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Selection of the Focusing Frequency in Wideband Array Processing – MUSIC and ESPRIT

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Abstract

Wide-band array processing using Coherent Signal-subspace Method (CSM) is discussed. It is shown that an optimal focusing subspace exists that improves the performance of the estimation. An error based on the subspace fitting is introduced. This error criterion gives the closest focused signal subspaces. Direct maximization of the criterion is very involved and the computational complexity increases with the number of frequency samples. A sub-optimal method is introduced that operates very close to the optimal case. This method is based on deriving tight bounds on the error. The computational complexity of the sub-optimal method is independent of the number of frequency samples. The sub-optimal method approaches the optimal case as the number of frequency samples increases. It is shown that the bias of the estimation is reduced by proper selection of the focusing subspace.

1. Introduction

Recently, array processing techniques have been widely used to locate wideband signals. A wideband signal has a bandwidth comparable to the center frequency. Different methods for the processing of the wideband signals using an array of sensors have been proposed in the literature. The first step in some of these techniques is to sample the signal in frequency domain. This sampling can be done through discreate Fourier transformation of the temporal samples of the signal or by using filter banks. The method which has been used to create the samples is not the concern of this paper. Here it is assumed only that J frequency samples of the signal are given. These samples can be uniformly or nonuniformly distributed in the frequency domain.

Most of the techniques in array processing use the concept of signal subspaces. The *signal subspace* is a space which is spanned by the location vectors of the array. Each location vector is a function of the observation frequency. Thus the signal subspace depends on the frequency of the observation. In wideband array processing the signal subspaces at different frequencies are different and do not overlap. This fact prevents the observation vectors in the frequency bins from being added to each other. Wang and Kaveh [1] proposed a method to transform the signal subspaces at different frequency bins into a predefined subspace and process them in this subspace. This is called *focusing*. They choose an arbitrary frequency, say f_0 the mid-band frequency, and transform all the subspaces of frequency bins into the subspace created by f_0 and then utilize the high resolution MUSIC algorithm to estimate the directions of arrival of the sources. They show that focusing reduces the threshold Signal-to-Noise Ratio (SNR). They also show that with very weak constraints on the signal covariance matrix, it is possible to handle coherent cases. Later, Hung and Kaveh, [2], proved that the best performance is obtained if and only if the mapping of the subspaces is done through a unitary transformation. They did not discuss how to choose the best focusing frequency, f_0. In [3] Hung and Kaveh extend the coherent subspace method to the ESPRIT algorithm of Roy *et al* [4]. They show that CSM can be framed based on the ESPRIT method. The selection of the focusing frequency is still arbitrary in their paper.

Recently, we introduced a method to select a sub-optimum focusing frequency [5]. The present paper is in the continuation of the previous work. Here we give more results for the focusing subspace selection method. Also we frame the proposed method for the ESPRIT algorithm. Simulation results show an improvement in the performance of the CSM algorithm by reducing the peak bias and the threshold SNR.

2. Problem formulation and CSM algorithm

Consider an array of p sensors exposed to $q < p$ far-field wideband sources that can be partially or fully correlated. The output of the sensors in the frequency domain is shown by

$$\mathbf{x}(\omega) = \mathbf{A}(\omega, \theta)\mathbf{s}(\omega) + \mathbf{n}(\omega), \tag{1}$$

where $\mathbf{x}(\omega), \mathbf{s}(\omega)$ and $\mathbf{n}(\omega)$ are the Fourier transforms of the observation, signal and noise vectors, respectively. The $p \times q$ matrix of location vectors is given by the full rank matrix $\mathbf{A}(\omega,\theta) = [\mathbf{a}(\omega,\theta_1)\ldots\mathbf{a}(\omega,\theta_q)]$. It is assumed that the signal and noise samples are independent identically distributed sequence of complex Gaussian random vectors with unknown covariance matrices $\mathbf{S}(\omega)$ and $\sigma^2\mathbf{I}$, respectively. With these assumptions the covariance matrix of the observation vector at the frequency ω_j is given by

