Bandwidth Efficient Transmultiplexers, Part 1: Synthesis

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***Abstract*—This paper develops a set of conditions which allow bandwidth efficient transmultiplexers to be synthesized. The synthesis procedure is based upon a generalized impulse response for the combining (modulating) and separation (demodulating) filters. In particular, the combining and separation filters are bandpass versions of one of two low-pass prototypes and are configured to cancel crosstalk by exploiting relationships between the center frequencies, delays, and phases in their impulse response. Based on the derived conditions, five different transmultiplexers are synthesized. Three of them implement multicarrier quadrature amplitude modulation (QAM). The other two accomplish multicarrier vestigial sideband modulation (VSB). Intersymbol interference is eliminated by appropriately designing the prototypes. The two band case is treated as a special case. For this case, the extra flexibility in choosing the center frequencies leads to the synthesis of additional transmultiplexers.**

I. Introduction

Multirate filter banks have been used in the realization of subband systems and transmultiplexers [1]-[3]. Two band systems have been realized through the use of quadrature mirror filter QMF banks [4]-[7]. For an arbitrary number of bands, modulated filter banks [8]-[11] use frequency shifted versions of a prototype filter. Also, the use of a matrix formalism [2], [3] and lossless structures [12] develop filter banks for an arbitrary number of bands.

A multi-input, multi-output transmultiplexer structure as shown in Fig. 1 is well suited for simultaneous transmission of many data signals across a single channel. In particular, it finds application in the design of multicarrier modems. At the transmitter, implicit modulation occurs in the interpolation step since the spectrum of the input signal is replicated with period $2\pi/N$. An implicit set of carrier frequencies at integer multiples of $2\pi/N$ results. The *combining* bank allocates different portions of the channel bandwidth to the various input signals by selecting a set of N center frequencies for the purposes of transmission. The outputs of the combining filters are multiplexed into one composite signal. At the receiver, the

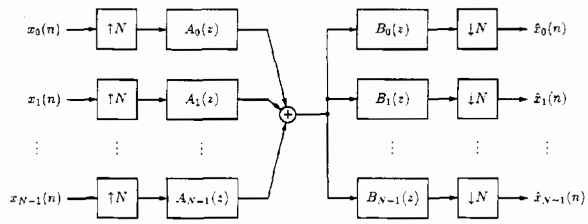

Fig. 1. A transmultiplexer system.

composite signal is passed through a parallel structure of *separation* filters whose outputs are individually decimated to yield the resultant output signals. The demodulation stage ensures that the resulting output signals depend only on their corresponding inputs. This eliminates the influence of other inputs (crosstalk). Note that the decimation and interpolation are performed synchronously at the same rate and in phase with each other.

In transmultiplexers, bandwidth efficiency is achieved by configuring the combining filters such that spectral overlap is present. This introduces crosstalk in transmultiplexers, that is, a particular output is influenced by many inputs. The separation filters are designed to cancel the crosstalk and hence, allow for the reconstruction of the inputs. Perfect reconstruction is achieved if the output signals are a scaled and delayed version of the inputs.

In this paper, five different transmultiplexers are synthesized based on a set of combining and separation filters that are formed from frequency shifted versions of a lowpass prototype (modulated filter banks). The impulse responses of the filters allow for the three free parameters—the center frequency, phase shift, and delay. Relationships among these parameters are derived such that crosstalk is canceled and an identical input-output transfer function between each pair of terminals is maintained. The transmultiplexers are synthesized based on these relationships.

Section II introduces the input-output description of transmultiplexers. The assumptions made for the present problem and the objectives of the synthesis procedure are outlined in Section III. The synthesis procedure is described in Sections IV-VI. Based on the formulated rules, five different transmultiplexers are configured in Section VII. Section VIII provides an interpretation of each of the systems from a communications point of view. The two band case is considered in Section IX.

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II. Transmultiplexers: Input–Output Descriptions

The input–output descriptions of transmultiplexers are examined in the context of critical sampling in which the sampling rate change N is equal to the number of frequency bands. An N band transmultiplexer as depicted in Fig. 1 generates input–output relations given by

$$\hat{X}_i(z) = \frac{1}{N}\sum_{k=0}^{N-1} X_k(z) \sum_{l=0}^{N-1} A_k(z^{1/N}W^{-l})B_i(z^{1/N}W^{-l})$$
$$\text{for } 0 \le i \le N-1 \quad (1)$$

or equivalently,

$$\hat{X}_i(z^N) = \frac{1}{N}\sum_{k=0}^{N-1} X_k(z^N) \sum_{l=0}^{N-1} A_k(zW^{-l})B_i(zW^{-l})$$
$$0 \le i \le N-1 \quad (2)$$

where $W = e^{-j(2\pi/N)}$. Each output signal $\hat{X}_i(z^N)$ is related to each input signal $X_k(z^N)$ via a transfer function $(1/N)T_{ki}(z^N)$ where $T_{ki}(z^N) = \Sigma_{l=0}^{N-1} A_k(zW^{-l})B_i(zW^{-l})$. When $k \neq i$, $T_{ki}(z^N)$ is called a crosstalk function and represents the contribution of the undesired input $X_k(z^N)$ to the output $\hat{X}_i(z^N)$. Crosstalk is eliminated when each output signal $\hat{X}_i(z)$ only depends on its corresponding input $X_i(z)$ and is not influenced by other input signals. In this case, $T_{ki}(z^N) = 0$ for $k \neq i$. Furthermore, if each of the input–output transfer functions $T_{ii}(z^N) = T(z^N)$, the output signals are given by $\hat{X}_i(z) = (1/N)T(z)X_i(z)$. Intersymbol interference is present if the samples at the output depend on past and future input samples. Intersymbol interference is eliminated if and only if $T(z)$ is of the form cz^{-p}. Then, perfect reconstruction is achieved in that the output samples are a scaled and delayed version of the input samples.

III. Filter Specification and Basic Assumptions

The investigation concentrates on modulated filter banks in a transmultiplexer. The main purpose is to find alternative configurations of modulated filter banks to those already described in the literature. This goal is achieved through the formulation of a synthesis procedure. The synthesis procedure allows for a systematic development in finding modulated filter banks. First, note that all the filters are modulated and delayed versions of one band-limited low-pass prototype $h(n)$. A filter $H(z)$ is a band-limited low-pass prototype if $H(e^{j\omega})$ is exactly equal to zero in the stopband region $\omega_s \le \omega \le \pi$. The filter banks consist of a set of bandpass filters that serve to allocate different portions of the channel bandwidth to the different inputs. The impulse responses of the combining filters $A_k(z)$ and the separation filters $B_k(z)$ are parameterized by a center frequency (ω_k), phase factor (α_k or β_k) and delay (n_k or p_k). Their impulse responses are given by

$$a_k(n) = h(n - n_k)\cos[\omega_k(n - n_k) + \alpha_k] \quad (3)$$

and

$$b_k(n) = h(n + p_k)\cos[\omega_k(n + p_k) + \beta_k] \quad (4)$$

respectively. In the z-transform domain, $A_k(z)$ and $B_k(z)$ are given by

$$A_k(z) = \tfrac{1}{2}z^{-n_k}[e^{j\alpha_k}H(e^{-j\omega_k}z) + e^{-j\alpha_k}H(e^{j\omega_k}z)] \quad (5)$$

and

$$B_k(z) = \tfrac{1}{2}z^{p_k}[e^{j\beta_k}H(e^{-j\omega_k}z) + e^{-j\beta_k}H(e^{j\omega_k}z)]. \quad (6)$$

The filter characterization provides an extra free parameter, namely, a delay factor in describing the impulse responses of the bandpass filters as compared to existing systems that only allow for a center frequency and phase factor.

We further assume that the center frequencies ω_k are equally spaced and lie between 0 and π (inclusive). In addition, two types of systems are considered. In one type, all the center frequencies are distinct. In the other case, center frequencies are repeated (with the exception of 0 and π) in that the same frequency is used for two bands. The idea of permitting center frequencies to repeat allows for two signals to be sent at the same frequency as compared to existing schemes in which all the center frequencies are distinct.

The synthesis procedure involves the following steps.

1) The bandwidth of the low-pass prototype is determined such that 1) spectral overlap occurs only between filters centered at adjacent center frequencies and at the same center frequency, and 2) the set of bandpass filters fill up the entire frequency range (0 to π).

2) Relationships among the three free parameters (center frequencies, phase factors, and delays) are derived such that the resulting transmultiplexers have the following properties.

a) The input-output transfer function is the same for every pair of corresponding terminals.

b) The crosstalk components in the output data signal that arise from other data signals due to the sharing of bandwidth are eliminated.

In step 1, we determine the stopband edge ω_s (thereby determining the bandwidth of the band-limited low-pass prototype) for the purposes of restricting spectral overlap and allowing for full bandwidth utilization. Step 2 consists of two parts each devoted to forming relationships among the center frequencies, phase factors and delays. First, the transfer function between each pair of corresponding terminals is made to be the same. In step 2b), the crosstalk components due to spectral overlap are canceled. The crosstalk between signals that do not share any bandwidth is zero for band-limited filters.

