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Selection of the Focusing Frequency in Wideband Array Processing

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Abstract

Wideband array systems can be decomposed into several narrowband systems by sampling in the frequency domain. Focusing is the combination of these narrowbands by transforming them into a focusing subspace. Corresponding to each focusing subspace there is a focusing frequency. So far, there has been no optimal way for choosing the focusing frequency – usually it is chosen to be the mid-band frequency. In this work we propose a technique to choose the focusing frequency. Our method is based on minimizing a subspace fitting error. The simulation results show that using the selected frequency for focusing improves the performance of the estimation by decreasing the resolution threshold and reducing the bias.

1. Introduction and Problem Formulation

In the literature, several methods have been proposed to process wideband signals. In this paper, we investigate the Coherent Signal-subspace Method (CSM), [1]. The method is based on transformation of the signal-subspaces for all the frequency bins into the signal-subspace created by f_0, the mid-band frequency. This transformation reduces the computational load and improves the efficiency of the estimation. It is shown that with very weak constraints on the signal covariance matrix, it is possible to handle coherent cases. Hung and Kaveh, [2], showed that the best performance is obtained if and only if the mapping is done through a unitary transformation. They did not discuss how to choose the best focusing frequency, f_0. In this paper we introduce a subspace fitting error and optimize it to find a suitable focusing frequency.

Consider an array of p sensors exposed to $q < p$ far-field wideband sources That can be partially or fully correlated. The output of the sensors in the frequency domain is shown by

$$\mathbf{x}(\omega) = \mathbf{A}(\omega, \theta)\mathbf{s}(\omega) + \mathbf{n}(\omega), \tag{1}$$

where $\mathbf{x}(\omega), \mathbf{s}(\omega)$ and $\mathbf{n}(\omega)$ are the Fourier transforms of the observation, signal and noise vectors, respectively. The $p \times q$ matrix of location vectors is given by the full rank matrix $\mathbf{A}(\omega, \theta) = [\mathbf{a}(\omega, \theta_1) \ldots \mathbf{a}(\omega, \theta_q)]$. It is assumed that the signal and noise samples are independent identically distributed sequence of complex Gaussian random vectors with unknown covariance matrices $\mathbf{S}(\omega)$ and $\sigma^2\mathbf{I}$, respectively. With these assumptions the covariance matrix of the observation vector at the frequency ω_j is given by

$$\mathbf{R}(\omega_j) = \mathbf{A}(\omega_j, \theta)\mathbf{S}(\omega_j)\mathbf{A}^H(\omega_j, \theta) + \sigma^2\mathbf{I}, \tag{2}$$

where the superscript H represents the Hermitian transpose. In the sequel, we suppress the frequency variable. Then $\mathbf{R}_j$ represents $\mathbf{R}(\omega_j)$, $\mathbf{x}_j$ represents $\mathbf{x}(\omega_j)$ and so on. The CSM algorithm is based on forming new observation vectors, $\mathbf{y}_j$, as

$$\mathbf{y}_j = \mathbf{T}_j\mathbf{x}_j \tag{3}$$

where $\mathbf{T}_j$'s are the unitary transformation matrices found from

$$\min \|\mathbf{A}_0 - \mathbf{T}_j\mathbf{A}_j\|, \quad j = 1, \ldots, J \tag{4}$$

where $\|.\|$ is the Frobenius matrix norm. The observation vectors that are formed by (3) are in the focusing subspace and their correlation matrices can be added together directly to make the universal correlation matrix. This universal correlation matrix is then used in the MUSIC algorithm to find the directions of arrival.

2. Selection of the focusing frequency

We find the optimized focusing frequency by minimizing

$$\min_{f_0} \min_{\mathbf{T}_j} \sum_{j=1}^{J} \|\mathbf{A}_0 - \mathbf{T}_j\mathbf{A}_j\|^2, \tag{5}$$

subject to $\mathbf{T}_j$ being a unitary matrix. For a fixed $\mathbf{A}_0$, it is already known, [2], that the optimal $\mathbf{T}_j$ is given by

$$\mathbf{T}_j = \mathbf{V}_j\mathbf{W}_j^H, \tag{6}$$

where $\mathbf{V}_j$ and $\mathbf{W}_j$ are the matrices of the left and right singular vectors of $\mathbf{A}_0\mathbf{A}_j^H$. It can be shown that the error is given by

$$\sum_{j=1}^{J} \|\mathbf{A}_0 - \mathbf{T}_j\mathbf{A}_j\|^2 = 2Jpq - 2\sum_{j=1}^{J}\sum_{i=1}^{q} \sigma_i(\mathbf{A}_0\mathbf{A}_j^H), \tag{7}$$

where $\sigma_i(\mathbf{B}), i = 1, \ldots, q$ are the singular values of the matrix $\mathbf{B}$ arranged in decreasing order. The optimization problem is the same as

$$\max_{f_0} \sum_{j=1}^{J}\sum_{i=1}^{q} \sigma_i(\mathbf{A}_0\mathbf{A}_j^H). \tag{8}$$

Direct minimization of (8) is very involved and the computational complexity increases with the number of the frequency samples. We have shown that

$$\sum_{j=1}^{J}\sum_{i=1}^{q} \sigma_i(\mathbf{A}_0\mathbf{A}_j^H) \le \sum_{j=1}^{J}\sum_{i=1}^{q} \sigma_i(\mathbf{A}_0)\sigma_i(\mathbf{A}_j^H). \tag{9}$$

Our proposed method is based on maximizing the right hand side of (9) subject to $\sum_{i=1}^{q} \sigma_i^2(\mathbf{A}_0) = pq$. We have shown that (9) is a tight bound in the vicinity of the optimum point. The optimization is done in two steps. First, the optimal singular values for the location matrix $\mathbf{A}_0$ are determined. Then, using the known structure of the location matrices, the optimized value of the focusing frequency, f_0, is found.

The classic Lagrange multiplier optimization method gives

$$\sigma_i^*(\mathbf{A}_0) = \frac{\mu_i\sqrt{pq}}{\sqrt{\sum_{i=1}^{q} \mu_i^2}} \qquad i = 1, \ldots, q \tag{10}$$

where $\mu_i = \sum_{j=1}^{J} \sigma_i(\mathbf{A}_j)$. It is important to notice that the only unknown in the location matrix is the frequency of the processing. We find a matrix that has the singular values close to $\sigma_i^*(\mathbf{A}_0), i = 1, \ldots, q$. This can be done by minimizing

$$\min_{f_0} \sum_{i=1}^{q} [\sigma_i(\mathbf{A}_0) - \sigma_i^*(\mathbf{A}_0)]^2, \tag{11}$$

subject to the matrix $\mathbf{A}_0$ being a location matrix.

In our simulation, we examined an array of six sensors exposed to two uncorrelated sources. Finding the error of subspace fitting for different frequencies, we observed that the mid-band frequency is not necessarily the best point for focusing. We used the optimized point found by the algorithm and the mid-band frequency for focusing. It was observed that with the selected point by the algorithm as the focusing frequency, some sources can be resolved that otherwise would not be. We also investigated the bias of estimation. The bias of the estimation was minimized for the optimized frequency selected by the proposed method.

REFERENCES

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