$$\mathbf{R}(\omega_j) = \mathbf{A}(\omega_j,\theta)\mathbf{S}(\omega_j)\mathbf{A}^H(\omega_j,\theta) + \sigma^2\mathbf{I}, \tag{2}$$

where the superscript H represents the Hermitian transpose. In the sequel, we suppress the frequency variable and represent $\mathbf{R}(\omega_j)$ by $\mathbf{R}_j$, $\mathbf{x}(\omega_j)$ by $\mathbf{x}_j$ and so on. The CSM algorithm is based on forming new observation vectors, $\mathbf{y}_j$, as

$$\mathbf{y}_j = \mathbf{T}_j\mathbf{x}_j \tag{3}$$

where $\mathbf{T}_j$'s are the unitary transformation matrices found from

$$\min_{\mathbf{T}_j} \|\mathbf{A}_0 - \mathbf{T}_j\mathbf{A}_j\|, \quad j = 1,\ldots,J \tag{4}$$

where $\|.\|$ is the Frobenius matrix norm. Then these transformed observation vectors are used to construct the sample correlation matrix for each frequency bin. An average of these aligned correlation matrices gives a universal sample correlation matrix that can be used for detection and estimation.

In (4) the focusing frequency, f_0, is chosen arbitrarily. This value can be anything in the set of positive real numbers. In past practice the center frequency has been used. It is worth noting that (4) is an almost periodic continuous function of f_0. The period depends on the angles of arrival. Once the angles of arrival are given the period can be computed. This period which is an interval of the set of real numbers forms a compact set. If the focusing frequency is restricted to take its values in one period, according to the Weierstrass' theorem there will be an optimum value for f_0 that minimizes (4). In the following section, we use this concept to find a sub-optimum focusing frequency.

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3. Focusing frequency selection

In the present section we define a criterion based on the error involved in the transformation of the signal subspaces. The optimization of this criterion gives the focusing frequency. Specifically, we seek an f_0 which minimizes

$$\min_{f_0} \min_{\mathbf{T}_j} \sum_{j=1}^{J} w_j \|\mathbf{A}_0 - \mathbf{T}_j \mathbf{A}_j\|^2. \tag{5}$$

subject to $\mathbf{T}_j$ being a unitary matrix. By $\|.\|$ we mean Frobenius norm. In (5), w_j is a weighting factor which is proportional to the signal-to-noise ratio at the j-th frequency bin.

For a fixed $\mathbf{A}_0$, it is already known, [2,6], that the optimal $\mathbf{T}_j$ is given by

$$\mathbf{T}_j = \mathbf{V}_j \mathbf{W}_j^H, \tag{6}$$

where $\mathbf{V}_j$ and $\mathbf{W}_j$ are the left and right singular vectors of $\mathbf{A}_0\mathbf{A}_j^H$, i.e.

$$\mathbf{A}_0\mathbf{A}_j^H = \mathbf{V}_j \Sigma_j \mathbf{W}_j^H.$$

In this case, it can be shown that the subspace fitting error is given by

$$\begin{aligned}\sum_{j=1}^{J} w_j \|\mathbf{A}_0 - \mathbf{T}_j\mathbf{A}_j\|^2 &= \sum_{j=1}^{J} w_j \left[\|\mathbf{A}_0\|^2 + \|\mathbf{A}_j\|^2 - 2\mathrm{Re}\ \mathrm{tr}(\mathbf{A}_0\mathbf{A}_j^H\mathbf{T}_j^H)\right] \\ &= 2Jpq - 2\sum_{j=1}^{J}\sum_{i=1}^{q} \sigma_i(\mathbf{A}_0\mathbf{A}_j^H).\end{aligned} \tag{7}$$

where $\sigma_i(\mathbf{B}), i = 1, \ldots, q$ are the singular values of the matrix $\mathbf{B}$ arranged in nonincreasing order. In (7) we have used the equality

$$\begin{aligned}\|\mathbf{A}\|^2 &= \mathrm{Re}\ \mathrm{tr}(\mathbf{A}\mathbf{A}^H), \\ &= \sum_{i=1}^{q} \|\mathbf{a}_i\|^2, \\ &= pq.\end{aligned} \tag{8}$$

In this equation the notation Re represents the real part of a complex number. It is worth mentioning that (8) also holds for sources in near field and any arbitrary array manifold.