IV. Bandwidth Constraints

The first step in the synthesis procedure is to impose a bandwidth constraint on the low-pass prototype. Consider the type of system in which all the center frequencies are distinct. The bandwidth of $h(n)$ (stopband response is exactly zero) is selected such that spectral overlap exists only between filters centered at adjacent center frequencies. In addition, the entire range 0 to π is utilized. Given $h(n)$, there are N bandpass filter responses centered at different frequencies and having the same bandwidth. The minimum bandwidth of the N bandpass filters such that their frequency responses are mutually exclusive (no spectral overlap), an equal bandwidth is allocated to each input and the full frequency range 0 to π is covered is π/N. Moreover, the center frequencies are odd multiples of $\pi/2N$. This translates to a minimum bandwidth of $\pi/2N$ for $h(n)$. Spectral overlap is restricted to bandpass filters centered at adjacent frequencies by allowing the low-pass prototype to have a bandwidth of no more than 100% in excess of its minimum bandwidth. The stopband of $h(n)$ extends from ω_s to π where $\pi/2N \leq \omega_s \leq \pi/N$.

Now, consider the type of system in which the center frequencies repeat. Two signals are transmitted at every repeating center frequency (0 and π excluded). The minimum bandwidth of the bandpass filters which allows for filters centered at different frequencies to have mutually exclusive frequency responses is $2\pi/N$. This translates to a minimum bandwidth of π/N for $h(n)$. Moreover, there are two possible sets of center frequencies. In one set, two of the center frequencies are 0 and π and the other repeating frequencies are integer multiples of $2\pi/N$. Another possibility is to have all the frequencies repeat and be odd multiples of π/N.[1] The idea is to allow for spectral overlap only between filters centered at the same frequency and at adjacent frequencies. For both sets of center frequencies, this is possible if the low-pass prototype $h(n)$ is band limited to no more than 100% over the minimum bandwidth. The stopband of $h(n)$ extends from ω_s to π where $\pi/N \leq \omega_s \leq 2\pi/N$.

The bandwidth constraint is different for repeated and distinct center frequencies. Given the above constraints on ω_s, the development of the synthesis procedure evolves by assuming that the low-pass prototype $h(n)$ has a stopband response that is exactly zero (band-limited prototype). The general frequency characteristic of the low-pass prototype with a tapered transition band is shown in Fig. 2 for the two types of systems.

We have established three sets of equally spaced center frequencies. For the case of repeated center frequencies, the two sets are

$$\text{Set 1: } 0, \frac{2\pi}{N}, \frac{2\pi}{N}, \frac{4\pi}{N}, \frac{4\pi}{N}, \cdots, \pi - \frac{2\pi}{N}, \pi - \frac{2\pi}{N}, \pi$$

and

$$\text{Set 2: } \frac{\pi}{N}, \frac{\pi}{N}, \frac{3\pi}{N}, \frac{3\pi}{N}, \cdots, \pi - \frac{\pi}{N}, \pi - \frac{\pi}{N}.$$

[1]We have implicitly considered the case when N is even. When N is odd, one of the center frequencies is 0 or π with the remaining center frequencies repeating. The spacing between adjacent frequencies is $2\pi/N$. The minimum bandwidth of the filters is the same as for N even.

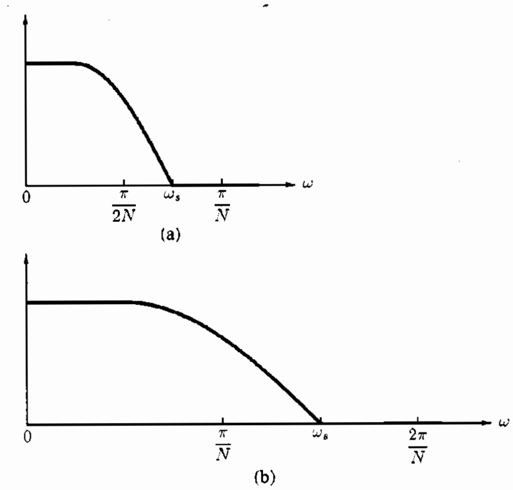

Fig. 2. General frequency characteristics of a band-limited low-pass prototype. (a) the case when all center frequencies are distinct. (b) The case of repeated center frequencies.

Both sets 1 and 2 ensure complete bandwidth utilization (frequency range 0 to π is covered) given a low-pass prototype with a stopband edge frequency $\omega_s \geq \pi/N$. Also, spectral overlap is restricted to filters centered at the same frequency and at adjacent center frequencies if $\omega_s \leq 2\pi/N$. Note that for sets 1 and 2, it is assumed that N is even. Later, we will see that this is necessary for realizing integral delay factors.

The set of N distinct equally spaced center frequencies is given by

$$\text{Set 3: } \frac{\pi}{2N}, \frac{3\pi}{2N}, \frac{5\pi}{2N}, \frac{7\pi}{2N}, \cdots, \pi - \frac{\pi}{2N}.$$

The center frequencies of set 3 are the same as those in [9]. Complete bandwidth utilization is achieved given a low-pass prototype with a stopband edge $\omega_s \geq \pi/2N$. Also, spectral overlap is restricted to bandpass filters centered at adjacent frequencies if $\omega_s \leq \pi/N$.

V. Input–Output Transfer Function

The next step is to make the input–output transfer function the same for every pair of corresponding terminals. The kth input–output terminal pair has a transfer function given by

$$\begin{aligned} T_{kk}(z^N) &= \sum_{i=0}^{N-1} A_k(zW^{-i})B_k(zW^{-i}) \\ &= \tfrac{1}{4} z^{-(n_k - p_k)} \sum_{i=0}^{N-1} W^{i(n_k - p_k)} \\ &\quad \cdot [\exp[j(\alpha_k + \beta_k)] H^2(e^{-j\omega_k} zW^{-i}) \\ &\quad + \exp[-j(\alpha_k + \beta_k)] H^2(e^{j\omega_k} zW^{-i}) \\ &\quad + 2\cos(\alpha_k - \beta_k) H(e^{-j\omega_k} zW^{-i}) H(e^{j\omega_k} zW^{-i})]. \end{aligned} \tag{7}$$

The strategy will be to try to make the transfer function

$T_{kk}(z^N)$ independent of k. To this end, it is assumed that $n_k - p_k = s$ for every k. The expression for the input-output transfer function consists of three terms. Note that the last term in (7) will be zero for center frequencies sufficiently away from 0 and π (the spectra in the $H(\cdot)$ terms do not overlap). Specifically, this will be true for $\omega_b \le \omega_k \le \pi - \omega_b$ where ω_b is the maximum allowable value of ω_s (π/N for distinct center frequencies and $2\pi/N$ for repeated center frequencies, see Fig. 2). For the center frequencies near 0 or π, choosing $\alpha_k - \beta_k$ to be an odd multiple of $\pi/2$ will suffice to set the last term to zero. We now formulate two sets of conditions for identical input-output transfer functions.

A. Difference Criterion

For the difference criterion, the difference between any two center frequencies is constrained to be an integer multiple of $2\pi/N$. We first note that the frequency response of $T_{kk}(z^N)$ is periodic in $2\pi/N$. Equation (7) remains unchanged if, in its first two terms, ω_k is replaced by $\omega_l = \omega_k + 2m\pi/N$ (where m is an integer) and $n_k - p_k = s$ is an integer multiple of N (recall that the last term is zero from the preceding discussion). Then, the same transfer functions at terminals k and l are achieved by adhering to the following set of rules.

1) If a particular ω_k does not satisfy the inequality $\omega_b \le \omega_k \le \pi - \omega_b$, then $\alpha_k - \beta_k$ must be an odd multiple of $\pi/2$. The same restriction holds for terminal l.

2) The phases are such that $\alpha_k + \beta_k = \alpha_l + \beta_l$.

3) The delay factors are such that $n_k - p_k = n_l - p_l$. Moreover, both $n_k - p_k$ and $n_l - p_l$ are integer multiples of N.

The above rules generate a reduced form of $T_{kk}(z^N) = T_{ll}(z^N)$ as given by

$$T_{kk}(z^N) = \tfrac{1}{4} z^{-(n_k-p_k)} \sum_{i=0}^{N-1} [\exp[j(\alpha_k+\beta_k)] H^2(e^{-j\omega_k} zW^{-i}) + \exp[-j(\alpha_k+\beta_k)] H^2(e^{j\omega_k} zW^{-i})]. \quad (8)$$

B. Sum Criterion

It can be shown that if we constrain the sum of the center frequencies $\omega_k + \omega_l = 2m\pi/N$ (where m is an integer), another set of rules for which $T_{kk}(z^N) = T_{ll}(z^N)$ emerges as follows.

1) If a particular ω_k does not satisfy the inequality $\omega_b \le \omega_k \le \pi - \omega_b$, then $\alpha_k - \beta_k$ must be an odd multiple of $\pi/2$. The same restriction holds for terminal l.

2) The phases are such that $\alpha_k + \omega_k = -(\alpha_l + \beta_l)$.

3) The delay factors are such that $n_k - p_k = n_l - p_l$. Moreover, both $n_k - p_k$ and $n_l - p_l$ are integer multiples of N.

