easy to show that $\hat{\sigma}^2 = (L+1)B$ is equal to the power of the interference. This results in a channel with a Raleigh fading of the average value G/A and an additive Gaussian noise of power $n^2 = \sigma^2 + \hat{\sigma}^2$. Figure (2) shows the block diagram of the equivalent channel.

3.1 Performance measure

For an average energy E, the minimum distance of an M-PSK is equal to,

$$d^2_{\min} = 8E \sin^2 \frac{\pi}{M}. \tag{5}$$

Assuming a Gaussian noise of power σ^2, the probability of error between two points of distance d is upperbounded by, [2],

$$P_e < (1/2)\exp(-d^2/8\sigma^2). \tag{6}$$

For an M-PSK, the decision regions are radial and consequently are insensitive to fading. In this case, assuming coherent demodulation, the probability of error is averaged over the statistics of the fading. Using these results and assuming a Raleigh fading of variance λ^2, the average error probability between nearest neighbors of an M-PSK is easily found as,

$$P_e = \int_0^{\infty} \frac{a}{\lambda^2} \exp\left[-\left(\frac{d^2_{\min}}{8\sigma^2} + \frac{1}{\lambda^2}\right)a^2\right] da, \tag{7}$$

which is equal to,

$$P_e = \frac{1}{2}\left(1 + \overline{SNR}\right)^{-1}, \tag{8}$$

where $\overline{SNR}$ is,

$$\overline{SNR} = \frac{\lambda^2 E}{\sigma^2} \sin^2 \frac{\pi}{M}. \tag{9}$$

Using a grey code, this is approximately equal to the bit error rate, *BER*.

Similarly, the outage probability is equal to,

$$p \equiv P[BER \geq \epsilon] = 1 - (2\epsilon)^{1/\overline{SNR}}. \tag{10}$$

In our case,

$$\lambda^2 = G/A = G[1 - \exp(-t_0/\tau)], \tag{11}$$

$$E = L + 1, \tag{12}$$

and,

$$M = 2^{R_0} = 2^{(L+1)R_t t_0}. \tag{13}$$

The value of L and t_0 are calculated to maximize the $\overline{SNR}$. We impose the additional constraint that the constellation rate, namely, $R_0 = (L+1)R_t t_0$ is an integer.

We can look at the optimization procedure from another point of view. For a given t_0, a larger L results in a lower $\hat{\sigma}^2$. But, at the same time, considering that the rate per two dimensional constellation is equal $R_t t_0(L+1)$, this results in higher rate per two dimensional constellation. These two phenomena have opposite effects on the error probability. The selection of L is based on providing the best compromise between these two effects.

4 Numerical results

The performance of the proposed scheme for $R_t = 1, 8$ megabits/second and are shown in Fig. (3) and (4). The effective signal to noise ratio is equal to G/σ^2. The time constant of the energy propagation is selected as $\tau = 60$ ns, [1]. The reference scheme corresponds to $L = 0$. The performance of the reference scheme is also optimized over the sampling interval (t_0). It is seen that the performance improvement, specially for lower values of R_t, is substantial.

Table (1) shows the corresponding values of the rate per two dimensional constellation, R_0. For $R_0 = 1$, the demodulation does not need to be coherent. The case $R_0 = 2$ corresponds to a biphase signaling.

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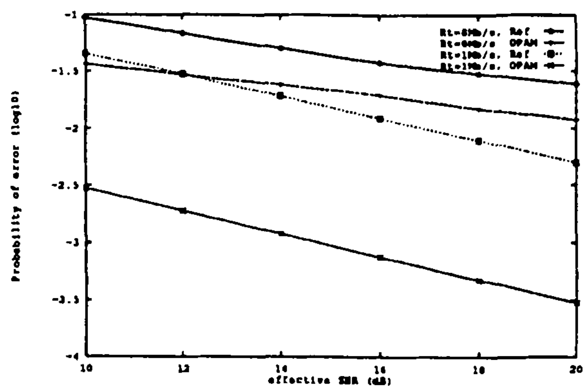

Fig. 3 Probability of error ($\log_{10}$) as a function of the effective signal to noise ratio (G/σ^2).

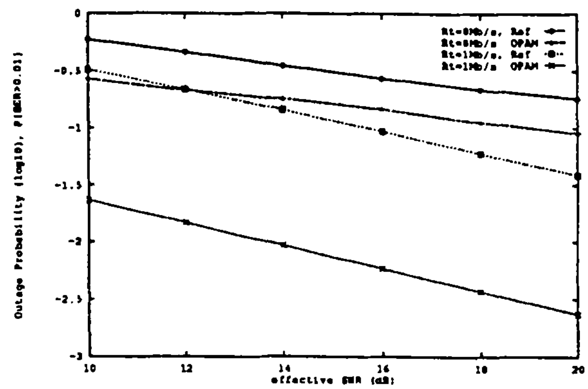

Fig. 4 Probability of outage ($\log_{10}$) for $\epsilon = 0.01$ as a function of the effective signal to noise ratio (G/σ^2).

	$R_t = 1$ Mb/s	$R_t = 8$ Mb/s
Ref	$R_0 = 1$	$R_0 = 2$
OPAM	$R_0 = 2$	$R_0 = 2$ ($G/\sigma^2 \leq 12$) $R_0 = 3$ ($G/\sigma^2 > 12$)

Table 1 The value of R_0 (rate per two dimensional constellation) for the reference scheme and the optimized PAM system (OPAM).