From (7) it is seen that the optimization problem (5) is identical to

$$\max_{f_0} \sum_{j=1}^{J}\sum_{i=1}^{q} w_j \sigma_i(\mathbf{A}_0\mathbf{A}_j^H). \tag{9}$$

Direct minimization of (9) is very involved and the computational complexity increases with the number of frequency samples. In the sequel, we present a suboptimum method. This suboptimal technique is based on maximizing an upper bound of (9). We show that in the vicinity of the optimum point the bound is tight. The tightness of the bound at the optimum point justifies the applicability of our method. We start by presenting a lemma that has been adopted from [6].

Lemma 1. *If $\mathbf{A}, \mathbf{B} \in \mathbf{M}_{m,n}$ are given matrices with respective ordered singular values $\sigma_1(\mathbf{A}) \geq \ldots \geq \sigma_q(\mathbf{A}) \geq 0$ and $\sigma_1(\mathbf{B}) \geq \ldots \geq \sigma_q(\mathbf{B}) \geq 0$, with $q = \min\{m, n\}$, then*

$$\|\mathbf{A} - \mathbf{B}\|^2 \geq \sum_{i=1}^{q} [\sigma_i(\mathbf{A}) - \sigma_i(\mathbf{B})]^2. \tag{10}$$

Application of Lemma 1 on (7) gives

$$\sum_{j=1}^{J}\sum_{i=1}^{q} w_j \sigma_i(\mathbf{A}_0\mathbf{A}_j^H) \leq \sum_{j=1}^{J}\sum_{i=1}^{q} w_j \sigma_i(\mathbf{A}_0)\sigma_i(\mathbf{A}_j^H). \tag{11}$$

Our proposed method is based on maximizing the right hand side of (11). The optimization is done in two steps. First, the optimal singular values for the location matrix $\mathbf{A}_0$ are determined. Then, using the known structure of the location matrices, the optimal value of the focusing frequency, f_0, is found. Let us define,

$$\mu_i \triangleq \sum_{j=1}^{J} w_j \sigma_i(\mathbf{A}_j). \tag{12}$$

Then, the optimization problem is represented by

$$\max_{f_0} \sum_{i=1}^{q} \mu_i \sigma_i(\mathbf{A}_0). \tag{13}$$

The optimum value of f_0 can be found in two steps. First, we find the singular values of the matrix $\mathbf{A}_0$ by

$$\begin{aligned}&\max_{\sigma_i(\mathbf{A}_0)} \sum_{i=1}^{q} \mu_i \sigma_i(\mathbf{A}_0) \\ &\text{s.t.} \sum_{i=1}^{q} \sigma_i^2(\mathbf{A}_0) = pq,\end{aligned} \tag{14}$$

where we have used the equation (8). The classic Lagrange multiplier optimization method gives

$$\sigma_i(\mathbf{A}_0) = \frac{\mu_i \sqrt{pq}}{\sqrt{\sum_{l=1}^{q} \mu_l^2}}, \qquad i = 1, \ldots, q \tag{15}$$

with the maximum value $\sqrt{pq\sum_{l=1}^{q}\mu_l^2}$.

The objective is to find the optimal focusing frequency, f_0. The singular values of the optimal location matrix, $\mathbf{A}_0^*$, are given by (15). We represent these optimal singular values by $\sigma_i^*, i = 1, \ldots, q$. It is important to notice that the only unknown in the location matrix is the frequency of the processing. The structure of the location matrix is known. In practice no matrix with the given structure exists which has the singular values $\sigma_i^*, i = 1, \ldots, q$. In such a case, we find a matrix that has the singular values close to $\sigma_i^*, i = 1, \ldots, q$. This can be done by minimizing the following one-variable nonlinear equation,