This generates a reduced form for the input–output transfer function as above.

C. Center Frequencies

The center frequencies of sets 1 and 2 satisfy both the difference and sum criteria. In fact, the conditions for the two criteria are equivalent for the frequencies of sets 1 and 2. Any two center frequencies of set 3 satisfy either the difference or the sum criterion. At this stage, we constrain $\alpha_k + \beta_k$ to be an integer multiple of π for sets 1–3. Appendix A elaborates on this aspect and justifies this choice. For the end center frequencies (those that do not satisfy the inequality $\omega_b \le \omega_k \le \pi - \omega_b$), the phase difference $\alpha_k - \beta_k$ is constrained to be an odd multiple of $\pi/2$. Combining this with the constraint on $\alpha_k + \beta_k$ gives the condition that the phases α_k and β_k are of the form $(2r + 1)\pi/4$, where r is an integer, for the end frequencies. The end frequencies are 0 and π for set 1, π/N and $\pi - \pi/N$ for set 2, and $\pi/2N$ and $\pi - \pi/2N$ for set 3.

VI. Analysis of Crosstalk

This section analyzes the crosstalk functions for signals sent at adjacent center frequencies and for signals sent at the same center frequency. We will adhere to the restrictions generated in Section V for the input-output transfer function and formulate additional conditions for canceling crosstalk. To start, we express the general crosstalk function for signals transmitted at two center frequencies ω_k and ω_l as

$$\begin{aligned} T_{kl}(z^N) &= \sum_{i=0}^{N-1} A_k(zW^{-i}) B_l(zW^{-i}) \\ &= \tfrac{1}{4} z^{-(n_k-p_l)} \sum_{i=0}^{N-1} W^{i(n_k-p_l)} [\exp[j(\alpha_k+\beta_l)] \\ &\quad \cdot H(e^{-j\omega_k} zW^{-i}) H(e^{-j\omega_l} zW^{-i}) \\ &\quad + \exp[-j(\alpha_k+\beta_l)] H(e^{j\omega_k} zW^{-i}) H(e^{j\omega_l} zW^{-i}) \\ &\quad + \exp[j(\alpha_k-\beta_l)] H(e^{-j\omega_k} zW^{-i}) H(e^{j\omega_l} zW^{-i}) \\ &\quad + \exp[-j(\alpha_k-\beta_l)] H(e^{j\omega_k} zW^{-i}) \\ &\quad \cdot H(e^{-j\omega_l} zW^{-i})]. \end{aligned} \quad (9)$$

The *crosstalk function* $T_{kl}(z^N)$ represents the contribution of the input $X_k(z^N)$ (transmitted at ω_k) to the output $\hat{X}_l(z^N)$. In the sequel, the four terms of which $T_{kl}(z^N)$ is comprised of are referred to as *crosstalk terms*.

A. Crosstalk: Different Center Frequencies of Sets 1 and 2

Consider the case of center frequencies belonging to sets 1 and 2. These frequencies are integer multiples of π/N. For now, it is assumed that the different positive frequencies ω_k and ω_l are in the closed interval $[2\pi/N, \pi - 2\pi/N]$. Two adjacent center frequencies ω_k and ω_l are related by $\omega_l - \omega_k = 2m\pi/N$ where $m = \pm 1$. Given two adjacent frequencies, the last two crosstalk terms of (9) are zero due to the band limitedness of $H(z)$. By substituting $\omega_l - \omega_k = 2m\pi/N$ ($m = \pm 1$) in the first two terms of (9), noting that $e^{j\omega_k} = W^p$ where p is an integer multiple of $1/2$ and performing algebraic manipulation to give identical crossterms in $H(\cdot)$, we get a simplified expression for the crosstalk function as

$$T_{kl}(z^N) = \frac{1}{4} z^{-(n_k - p_l)} \sum_{i=0}^{N-1} [[W^{(m-2p)(n_k - p_l)} \cdot \exp[j(\alpha_k + \beta_l)] + \exp[-j(\alpha_k + \beta_l)]] \cdot W^{i(n_k - p_l)} H(zW^{-i+p}) H(zW^{-i-m+p})]. \quad (10)$$

From (10), we develop a general rule relating the phases, delays, m and p as given by (discussion in Appendix B)

$$\alpha_k + \beta_l = \pi \left[\frac{(m - 2p)(n_k - p_l)}{N} + \frac{1}{2} \right]. \quad (11)$$

Since $m = \pm 1$, we have considered crosstalk due to spectral overlap between signals transmitted at any two adjacent center frequencies in the closed interval $[2\pi/N, \pi - 2\pi/N]$. Then, (11) becomes

$$\alpha_k + \beta_l = \pi \left[\frac{(\pm 1 - 2p)(n_k - p_l)}{N} + \frac{1}{2} \right]. \quad (12)$$

1) Set 1: In set 1, p is an even multiple of $1/2$ (center frequencies are even multiples of π/N). Two solutions to (12) are given below.

a) Solution one: 1) The delays are such that $n_k - p_l$ is an integer multiple of N. 2) The phases are such that $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$.

b) Solution two: 1) The delays are such that $m_k - p_l$ is an odd multiple of $N/2$. 2) The phases are such that $\alpha_k + \beta_l$ is an integer multiple of π.

The only remaining crosstalk due to spectral overlap occurs between the center frequencies $\omega_k = 0$ and $\omega_l = 2\pi/N$. Retaining the restriction on α_k and β_k for the end center frequencies, two ways of eliminating crosstalk are as follows.

1) The delays are such that $n_k - p_l$ and $n_l - p_k$ are integer multiples of N. The phases α_k and β_k are either $\pm\pi/4$ or $\pm 3\pi/4$. The phases α_l and β_l are odd multiples of $\pi/2$.

2) The delays are such that $n_k - p_l$ and $n_l - p_k$ are odd multiples of $N/2$. The phases α_k and β_k are either $\pm\pi/4$ or $\pm 3\pi/4$. The phases α_l and β_l are integer multiples of π.

The same techniques result in canceling crosstalk between signals sent at the other center frequencies of $\pi - 2\pi/N$ and π.

The preceding analysis generates two solutions. Two different solutions are needed since two signals are sent with the same center frequency.

2) Set 2: For set 2, p is an odd multiple of $1/2$ (center frequencies are odd multiples of π/N). A solution to (12) is given below.

a) Solution: 1) The delays are such that $n_k - p_l$ is an integer multiple of $N/2$. 2) The phases are such that $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$.

For the end center frequency $\omega_k = \pi/N$, spectral overlap occurs with $\omega_l = 3\pi/N$. By substituting these frequencies in (9), it is found that the elimination of crosstalk is feasible if both of the conditions below are satisfied.

1) The delays are such that $n_k - p_l$ and $n_l - p_k$ are integer multiples of $N/2$.

2) The phases are such that (α_k, β_l) and (β_k, α_l) are $(\pi/4, \pi/4 \pm l\pi)$, $(-\pi/4, -\pi/4 \pm l\pi)$, $(3\pi/4, 3\pi/4 \pm l\pi)$ or $(-3\pi/4, -3\pi/4 \pm l\pi)$.

The same conditions result for canceling the crosstalk between signals sent at a center frequency of $\pi - 3\pi/N$ and the other end frequency $\pi - \pi/N$.

Although the preceding analysis generates only one solution, there are in fact two embedded solutions that arise by making the delay factor an odd or even multiple of $N/2$.

B. Crosstalk: Repeated Center Frequencies

Here, we examine the crosstalk function associated with two signals transmitted with the same center frequency. We return to the original expression for the crosstalk function as in (9) and let ω_l be equal to ω_k to get

$$T_{kl}(z^N) = \frac{1}{4} z^{-(n_k - p_l)} \sum_{i=0}^{N-1} W^{i(n_k - p_l)} \cdot [\exp[j(\alpha_k + \beta_l)] H^2(e^{-j\omega_k} zW^{-i}) + \exp[-j(\alpha_k + \beta_l)] H^2(e^{j\omega_k} zW^{-i}) + 2\cos(\alpha_k - \beta_l) H(e^{j\omega_k} zW^{-i}) H(e^{-j\omega_k} zW^{-i})]. \quad (13)$$

In this specific case, the crosstalk function $T_{kl}(z^N)$ is comprised of three crosstalk terms. For $2\pi/N \leq \omega_k \leq \pi - 2\pi/N$, the third crosstalk term in the above equation is zero due to the band limitedness of $H(z)$. The crosstalk function is reduced to

$$T_{kl}(z^N) = \frac{1}{4} z^{-(n_k - p_l)} \sum_{i=0}^{N-1} W^{i(n_k - p_l)} \cdot [\exp[j(\alpha_k + \beta_l)] H^2(e^{-j\omega_k} zW^{-i}) + \exp[-j(\alpha_k + \beta_l)] H^2(e^{j\omega_k} zW^{-i})]. \quad (14)$$

We have many degrees of freedom with which to force a zero crosstalk function. To maintain compatibility with the solutions formulated earlier, we restrict the differences in the delays to be integer multiples of $N/2$ and the sum of the phases to be integer multiples of $\pi/2$. Otherwise, we would admit the possibility of deriving conditions which when united with the solutions in Section V and VI-A become contradictory in that no combination of the parameters would satisfy the entire set. Given the delays and phases as above, the analysis procedure investigates the question of which center frequencies can be utilized for transmitting more than one signal. The details are laid out in Appendix C. Given the derivations in Appendix C, we have the following restrictions on the center frequencies.