$$\min_{f_0} \sum_{i=1}^{q} [\sigma_i(\mathbf{A}_0) - \sigma_i^*]^2, \tag{16}$$

subject to the matrix $\mathbf{A}_0$ being a location matrix with the known structure. It is important to note that (16) is a convex function of the singular values $\sigma_i(\mathbf{A}_0)$. The optimum point $\sigma^* = (\sigma_1^* \ \ldots \ \sigma_q)$ is on the sphere with radius $\sqrt{pq}$. The singular values of $\mathbf{A}_0$ are continous functions of the frequency, f_0. A procedure to solve (16) is to increase the focusing frequency by fine steps and compute the singular values. The complexity of this minimization is independent of the number of frequency samples. If more than one solution for (16) is found, the optimum value is selected by evaluating the criterion (5) for the candidate points.

4. Bias of the estimation

Despite the fact that the CSM algorithm is very effective in wideband array processing, it suffers from asymptotic peak bias. The bias increases with the bandwidth of the sources and the deviation of the focusing points from the true DOA. Two alternative methods are introduced in the literature to reduce the bias of the estimation [7, 8]. In this section we show that there is an optimal point for the focusing frequency that minimizes the bias of the estimation.

It is important to notice that MUSIC estimator is asymtotically unbiased. Thus the peak bias which is created in the CSM algorithm is a consequence of the focusing. A proper selection of the focusing frequency minimizes the bias of the estimation. We show that the proposed method for the focusing frequency selection also minimizes the peak bias. We start by discussing the mechanism that generates the bias.

The array manifold is a curve in the p-dimensional complex space that is created from the location vectors $\mathbf{a}(\omega, \theta)$, for all θ and all ω in the source bandwidth. For every pure delay environment the norm of the location vector is equal to the square root of the number of the sensors. Thus the array manifold lies on the surface of a sphere with the radius $\sqrt{p}$. We represent this sphere by $\mathcal{S}$. It is also important to note that the array manifold is continous on ω and θ. The MUSIC algorithm finds the intersection of the signal subspace (the space spanned by the eigenvectors of the correlation matrix corresponding to q largest eigenvalues) with the array manifold. If the true correlation matrix is applied in MUSIC, the directions of arrival are estimated without bias. However, deviation from the true signal subspace will cause bias in the estimation.

Now consider the case in which several location matrices, $\mathbf{A}_j$, are transformed by the unitary matrices, $\mathbf{T}_j$, to the vicinity of the focusing location matrix, $\mathbf{A}_0$. The transformed location matrices form a cluster around $\mathbf{A}_0$. It is quite clear that the closer the transformed matrices are, the better the performance is. In an ideal case all the transformed location matrices superimpose on $\mathbf{A}_0$. We call this case *complete focusing*. In complete focusing the column vectors of the location matrix $\mathbf{A}_j$ are transformed to the corresponding column vectors of $\mathbf{A}_0$. This is also seen from the characteristic of the Frobenius norm – the square of the Frobenius norm of a matrix is equal to addition of the square of the Euclidean norm of the column vectors. Hung and Kaveh [2] show that for successful application of the unitary transformation method it is necessary to add two extra directions of arrival at $\pm 0.25 B_W$ (BeamWidth) of the estimated DOA. For instance if the i-th DOA is found at $\hat{\theta}_i$ by the pre-process, the focusing points for the i-th angle are chosen at $(\hat{\theta}_i - 0.25B_W, \hat{\theta}_i, \hat{\theta}_i + 0.25B_W)$. These angles determine an interval on the array manifold. If this interval is small, it is transformed to a corresponding interval at the array manifold with the processing frequency ω_0. This is attributed to the continuity of the array manifold and the unitary transformation. Thus in complete focusing the location vectors of each frequency bin that are located at the true DOA are transformed to the corresponding vectors at the focusing manifold; hence unbiased estimation. In practice complete focusing is not satisfied. The transformed matrices are clustered around $\mathbf{A}_0$. The closest distance between these matrices is obtained if and only if $\mathbf{A}_0$ is on the centroid of the transformed matrices. To see this take any $p \times q$ matrix $\mathbf{C}$ that has column vectors on the sphere $\mathcal{S}$. To have a tight cluster, we should minimize

$$\min_{\mathbf{C}\in\mathcal{S}} \min_{\mathbf{T}_j} \sum_{j=1}^{J} w_j \|\mathbf{C} - \mathbf{T}_j\mathbf{A}_j\|^2. \tag{17}$$

subject to $\mathbf{T}_j$ being unitary. The minimum of (17) is obtained for

$$\mathbf{C} = \left[\sum_{j=1}^{J} w_j \mathbf{T}_j \mathbf{A}_j\right]\mathbf{K} \tag{18}$$

where $\mathbf{K}$ is a diagonal normalization matrix. It is seen that $\mathbf{C}$ is formed by the centroid of the transformed location vectors for each source. The method that we proposed gives the closest $\mathbf{A}_0$ to the matrix $\mathbf{C}$. This suggests that the bias is also minimized at the selected focusing frequency.

5. Tightness of the upper bound

Earlier we stated that the upper bound of (9) is tight in the vicinity of the optimum point. In this section we discuss this issue. It is already shown that

$$\sum_{j=1}^{J}\sum_{i=1}^{q} w_j \sigma_i(\mathbf{A}_0\mathbf{A}_j^H) \le \sum_{j=1}^{J}\sum_{i=1}^{q} w_j \sigma_i(\mathbf{A}_0)\sigma_i(\mathbf{A}_j) \le \left(pq\sum_{i=1}^{q}\mu_i^2\right)^{\frac{1}{2}} \tag{19}$$

where $\mu_i, i = 1, \ldots, q$, are given by (12). We make some observations on (19).

1. The right-hand-side of (19) is independent of the focusing frequency.
2. If $\mathbf{A}_0 = \mathbf{T}_j\mathbf{A}_j, j = 1, \ldots, J$, then the bound is achieved. This case corresponds to complete focusing. It is seen that such $\mathbf{A}_0$ is an optimum focusing matrix. In other words, in a complete focusing the left-hand-side of (19) attains its maximum value. However, in practice complete focusing is not fulfilled. The criterion that we defined in (5) gives the closest case to the complete focusing. Thus the optimum value that is obtained from (5) is very close to the bound.
3. The closeness of the left-hand-side of (19) to the bound is a function of the number of frequency samples. Fig. 1 represents the left-hand-side of (19) normalized with respect to the norm of the vector $\mu = (\mu_1, \ldots, \mu_J)$, as a function of the number of frequency samples for a configuration of two sources arriving at eight sensors. As it is seen, increasing the number of frequency samples gives a tighter bound.

Based on these observations, we claim that the proposed method operates very close to the optimal case. This performance is, however, achieved with a considerable reduction in the computational complexity.

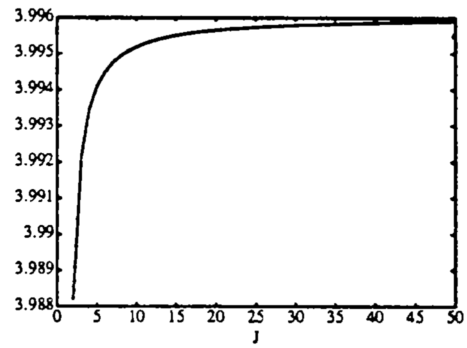

Fig. 1 The left-hand-side of (19) normalized with respect to the norm of the vector $\mu = (\mu_1, \ldots, \mu_J)$, as a function of the number of frequency samples.

6. Focusing and the ESPRIT Algorithm

Suppose that q far-field wideband sources are received by $2p > q$ wide-band matched sensor doublets. The sensors of the doublets are pairwise identical and the arrays are displaced with a known directional vector $\mathbf{d}$. The statistics of the signals and noises are as before.