1) If $n_k - p_l$ is an integer multiple of N and $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$, the center frequency must be an integer multiple of π/N.

2) If $n_k - p_l$ is an odd integer of $N/2$ and $\alpha_k + \beta_l$ is

an integer multiple of π, the center frequency must be an odd multiple of π/N.

3) If $n_k - p_l$ is an odd multiple of $N/2$ and $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$, the center frequency must be an even multiple of π/N.

The crosstalk cannot be made zero if $n_k - p_l$ is an integer multiple of N and $\alpha_k + \beta_l$ is an integer multiple of π.

It was initially established that the repeated center frequencies are integer multiples of π/N. Here, we have an additional result that fixes these frequencies. It has been shown that with appropriate limitations on the delays and phases, the repeated center frequencies must be integer multiples of π/N to ensure zero crosstalk.

The preceding analysis is specifically devoted to the center frequencies is the closed interval $[2\pi/N, \pi - 2\pi/N]$. The remaining case is to consider the end center frequency π/N in set 2. Two signals can be transmitted at this frequency without crosstalk subject to both of the following conditions. 1) The delays are such that $n_k - p_l$ is an odd multiple of $N/2$. 2) The phases are such that $(\alpha_k, \beta_l) = (\pi/4, -\pi/4), (-\pi/4, \pi/4), (3\pi/4, -3\pi/4)$ or $(-3\pi/4, 3\pi/4)$.

The same conditions hold for the other end frequency of $\pi - \pi/N$ in set 2.

C. Distinct Center Frequencies of Set 3

Now, we consider the distinct center frequencies of set 3. Crosstalk due to spectral overlap occurs only between two signals transmitted at adjacent center frequencies. In set 3, let two adjacent center frequencies be given by $\omega_k = (2r + 1)\pi/2N$ and $\omega_l = (2r + 3)\pi/2N$ for $r = 0, 1, \cdots, N - 2$. By substituting these frequencies in (9), invoking the band-limitedness assumptions for $H(z)$ and performing algebraic manipulation gives a relationship similar to (12) as

$$\alpha_k + \beta_l = \pi\left[\frac{(r+1)(n_k - p_l)}{N} + \frac{1}{2}\right]. \tag{15}$$

Note that the same relationship holds between $\alpha_l + \beta_k$ and $n_l - p_k$. Two solutions to (15) lead to two different transmultiplexers.

1) Solution One: 1) The delays are such that $n_k - p_l$ and $n_l - p_k$ are integer multiples of N. 2) The phases are such that $\alpha_k + \beta_l$ and $\alpha_l + \beta_k$ are odd multiples of $\pi/2$.

2) Solution Two: 1) The delays are such that $n_k - p_l$ and $n_l - p_k$ are odd multiples of $N/2$. 2) If r is odd, $\alpha_k + \beta_l$ and $\alpha_l + \beta_k$ are odd multiples of $\pi/2$. If r is even, $\alpha_k + \beta_l$ and $\alpha_l + \beta_k$ are integer multiples of π.

VII. Synthesized Transmultiplexers

Given the above guidelines, we establish values for the free parameters and synthesize five different transmultiplexers. The first three use repeated center frequencies (set 1 or 2). The other two use the distinct frequencies of set 3. In four of the five systems, it is necessary to implement delays which are odd multiples of $N/2$. For these cases, the parameter N is constrained to be even. Note that Appendix D gives general expressions for $a_k(n)$ and $b_k(n)$.

A. System T1

In the first system T1, we use center frequencies in set 1. The combining and separation filters corresponding to the end frequencies $\omega_0 = 0$ are associated with parameters $n_0 = p_0 = 0$ and $\alpha_0 = -\beta_0 = \pi/4$. The next center frequency, $\omega_1 = \omega_2 = 2\pi/N$ is used to transmit two signals. Crosstalk is eliminated between these two signals and with the signal sent at zero frequency by setting $n_1 = p_1 = N/2$, $\alpha_1 = -\beta_1 = \pi$, $n_2 = p_2 = 0$, and $\alpha_2 = -\beta_2 = -\pi/2$. Now, we proceed to the frequency $\omega_3 = \omega_4 = 4\pi/N$. To cancel crosstalk between signals sent at $2\pi/N$ and $4\pi/N$, we set $n_3 = p_3 = 0$, $\alpha_3 = -\beta_3 = 0$, $n_4 = p_4 = N/2$ and $\alpha_4 = -\beta_4 = 3\pi/2$. These parameters eliminate crosstalk between the two signals sent at $4\pi/N$ due to the compatibility in the rules formed for canceling crosstalk due to spectral overlap between adjacent and repeated frequencies. We continue this procedure in a sequential fashion for each center frequency. This establishes the combining and separation filters of T1 as follows:

$$\begin{aligned}
a_0(n) &= h(n)\cos\frac{\pi}{4} & b_0(n) &= h(n)\cos\frac{\pi}{4}\\
a_1(n) &= h\left(n - \frac{N}{2}\right)\cos\frac{2\pi}{N}n & b_1(n) &= h\left(n + \frac{N}{2}\right)\cos\frac{2\pi}{N}n\\
a_2(n) &= h(n)\sin\frac{2\pi}{N}n & b_2(n) &= -h(n)\sin\frac{2\pi}{N}n\\
a_3(n) &= h(n)\cos\frac{4\pi}{N}n & b_3(n) &= h(n)\cos\frac{4\pi}{N}n\\
a_4(n) &= h\left(n - \frac{N}{2}\right)\sin\frac{4\pi}{N}n & b_4(n) &= -h\left(n + \frac{N}{2}\right)\sin\frac{4\pi}{N}n.\\
&\vdots\quad\vdots & &\vdots\quad\vdots
\end{aligned} \tag{16}$$

It is noted that for T1, the delay elements of $N/2$ alternate between the cosine and sine carriers and that the separation filters associated with the sine carriers have a minus sign associated with $h(n)$. It is also observed that a delay element $N/2$ is associated with a center frequency of π only if $N = 2, 6, 10, \cdots$. The input–output transfer function for any pair of corresponding terminals is

$$T(z^N) = \frac{1}{2}\sum_{i=0}^{N-1} H^2(zW^{-i})$$
$$= \frac{N}{2}[\cdots + v(-2N)z^{2N} + v(-N)z^N + v(0) + v(N)z^{-N} + v(2N)z^{-2N} + \cdots] \tag{17}$$

where $v(n)$ is the inverse z transform of $H^2(z)$.

B. System T2

In the second system T2, we use center frequencies in set 2. The combining and separation filters for the first signal sent with the end center frequency π/N have parameters $n_0 = p_0 = 0$, $\alpha_0 = -\beta_0 = \pi/4$, $n_1 = p_1 = N/2$, and $\alpha_1 = -\beta_1 = \pi/4$. For a frequency of $3\pi/N$, crosstalk due to spectral overlap with π/N is canceled by setting $n_2 = p_2 = 0$, $\alpha_2 = -\beta_2 = -\pi/4$, $n_3 = p_3 = N/2$, and $\alpha_3 = -\beta_3 = 7\pi/4$. We observe that these parameters ensure no crosstalk between the signals sent at $3\pi/N$. For the next frequency $5\pi/N$, crosstalk due to spectral overlap with $3\pi/N$ is canceled by invoking the solution derived in Section VI-A. Again, these parameters eliminate crosstalk arising from frequency repetition due to the compatibility of the derived conditions. This process continues in a sequential fashion. This establishes the combining and separation filters of T2 as follows:

$$\begin{aligned}
a_0(n) &= h(n)\cos\left(\frac{\pi}{N}n + \frac{\pi}{4}\right) & b_0(n) &= h(n)\cos\left(\frac{\pi}{N}n - \frac{\pi}{4}\right)\\
a_1(n) &= h\left(n - \frac{N}{2}\right)\cos\left(\frac{\pi}{N}n - \frac{\pi}{4}\right) & b_1(n) &= h\left(n + \frac{N}{2}\right)\cos\left(\frac{\pi}{N}n + \frac{\pi}{4}\right)\\
a_2(n) &= h(n)\cos\left(\frac{3\pi}{N}n - \frac{\pi}{4}\right) & b_2(n) &= h(n)\cos\left(\frac{3\pi}{N}n + \frac{\pi}{4}\right)\\
a_3(n) &= h\left(n - \frac{N}{2}\right)\cos\left(\frac{3\pi}{N}n + \frac{\pi}{4}\right) & b_3(n) &= h\left(n + \frac{N}{2}\right)\cos\left(\frac{3\pi}{N}n - \frac{\pi}{4}\right).\\
&\vdots \quad \vdots & &\vdots \quad \vdots
\end{aligned} \tag{18}$$

The delay element of $N/2$ alternates between the cosine carriers having a resultant phase of $\pi/4$ and $-\pi/4$. When no delay element is present, the resultant phase of the cosine carriers alternates between $\pi/4$ and $-\pi/4$. For the last center frequency $\pi - \pi/N$, the delay at the transmitter is associated with a resultant phase of $\pi/4$ only if $N = 4, 8, 12, \cdots$. Otherwise, it is associated with a resultant phase of $-\pi/4$. The input–output transfer function for any pair of corresponding terminals is

$$T(z^N) = \frac{1}{2}\sum_{i=0}^{N-1} H^2(zW^{-i+1/2})$$
$$= \frac{N}{2}[\cdots + v(-2N)z^{2N} - v(-N)z^N + v(0) - v(N)z^{-N} + v(2N)z^{2N} - \cdots]. \tag{19}$$

C. System T3

A third transmultiplexer is synthesized by relaxing the assumption of using only a single low-pass prototype. System T3 uses two low-pass prototypes $h(n)$ and $g(n)$ which are each band limited to no less than π/N and no more than $2\pi/N$. Each of the combining and separation filters are modulated and delayed versions of one of the low-pass prototypes just as in (3) and (4).