The outputs of the subarrays are shown by p-vectors $\mathbf{x}(t)$ and $\mathbf{y}(t)$ with the i-th components

$$x_i(t) = \sum_{l=1}^{q} s_l(t - \tau_i(\theta_l)) + n_{x_i}(t), \qquad 1 \le i \le p, \tag{20}$$

$$y_i(t) = \sum_{l=1}^{q} s_l(t - \tau_i(\theta_l) - \tau_d(\theta_l)) + n_{y_i}(t), \qquad 1 \le i \le p, \tag{21}$$

where s_l is the l-th source signal, θ_l is the angle of arrival for the l-th source, $\tau_i(\theta_l)$ is the propagation delay for the l-th source at sensor i of the first subarray with respect to the reference point and $\tau_d(\theta_l) = \frac{d}{c}\sin\theta_l$ is the delay in the propagation between the two subarrays, where c is the propagation velocity and d is the distance between two subarrays. For linear array with uniform spacing, $\tau_i(\theta_l) = (i-1)\frac{\Delta}{c}\sin\theta_l$, where Δ is the spacing between two consecutive sensors and the reference point is at the first sensor of the first subarray. The noise components of the subarrays are represented by $n_{x_i}(t)$ and $n_{y_i}(t)$.

In the frequency domain (20) and (21) can be shown as

$$\mathbf{x}(\omega) = \mathbf{A}(\omega,\theta)\mathbf{s}(\omega) + \mathbf{n}_x(\omega). \quad (22)$$

$$\mathbf{y}(\omega) = \mathbf{A}(\omega,\theta)\Phi(\omega,\theta)\mathbf{s}(\omega) + \mathbf{n}_y(\omega). \quad (23)$$

where $\mathbf{A}(\omega,\theta) = [\mathbf{a}(\omega,\theta_1)\ldots\mathbf{a}(\omega,\theta_q)]$ is the $p \times q$ matrix of the steering vectors and assumed to be of full rank for every set of frequency and directions of arrival and

$$\Phi(\omega,\theta) = \mathrm{diag}(e^{-j\omega(d\sin\theta_1/c)},\ldots,e^{-j\omega(d\sin\theta_q/c)}) \quad (24)$$

is the rotation matrix of the phase delays between the two subarrays. From (22) and (23), the spatial auto-covariance matrix of the first subarray and the spatial cross-covariance matrix of the two subarrays are represented as

$$\mathbf{R}_{xx}(\omega) = \mathbf{A}(\omega,\theta)\mathbf{S}(\omega)\mathbf{A}^H(\omega,\theta) + \sigma^2\mathbf{I}, \quad (25)$$

$$\mathbf{R}_{xy}(\omega) = \mathbf{A}(\omega,\theta)\mathbf{S}(\omega)\Phi^H(\omega,\theta)\mathbf{A}^H(\omega,\theta), \quad (26)$$

where superscript H is the Hermitian transpose.

Hung and Kaveh [3] framed the CSM technique for the ESPRIT algorithm. The method is based on finding the transformation matrices, $\mathbf{T}_j$ and $\mathbf{Q}_j$, by

$$\min_{\mathbf{T}_j} \|\mathbf{A}_0 - \mathbf{T}_j\mathbf{A}_j\| \qquad j = 1,\ldots,J \quad (27)$$
$$\text{s.t.} \qquad \mathbf{T}_j^H\mathbf{T}_j = \mathbf{I},$$

and

$$\min_{\mathbf{Q}_j} \|\mathbf{A}_0\Phi_0 - \mathbf{Q}_j\mathbf{A}_j\Phi_j\| \qquad j = 1,\ldots,J \quad (28)$$
$$\text{s.t.} \qquad \mathbf{Q}_j^H\mathbf{Q}_j = \mathbf{I},$$

where for the simplicity of notation we represent $\mathbf{A}(\omega_j,\theta)$ and $\Phi(\omega_j,\theta)$ by $\mathbf{A}_j$ and Φ_j. Then new observation vectors are formed by

$$\mathbf{w}_j = \mathbf{T}_j\mathbf{x}_j, \qquad j = 1,\ldots,J. \quad (29)$$

$$\mathbf{z}_j = \mathbf{Q}_j\mathbf{y}_j, \qquad j = 1,\ldots,J. \quad (30)$$

These transformed observation vectors are used to construct the sample auto-correlation and cross-correlation matrices for frequency bin j. An average of these aligned correlation matrices gives universal sample correlation matrices that can be used for detection and localization of the sources.

In (27) and (28), the focusing frequency, f_0, is chosen arbitrarily. We introduce a criterion to obtain the optimum value of the focusing frequency. We show that the optimized focusing frequency that is found for the ESPRIT algorithm is the same as the one for the MUSIC method. This suggests that the optimum focusing can be done regardless of the procedure used to estimate the directions of arrival. We define a criterion based on the error involved in the transformation of the signal subspaces. The optimization of this criterion yields the focusing frequency. Specifically, we seek an f_0 which minimizes

$$\min_{f_0}\min_{\mathbf{T}_j}\min_{\mathbf{Q}_j}\sum_{j=1}^{J} w_j\left[\|\mathbf{A}_0 - \mathbf{T}_j\mathbf{A}_j\|^2 + \|\mathbf{A}_0\Phi_0 - \mathbf{Q}_j\mathbf{A}_j\Phi_j\|^2\right] \quad (31)$$
$$\text{s.t.} \qquad \mathbf{T}_j^H\mathbf{T}_j = I, \qquad j = 1,\ldots,J$$
$$\mathbf{Q}_j^H\mathbf{Q}_j = I.$$

For a fixed $\mathbf{A}_0$, it is already known, [2,6], that the optimal $\mathbf{T}_j$ and $\mathbf{Q}_j$ are given by

$$\mathbf{T}_j = \mathbf{V}_j\mathbf{U}_j^H, \quad (32)$$

$$\mathbf{Q}_j = \mathbf{E}_j\mathbf{F}_j^H, \quad (33)$$

where $\mathbf{V}_j$ and $\mathbf{U}_j$ are the left and right singular vectors of $\mathbf{A}_0\mathbf{A}_j^H$ and $\mathbf{E}_j$ and $\mathbf{F}_j$ are the left and right singular vectors of $\mathbf{A}_0\Phi_0\Phi_j^H\mathbf{A}_j^H$.

Thus, it is seen that in (31) the only argument of the minimization is the focusing frequency, f_0. In this case the subspace fitting error is given by

$$\sum_{j=1}^{J} w_j\left[\|\mathbf{A}_0 - \mathbf{T}_j\mathbf{A}_j\|^2 + \|\mathbf{A}_0\Phi_0 - \mathbf{Q}_j\mathbf{A}_j\Phi_j\|^2\right]$$
$$= \sum_{j=1}^{J} w_j\Big[\|\mathbf{A}_0\|^2 + \|\mathbf{A}_j\|^2 - 2\mathrm{Re}\ \mathrm{tr}(\mathbf{A}_0\mathbf{A}_j^H\mathbf{T}_j^H) +$$
$$\|\mathbf{A}_0\Phi_0\|^2 + \|\mathbf{A}_j\Phi_j\|^2 - 2\mathrm{Re}\ \mathrm{tr}(\mathbf{A}_0\Phi_0\Phi_j^H\mathbf{A}_j^H\mathbf{Q}^H)\Big]$$
$$= 4Jpq - 2\sum_{j=1}^{J}\sum_{i=1}^{q} w_j\left[\sigma_i(\mathbf{A}_0\mathbf{A}_j^H) + \sigma_i(\mathbf{A}_0\Phi_0\Phi_j^H\mathbf{A}_j^H)\right]. \quad (34)$$

Using Lemma 1, it is possible to show that

$$\sum_{j=1}^{J}\sum_{i}^{q} w_j\sigma_i(\mathbf{A}_0\Phi_0\Phi_j^H\mathbf{A}_j^H) \le \sum_{j=1}^{J}\sum_{i}^{q} w_j\sigma_i(\mathbf{A}_0)\sigma_i(\mathbf{A}_j). \quad (35)$$

Therefore the method that was proposed before can be used for ESPRIT algorithm as well.