Suppose T1 is modified to include two prototypes by alternating the positions of $h(n)$ and $g(n)$ between the combining and separation filters for each center frequency. This leads to a new transmultiplexer T3 described as follows:

$$\begin{aligned}
a_0(n) &= h(n)\cos\frac{\pi}{4} & b_0(n) &= g(n)\cos\frac{\pi}{4}\\
a_1(n) &= g\left(n - \frac{N}{2}\right)\cos\frac{2\pi}{N}n & b_1(n) &= h\left(n + \frac{N}{2}\right)\cos\frac{2\pi}{N}n\\
a_2(n) &= g(n)\sin\frac{2\pi}{N}n & b_2(n) &= -h(n)\sin\frac{2\pi}{N}n\\
a_3(n) &= h(n)\cos\frac{4\pi}{N}n & b_3(n) &= g(n)\cos\frac{4\pi}{N}n
\end{aligned} \tag{20}$$

$$a_4(n) = h\left(n - \frac{N}{2}\right) \sin \frac{4\pi}{N} n \qquad b_4(n) = -g\left(n + \frac{N}{2}\right) \sin \frac{4\pi}{N} n. \tag{20}$$

$$\vdots \quad \vdots \qquad\qquad \vdots \quad \vdots$$

The crosstalk between two signals sent at adjacent frequencies is eliminated as in T1. Moreover, it can be shown that the crosstalk between two signals transmitted at the same center frequency ω_k where $2\pi/N \leq \omega_k \leq \pi - 2\pi/N$ is eliminated by the same approaches as derived in Section VI-B even when two prototypes are used. Therefore, system T3 is crosstalk-free. The input–output transfer function for any pair of corresponding terminals is

$$\begin{aligned} T(z^N) &= \frac{1}{2} \sum_{i=0}^{N-1} H(zW^{-i}) G(zW^{-i}) \\ &= \frac{N}{2} [\cdots + w(-2N)z^{2N} + w(-N)z^N + w(0) \\ &\quad + w(N)z^{-N} + w(2N)z^{-2N} + \cdots]. \end{aligned} \tag{21}$$

where $w(n)$ is the inverse z transform of $H(z)\,G(z)$.

Consider modifying T2 to include two prototypes. Again, we alterate the positions of $h(n)$ and $g(n)$ between the combining and separation filters for each center frequency. In this case, the crosstalk between two signals sent at an end center frequency (π/N or $(N-1)\pi/N$) is not canceled with two prototypes. Therefore, T2 cannot be modified to include two prototypes.

Note that there are other ways of modifying the transmultiplexers to allow for two prototypes. However, any other arrangement leads to a crosstalk function $T_{kl}(z^N)$ for two signals sent at adjacent frequencies to be expressed in terms of $H(z)$ and $G(z)$. Then, the crosstalk terms in $T_{kl}(z^N)$ that involve spectral overlap cannot be canceled. To conclude, we observe that only T1 can be modified to allow for two prototypes. Moreover, the modification must be performed in the unique way described above.

D. System T4

The center frequencies of set 3 are used to synthesize system T4. A low-pass prototype with a maximum bandwidth of π/N is used. One of the solutions formulated in Section VI-C in invoked to establish the parameters. All the delay factors n_k and p_k equal zero. The phases are set such that $(\alpha_k, \beta_k) = (-\pi/4, \pi/4)$ for $k = 0, 2, \cdots$ and $(\alpha_k, \beta_k) = (\pi/4, -\pi/4)$ for $k = 1, 3, \cdots$. The filter bank description of T4 is as follows:

$$\begin{aligned} a_0(n) &= h(n) \cos\left(\frac{\pi}{2N} n - \frac{\pi}{4}\right) & b_0(n) &= h(n) \cos\left(\frac{\pi}{2N} n + \frac{\pi}{4}\right) \\ a_1(n) &= h(n) \cos\left(\frac{3\pi}{2N} n + \frac{\pi}{4}\right) & b_1(n) &= h(n) \cos\left(\frac{3\pi}{2N} n - \frac{\pi}{4}\right) \\ a_2(n) &= h(n) \cos\left(\frac{5\pi}{2N} n - \frac{\pi}{4}\right) & b_2(n) &= h(n) \cos\left(\frac{5\pi}{2N} n + \frac{\pi}{4}\right). \\ &\vdots \quad \vdots & &\vdots \quad \vdots \end{aligned} \tag{22}$$

The input–output transfer function for any pair of corresponding terminals is

$$\begin{aligned} T(z^N) &= \frac{1}{4} \sum_{i=0}^{N-1} [H^2(zW^{-i+1/4}) + H^2(zW^{-i-1/4})] \\ &= \frac{N}{2} [\cdots + v(-4N)z^{4N} - v(-2N)z^{2N} + v(0) \\ &\quad - v(2N)z^{-2N} + v(4N)z^{-4N} - \cdots]. \end{aligned} \tag{23}$$

Note that the transfer function is in fact a function of z^{2N}. In fact, system T4 is the same as the transmultiplexer formed as the complement of the subband system in [9] except for the phase factors. The complement of the system in [9] has phase factors which satisfy the same solution in Section VI-C that was invoked in forming T4 and which are either the same as those in T4 or differ from those in T4 by an integer multiple of π.

Just as T1 was modified to get T3, we attempt to modify T4 by alternating the positions of two prototypes (both band limited to no more than π/N) for each center frequency. With this arrangement, the crosstalk between two signals sent at adjacent center frequencies is canceled as in T4. However, it can be shown that the input–output transfer function will not be the same for all pairs of terminals.

E. System T5

In system T5, we again use the center frequencies of set 3 and band limit the prototype to no more than π/N. Another solution formulated in Section VI-C is used to configure T5. For the end center frequency $\pi/2N$, the parameters are $n_0 = p_0 = 0$ and $\alpha_0 = -\beta_0 = -\pi/4$. When establishing the combining and separation filters for the frequency $3\pi/2N$, crosstalk due to spectral overlap with $\pi/2N$ is canceled by setting $n_1 = p_1 = N/2$ and $\alpha_1 = -\beta_1 = 3\pi/4$. This process continues in a sequential fashion such that the filter bank of T5 is described by

$$\begin{aligned}
a_0(n) &= h(n)\cos\left(\frac{\pi}{2N}n - \frac{\pi}{4}\right) & b_0(n) &= h(n)\cos\left(\frac{\pi}{2N}n + \frac{\pi}{4}\right)\\
a_1(n) &= h\left(n - \frac{N}{2}\right)\cos\frac{3\pi}{2N}n & b_1(n) &= h\left(n + \frac{N}{2}\right)\cos\frac{3\pi}{2N}n\\
a_2(n) &= h(n)\cos\left(\frac{5\pi}{2N}n + \frac{\pi}{4}\right) & b_2(n) &= h(n)\cos\left(\frac{5\pi}{2N}n - \frac{\pi}{4}\right)\\
a_3(n) &= h\left(n - \frac{N}{2}\right)\sin\frac{7\pi}{2N}n & b_3(n) &= -h\left(n + \frac{N}{2}\right)\sin\frac{7\pi}{2N}n.\\
&\vdots \quad \vdots & &\vdots \quad \vdots
\end{aligned} \tag{24}$$

System T5 is a new alternative to T4. A delay element of $N/2$ is present for every other center frequency. In addition, the modulating function alternates between a cosine and a sine for the cases in which a delay element is present. When no delay element is present, the resultant phase of the cosine carrier alternates between $-\pi/4$ and $\pi/4$. The input–output transfer function for T5 is the same as that for T4 and is given by (23).

As in T4, we attempt to modify T5 to accommodate two prototypes. In general, the transfer function is not the same between every pair of corresponding terminals. However, an exception occurs for the case $N = 2$ (see Section IX).

F. Modification of the Parameters

Note that the configured transmultiplexers are not unique in that the delay factors and phases can be changed and still satisfy the derived conditions. For the cases when the delay element is 0, alternative delay factors which are an integer multiple of N could be used. Similarly, delay factors of $N/2$ can be replaced by any odd multiple of $N/2$. Any set of phase factors (α_k, β_k) can be replaced by $(\alpha_k + m\pi, \beta_k - m\pi)$ where m is an integer. Also, note that the transmultiplexers T1 though T5 were developed by starting with phase factors $\pi/4$ and $-\pi/4$ for the lowest end frequency. However, any odd multiple of $\pi/4$ could be used as the starting point. In this case, the filters in T1 and T3 are either the same as or negatives of those presented above. In systems T2, T4, and T5, the phase factors used in all the filters would still remain to be odd multiples of $\pi/4$. The cosine and sine carriers at the odd-indexed terminals of T5 could become sine and cosine carriers respectively. To conclude, note that any changes in the parameters render only a trivial modification of the systems.

G. Elimination of Intersymbol Interference

The five preceding transmultiplexers have been synthesized to eliminate crosstalk. The input–output transfer function $T(z^N)$ still admits intersymbol interference. Intersymbol interference is eliminated in T1 and T2 if $H^2(z)$ satisfies the Nyquist criterion in which every Nth sample of its impulse response (except for a reference sample) is equal to zero. In T3, the product $H(z)G(z)$ must satisfy the Nyquist criterion. Intersymbol interference is eliminated in T4 and T5 if $H^2(z)$ satisfies the Nyquist criterion in which every $2N$th sample of its impulse response (except for a reference sample) is equal to zero.

For T1, T2, and T3, the prototypes $H(z)$ and $G(z)$ must be band limited to no less than π/N for the Nyquist criterion to be satisfied. Similarly, for T4 and T5, $H(z)$ must be band limited to no less than $\pi/2N$ for the Nyquist criterion to be satisfied. The minimum bandwidth of the prototypes that is required to get a Nyquist characteristic corresponds to the minimum bandwidth found in Section IV. Perfect reconstruction is achieved if the prototypes are band limited (up to 100% above the minimum bandwidth) and the Nyquist criterion is satisfied.

VIII. Multicarrier QAM and VSB Systems

An interpretation of what exactly each system implements is made clear by examining the input signal spectrum and the filter responses as shown in Fig. 3. As shown in Fig. 3(a), modulation is implicitly accomplished by the interpolation step in the copies of the input signal spectrum appear at intervals of $2\pi/N$. The three systems T1, T2, and T3 accomplish multicarrier quadrature amplitude modulation (QAM) in the form of a digital multirate filter structure as in Fig. 1. For each unique center frequency (except 0 and π), two signals are sent in quadrature. Systems T1 and T3 explicitly accomplish QAM in that a particular combining filter extracts one of the replicated copies of the input spectrum around carrier frequencies at integer multiples of $2\pi/N$ (see Fig. 3). The same is not true of T2 in that the combining filters, whose center frequencies are odd multiples of π/N, extract a portion of two adjacent copies of the input spectrum. System T2 can be converted to a true QAM scheme as follows. Suppose each of the signals is multiplied by $(-1)^n$ prior to interpolation. Then, the input spectrum shifts in such a way that after interpolation, the replicated copies are centered at implicit carriers equal to odd multiples of π/N (shown in Fig. 3(b)). Now, each of the combining filters will extract a replicated copy centered at an odd multiple of π/N.

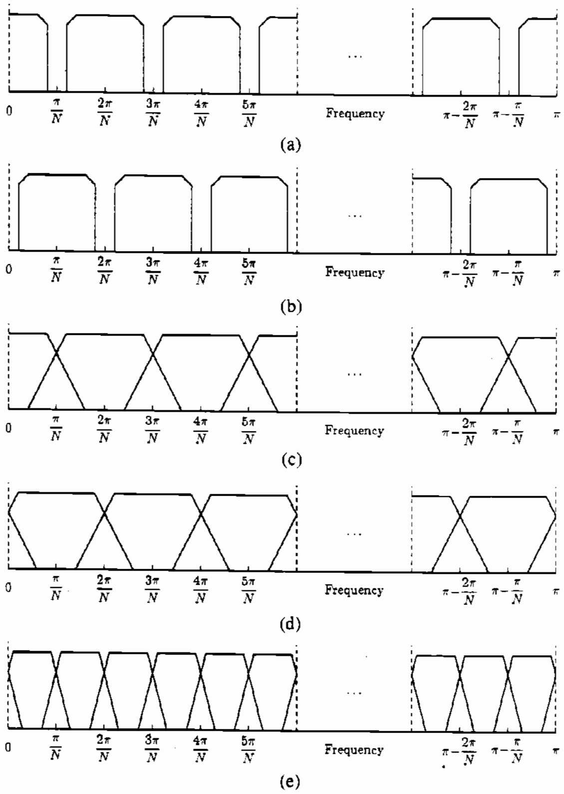

Fig. 3. Input signal spectrum and responses of the filters used in systems T1 to T5 (shown for N even). (a) Input signal spectrum after interpolation. (b) Spectrum of input signal multiplied by $(-1)^n$, after interpolation. (c) Filter responses for systems T1 and T3. (d) Filter responses for system T2. (e) Filters responses for systems T4 and T5.

The original signals can be recovered by multiplying each of the outputs by $(-1)^n$.

Multicarrier quadrature amplitude modulation systems have been realized in continuous time [13] and in discrete time [14]. Also, a data modem based on the QAM technique is described in [15]. The system in [14] uses one low-pass prototype and a set of equally spaced frequencies for transmission. Also, it is oversampled as opposed to the critically sampled systems that we consider. In an oversampled system, the interpolation/decimation factor is greater than the number of frequency bands. This gives additional freedom in choosing the repeated center frequencies but does not generally result in the utilization of the entire range 0 to π. In every band, the low-pass prototype extracts the copy of the input spectrum around the 0 frequency. Each of the filtered outputs is then explicitly modulated by multiplication with a sinusoid at the corresponding carrier frequency. Two signals are sent in quadrature at each carrier frequency through modulation by a cosine and sine carrier. Our system T1 is related to the system proposed in [14]. The system in [14] can be converted to our system T1 with the restriction that the carrier frequencies are integer multiples of $2\pi/N$.

In contrast, system T4 and system T5 do not implement QAM. Systems T4 and T5 can be thought of as being multicarrier vestigial sideband (VSB) schemes. Given an implicit set of carriers at integer multiples of $2\pi/N$, there are both lower and upper sidebands at these carrier frequencies. A combining filter extracts either an upper or lower sideband of a particular copy of the input spectrum and a vestige of a suppressed sideband for transmission. Multiplication of the input signal by $(-1)^n$ prior to sampling rate expansion results in an implicit set of carriers at odd multiples of π/N. Again, one upper or lower sideband and a vestige of a suppressed sideband is extracted for transmission. In contrast to conventional frequency division multiplexing (FDM) schemes which avoid spectral overlap by using guard bands, the VSB systems allow overlap between the transmitted sidebands of different input signals.

Another multirate system described in [8] is not a regular structure in that the center frequencies are not equally spaced and two prototypes of different bandwidths are used to derive the filter bank. For the transmultiplexer form of the system in [8], VSB is used for all carriers other than 0 and π.

A synthesis procedure that establishes a set of analog transmitter filters for the simultaneous transmission of data is developed in [16]. Transmultiplexer T4 is a digital counterpart to the system configured in [16].

IX. The Two Band Case

This section examines two band systems as a separate case. Although two band versions of transmultiplexers T1 to T5 exist, we anticipate that a synthesis procedure devoted only to the $N = 2$ case will lead to more flexible conditions than the N band case and consequently, lead to many transmultiplexers. As before, the combining filters $A_k(z)$ have parameters ω_k, n_k, and α_k for $k = 0$ and 1. The separation filters $B_k(z)$ have parameters ω_k, p_k, and β_k for $k = 0$ and 1. We do not impose any bandwidth restriction on the low-pass prototypes in formulating a synthesis procedure for crosstalk-free transmultiplexers with two identical input–output transfer functions.

For systems based on one prototype filter and with two distinct center frequencies, the following conditions must hold.

1) The two center frequencies must satisfy the relation $\omega_0 + \omega_1 = \pi$.

2) The delays are chosen such that:

i) The relationship $n_0 - p_0 = n_1 - p_1$ must be satisfied. Moreover, both $n_0 - p_0$ and $n_1 - p_1$ are even.

ii) Both $n_0 - p_1$ and $n_1 - p_0$ are odd.

3) The phases are chosen such that;

i) If $\omega_0 \neq 0$ and $\omega_1 \neq \pi$, then $\alpha_0 + \beta_0 = -(\alpha_1 + \beta_1)$. If $\omega_0 = 0$ and $\omega_1 = \pi$, then $\alpha_0 + \beta_0 = \pm(\alpha_1 + \beta_1)$.

ii) The relationship $\alpha_0 - \beta_0 = \pm(\alpha_1 - \beta_1)$ must hold.

iii) If $\omega_0 \neq 0$ and $\omega_1 \neq \pi$, both $\alpha_0 + \beta_1$ and $\alpha_1 + \beta_0$ are integer multiples of π.