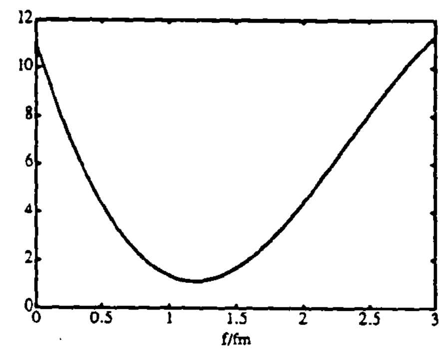

Fig. 2 The subspace fitting error.

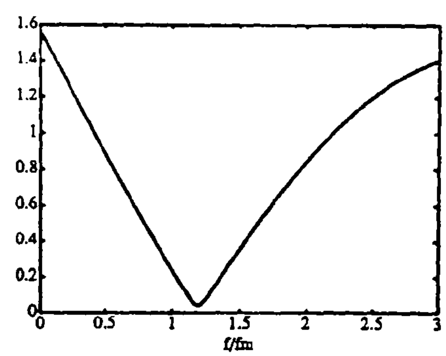

Fig. 3 The difference between the centroid matrix **C** and the focusing location matrix.

7. Simulation Results

Using a very simple scenario, we show that the mid-band frequency is not necessarily the best choice for focusing. In our example, we investigate a configuration of six sensors exposed to two sources impinging from 10 and 15 degrees (correspondingly, 0.1745 and 0.2618 radians). The signals of these sources are uncorrelated and compose of four frequencies, 0.5, 1.35, 1.45 and 1.5. These frequencies can also be considered as nonuniform samples of a wide-band signal. The signal-to-noise ratio at each sub-band is 20 dB. It is assumed that 20 batches of data are constructed. The error of the focusing is shown in Fig. 2. The optimum focusing frequency is 1.2 which has been found by the proposed method.

Fig. 3 represents the difference between the centroid matrix **C** and the focusing location matrix, $\mathbf{A}_0$, as a function of the focusing frequency. The closest $\mathbf{A}_0$ to the centroid is obtained for optimized frequency. For the focusing frequencies 1 and 1.2 we applied the MUSIC algorithm to estimate the angles of arrival. The results are shown in Fig. 4 and Fig. 5. As it is seen, when the mid-band frequency is chosen for focusing, the source at 10 degrees can not be estimated.

8. Summary

In this paper we have proposed a method to determine the focusing frequency for the Coherent Signal-subspace Method. The transformation of the subspaces into the focusing subspace is done by a unitary transformation. We defined a criterion based on the subspace fitting error and optimized a tight bound of it. The optimization is done in two steps. First, the singular values of the optimum location matrix are obtained. Then a one-variable nonlinear minimization problem is solved to get the focusing frequency. The simulation results show that the method successfully finds the global optimum value and improves the performance of the estimation.

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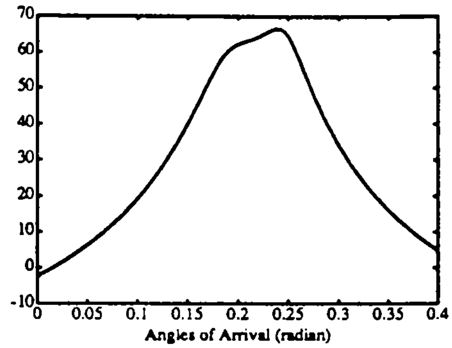

Fig. 4 MUSIC spatial spectrum for the focussed subspace. The focusing frequency is 1, the mid-band frequency.

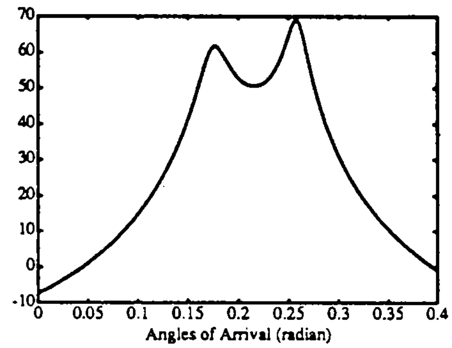

Fig. 5 MUSIC spatial spectrum for the focussed subspace. The focusing frequency is 1.2, the optimized frequency.