For the case in which both center frequencies are the same, we have the same restrictions on the delays as given

TABLE I
SYNTHESIZED TWO BAND SYSTEMS

System	Center Frequencies	Combining Filters	Separation Filters
A	0	$a_0(n) = h(n)$	$b_0(n) = g(n)$
	π	$a_1(n) = (-1)^n g(n-1)$	$b_1(n) = (-1)^n h(n+1)$
B	$\pi/2$	$a_0(n) = h(n) \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$	$b_0(n) = g(n) \cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)$
	$\pi/2$	$a_1(n) = g(n-1) \cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)$	$b_1(n) = h(n+1) \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$
C	$\pi/4$	$a_0(n) = h(n) \cos\left(\frac{\pi}{4}n - \frac{\pi}{4}\right)$	$b_0(n) = g(n) \cos\left(\frac{\pi}{4}n + \frac{\pi}{4}\right)$
	$3\pi/4$	$a_1(n) = g(n-1) \cos\left(\frac{3\pi}{4}n\right)$	$b_1(n) = h(n+1) \cos\left(\frac{3\pi}{4}n\right)$
D	$\pi/3$	$a_0(n) = h(n) \cos\left(\frac{\pi}{3}n + \frac{\pi}{3}\right)$	$b_0(n) = g(n) \cos\left(\frac{\pi}{3}n - \frac{\pi}{3}\right)$
	$2\pi/3$	$a_1(n) = g(n-1) \cos\left(\frac{2\pi}{3}n - \frac{\pi}{3}\right)$	$b_1(n) = h(n+1) \cos\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right)$

above. It can be shown that the only possible center frequency that can be repeated is $\pi/2$. The restrictions on the phases are as above except that 3i) becomes $\alpha_0 + \beta_0 = \pm(\alpha_1 + \beta_1)$.

Now, consider the case when two prototypes $H(z)$ and $G(z)$ are used. The filters $A_0(z)$ and $B_1(z)$ are frequency shifted versions of $H(z)$. Similarly, $A_1(z)$ and $B_0(z)$ are frequency shifted versions of $G(z)$. The conditions for the cancellation of crosstalk remain the same as above. The input–output transfer function is examined to establish any further requirements. For distinct center frequencies except 0 and π, the rules are the same as for the single prototype case except that 3ii) changes to $\alpha_0 - \beta_0 = \alpha_1 - \beta_1$. If $\omega_0 = 0$ and $\omega_1 = \pi$, the rules are the same as the single prototype case. For this case in which the center frequencies are the same, the rules are again the same as the single prototype case.

As anticipated, the above rules permit the synthesis of many two band transmultiplexers. There is no bandwidth restriction on the prototypes for the two band case. This allows for more freedom in choosing the center frequencies for the 2 band case as compared to the N band case and yet ensures complete bandwidth utilization. Table I shows some two band systems that are synthesized from the formulated rules.

The systems depicted in Table I involve two prototypes. One prototype versions occur as a special case. System A is a two band version of T3 (the two band version of T1 is the special case). When $G(z) = H(z)$, system B is a two band version of T2. Although many two band systems can be developed, they cannot necessarily be extended to the N band case for our objectives. An N band version of system B cannot be configured since the crosstalk function for two signals sent at adjacent center frequencies will involve two prototypes and cannot be made equal to zero. If $G(z) = H(z)$, an N band version of system C results if the bandwidth of the prototype is reduced to π/N (system T5). However, an N band system with two prototypes cannot be formed even with the reduced bandwidth since the input–output transfer function is not the same for every pair of terminals. System D is synthesized by taking advantage of the flexibility in choosing the center frequencies specifically for the two band case. The general synthesis procedure in this paper does not lead to an N band version of system D even if $G(z) = H(z)$.

X. SUMMARY AND CONCLUSIONS

This paper develops a synthesis procedure for transmultiplexers that use modulated filter banks. The combining and separation filters are bandpass versions of a low-pass prototype. The impulse responses of the filters are described by the impulse response of the low-pass prototype, a center frequency, delay, and phase factor. The objectives of the synthesis procedure are to allow for complete bandwidth utilization by allowing spectral overlap among the filters, achieve an identical input–output transfer function between every corresponding pair of terminals and eliminate crosstalk. As a result, five different transmultiplexers are configured. Three accomplish QAM and the other two implement VSB. Intersymbol interference can be eliminated by appropriately designing the low-pass prototype.

Each of the transmultiplexers implements a form of frequency division multiplexing (FDM) without the use of guard bands. Consider the case in which each input signal to the transmultiplexer is sampled at f_0 Hz. Then, the total information rate is $f_s = Nf_0$ samples/second where f_s is the sampling rate of the composite signal which occupies a bandwidth of $f_s/2$ Hz. The bandwidth efficiency of each of the systems is the ratio of the information rate (f_s samples/second) to the total bandwidth (f_s/2Hz) and is equal to 2 samples/s/Hz. The synthesized transmultiplexers are bandwidth efficient in that the full information in each input is transmitted and the inputs are recovered.

The two band case is dealt with separately. Many transmultiplexers can be synthesized due to the flexibility in choosing the center frequencies.

Appendix A
Phase Factors in Relation to the Synthesis Procedure

Given the sum and difference criteria and the three sets of center frequencies, the sum of the phase factors $\alpha_k + \beta_k$ was constrained to be an integer multiple of π for every terminal k. Here, we justify this choice based on a crosstalk analysis and design constraints. Consider the center frequencies in set 1 which lead to system T1. For crosstalk cancellation between two signals sent at $\omega_k = 0$ and $\omega_l = 2\pi/N$, the condition that α_l and β_l be odd multiples of $\pi/2$ emerges if $n_k - p_l$ and $n_l - p_k$ are integer multiples of N. Then, $\alpha_l + \beta_l$ is an integer multiple of π. Considering either the sum or difference criterion reveals that the sum of the phase factors should be an integer multiple of π for each terminal.

Consider the frequencies of set 2 which leads to system T2. In particular, we examine the crosstalk function relating two signals transmitted at $\omega_k = \omega_l = \pi/N$ (an end frequency). If the difference in the delay factors $n_k - p_l$ is an odd multiple of $N/2$, $\alpha_k + \beta_l$ should be an integer multiple of π and $\alpha_k - \beta_l$ should be an odd multiple of $\pi/2$ for canceling the crosstalk. Combining these restrictions with those for either the sum or difference criterion and noting the conditions on the phase factors for the end frequencies leads us to constrain the sum of the phase factors $\alpha_k + \beta_k$ and $\alpha_l + \beta_l$ to be an integer multiple of π. This restriction on the sum of the phase factors will then hold for every terminal.

In the case of the frequencies of set 3, the arbitrary nature of the sum of the phase factors allows us to synthesize systems other than T4 and T5. The phase factors α_k and β_k of these systems will be different from those in T4 and T5. Also, the input-output transfer functions of these systems will differ from that of T4 and T5 in that they will be a function of z^N as opposed to z^{2N} as in T4 and T5 (see (23)). Then, the condition for cancelling intersymbol interference is that $H^2(z)$ should be a Nyquist filter with an impulse response having zero crossings every Nth sample (except for a reference sample). This requires a minimum bandwidth of π/N (see Section VII-G) which corresponds to the maximum bandwidth allowed for the low-pass prototype $H(z)$. Hence, there is a conflict in the bandwidth constraints. By restricting $\alpha_k + \beta_k$ to be an integer multiple of π for every terminal, the input-output transfer function is a function of z^{2N}. Then, the minimum bandwidth that is required to satisfy the Nyquist criterion corresponds to the minimum bandwidth found in Section IV.

Appendix B
Derivation of (11)

The crosstalk function specified by (10) is

$$T_{kl}(z^N) = \tfrac{1}{4} z^{-(n_k - p_l)} \sum_{i=0}^{N-1} [[W^{(m-2p)(n_k-p_l)} \exp[j(\alpha_k + \beta_l)] + \exp[-j(\alpha_k + \beta_l)]] \cdot W^{i(n_k-p_l)} H(zW^{-i+p}) H(zW^{-i-m+p})]. \quad \text{(B.1)}$$

For notational convenience, let $n_k - p_l = s$, $m - 2p = l$, and $\alpha_k + \beta_l = \theta$. The crosstalk function is zero if

$$W^{ls} e^{j\theta} + e^{-j\theta} = 0 \quad \text{(B.2)}$$

or equivalently

$$e^{j2\theta} = -\frac{1}{W^{ls}} = -\exp\left(j\frac{2\pi}{N} ls\right) = \exp\left[j\left(\frac{2\pi}{N} ls + \pi\right)\right]. \quad \text{(B.3)}$$

This implies that

$$\theta = \pi\left[\frac{ls}{N} + \frac{1}{2}\right]. \quad \text{(B.4)}$$

Appendix C
Examination of (14)

For notational convenience, let $a = \exp[j(\alpha_k + \beta_l)]$ and a^* is its complex conjugate. The first step in analyzing (14) is to substitute $\omega_k = (2\pi/N)q + \Delta\omega$ to get

$$4z^{(n_k-p_l)} T_{kl}(z^N) = a \sum_{i=0}^{N-1} W^{i(n_k-p_l)} H^2(e^{-j\Delta\omega} z W^{-i+q}) + a^* \sum_{i=0}^{N-1} W^{i(n_k-p_l)} H^2(e^{j\Delta\omega} z W^{-i-q}). \quad \text{(C.1)}$$

Note that q is an integer and $0 \le \Delta\omega < 2\pi/N$. The limitations on $\Delta\omega$ are determined in order to fix the frequencies at which two signals can be transmitted without crosstalk. Letting $e^{j\Delta\omega} = W^p$ where $-1 < p \le 0$. Then

$$4z^{(n_k-p_l)} T_{kl}(z^N) = a \sum_{i=0}^{N-1} W^{i(n_k-p_l)} H^2(zW^{-i+q-p}) + a^* \sum_{i=0}^{N-1} W^{i(n_k-p_l)} W^{-2q(n_k-p_l)} H^2(zW^{-i+q+p}). \quad \text{(C.2)}$$

It is desired to have the two terms in the above equation cancel each other.

Consider the case when $n_k - p_l$ is an integer multiple of N and $a = -a^*$. Then, the exponential indices of W in the arguments of $H^2(\cdot)$ of the two terms must differ by an integer to make the crosstalk zero. Therefore, p is fixed at either 0 or $-(1/2)$ thereby forcing the center frequencies to be integer multiples of π/N. Since $a = -a^*$, $\alpha_k + \beta_l$ is an odd multiple of $\pi/2$.

Suppose $n_k - p_l$ is an odd multiple of $N/2$. Then, we get

$$4z^{(n_k - p_l)} T_{kl}(z^N) = a \sum_{i=0}^{N-1} (-1)^i H^2(zW^{-i+q-p}) + a^* \sum_{i=0}^{N-1} (-1)^i H^2(zW^{-i+q+p}). \tag{C.3}$$

Algebraic substitution for the second term only yields

$$4z^{(n_k - p_l)} T_{kl}(z^N) = a \sum_{i=0}^{N-1} (-1)^i H^2(zW^{-i+q-p}) + a^* \sum_{i=-2p}^{N-1-2p} (-1)^i \cdot (-1)^{2p} H^2(zW^{-i+q-p}). \tag{C.4}$$

If $a = -a^*$, $2p$ must be an even integer for the two terms to cancel. Therefore, $p = 0$ and the center frequencies are integer multiples of $2\pi/N$. If $a = a^*$, $2p$ must be an odd integer for the two terms to cancel. Hence, $p = -(1/2)$ and the center frequencies are odd multiples of π/N. This development generates the various approaches as outlined in Section VI-B.

Appendix D
General Expressions for the Combining and Separation Filters

1. *System T1*

$$a_0(n) = h(n) \cos \frac{\pi}{4} \qquad b_0(n) = h(n) \cos \frac{\pi}{4}.$$

For $k = 0, 1, 2, \cdots, \lfloor (N-4)/4 \rfloor$

$$a_{4k+1}(n) = h\left(n - \frac{N}{2}\right) \cos \frac{2\pi}{N} (2k+1)n \qquad b_{4k+1}(n) = h\left(n + \frac{N}{2}\right) \cos \frac{2\pi}{N} (2k+1)n$$

$$a_{4k+2}(n) = h(n) \sin \frac{2\pi}{N} (2k+1)n \qquad b_{4k+2}(n) = -h(n) \sin \frac{2\pi}{N} (2k+1)n.$$

For $k = 0, 1, 2, \cdots, \lfloor (N-6)/4 \rfloor$

$$a_{4k+3}(n) = h(n) \cos \frac{2\pi}{N} (2k+2)n \qquad b_{4k+3}(n) = h(n) \cos \frac{2\pi}{N} (2k+2)n$$

$$a_{4k+4}(n) = h\left(n - \frac{N}{2}\right) \sin \frac{2\pi}{N} (2k+2)n \qquad b_{4k+4}(n) = -h\left(n + \frac{N}{2}\right) \sin \frac{2\pi}{N} (2k+2)n$$

$$a_{N-1}(n) = \begin{cases} (-1)^n h(n) \cos \frac{\pi}{4} & N = 4, 8, 12, \cdots \\ (-1)^n h\left(n - \frac{N}{2}\right) \cos \frac{\pi}{4} & N = 2, 6, 10, \cdots \end{cases}$$

$$b_{N-1}(n) = \begin{cases} (-1)^n h(n) \cos \frac{\pi}{4} & N = 4, 8, 12, \cdots \\ (-1)^n h\left(n + \frac{N}{2}\right) \cos \frac{\pi}{4} & N = 2, 6, 10, \cdots. \end{cases}$$

2. *System T2*

For $k = 0, 1, 2, \cdots, (N-2)/2$

$$a_{2k}(n) = h(n) \cos \left[\frac{\pi}{N} (2k+1)n + \frac{\pi}{4} (-1)^k\right] \qquad b_{2k}(n) = h(n) \cos \left[\frac{\pi}{N} (2k+1)n + \frac{\pi}{4} (-1)^{k+1}\right]$$

$$a_{2k+1}(n) = h\left(n - \frac{N}{2}\right) \cos \left[\frac{\pi}{N} (2k+1)n + \frac{\pi}{4} (-1)^{k+1}\right] \qquad b_{2k+1}(n) = h\left(n + \frac{N}{2}\right) \cos \left[\frac{\pi}{N} (2k+1)n + \frac{\pi}{4} (-1)^k\right].$$

3. *System T3*

$$a_0(n) = h(n) \cos \frac{\pi}{4} \qquad b_0(n) = g(n) \cos \frac{\pi}{4}.$$

For $k = 0, 1, 2, \cdots, \lfloor (N-4)/4 \rfloor$

$$a_{4k+1}(n) = g\left(n - \frac{N}{2}\right) \cos \frac{2\pi}{N} (2k+1)n \qquad b_{4k+1}(n) = h\left(n + \frac{N}{2}\right) \cos \frac{2\pi}{N} (2k+1)n$$

$$a_{4k+2}(n) = g(n) \sin \frac{2\pi}{N} (2k+1)n \qquad b_{4k+2}(n) = -h(n) \sin \frac{2\pi}{N} (2k+1)n.$$

For $k = 0, 1, 2, \cdots, \lfloor (N-6)/4 \rfloor$

$$a_{4k+3}(n) = h(n) \cos \frac{2\pi}{N} (2k+2)n \qquad b_{4k+3}(n) = g(n) \cos \frac{2\pi}{N} (2k+2)n$$

$$a_{4k+4}(n) = h\left(n - \frac{N}{2}\right) \sin \frac{2\pi}{N} (2k+2)n \qquad b_{4k+4}(n) = -g\left(n + \frac{N}{2}\right) \sin \frac{2\pi}{N} (2k+2)n$$

$$a_{N-1}(n) = \begin{cases} (-1)^n h(n) \cos \frac{\pi}{4} & N = 4, 8, 12, \cdots \\ (-1)^n g\left(n - \frac{N}{2}\right) \cos \frac{\pi}{4} & N = 2, 6, 10, \cdots \end{cases}$$

$$b_{N-1}(n) = \begin{cases} (-1)^n g(n) \cos \frac{\pi}{4} & N = 4, 8, 12, \cdots \\ (-1)^n h\left(n + \frac{N}{2}\right) \cos \frac{\pi}{4} & N = 2, 6, 10, \cdots. \end{cases}$$

4. System T4

For $k = 0, 1, 2, \cdots, N-1$

$$a_k(n) = h(n) \cos \left[\frac{\pi}{2N} (2k+1)n + \frac{\pi}{4} (-1)^{k+1}\right] \qquad b_k(n) = h(n) \cos \left[\frac{\pi}{2N} (2k+1)n + \frac{\pi}{4} (-1)^k\right].$$

5. System T5

For $k = 0, 1, 2, \cdots, (N-2)/2$

$$a_{2k}(n) = h(n) \cos \left[\frac{\pi}{2N} (4k+1)n + \frac{\pi}{4} (-1)^{k+1}\right] \qquad b_{2k}(n) = h(n) \cos \left[\frac{\pi}{2N} (4k+1)n + \frac{\pi}{4} (-1)^k\right].$$

For $k = 0, 1, 2, \cdots, \lfloor (N-2)/4 \rfloor$

$$a_{4k+1}(n) = h\left(n - \frac{N}{2}\right) \cos \frac{\pi}{2N} (8k+3)n \qquad b_{4k+1}(n) = h\left(n + \frac{N}{2}\right) \cos \frac{\pi}{2N} (8k+3)n.$$

For $k = 0, 1, 2, \cdots, \lfloor (N-4)/4 \rfloor$

$$a_{4k+3}(n) = h\left(n - \frac{N}{2}\right) \sin \frac{\pi}{2N} (8k+7)n \qquad b_{4k+3}(n) = -h\left(n + \frac{N}{2}\right) \sin \frac{\pi}{2N} (8k+7)n.$$

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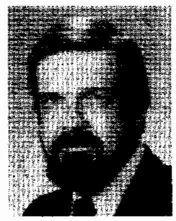

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