Fixed-Rate Entropy Coded Vector-Quantization

A. K. Khandani[†], P. Kabal[†,‡] and E. Dubois[†]

We introduce two methods for the fixed rate entropy coding of a memoryless source using a dynamic programming approach. The core of the schemes is a recursive relationship which is built in a hierarchy of steps. Each step involves the cartesian product of two lower dimensional subspaces. For $N=2^u$, the corresponding hierarchy is composed of u steps where the ith step, $i=0,\ldots,u-1$, is based on the (pair-wise) cartesian product of the 2^i-D subspaces.

A shell $F_N(C)$ is defined as the set of N-D points of self information C. We have the following recursive relationship,

$$F_N(C)=\bigcup[F_{N_1}(C_1)\otimes F_{N_2}(C_2)] \qquad (1)$$

where $\otimes$ denotes the cartesian product, $N=N_1+N_2$, and the union is computed over all the pairs (C_1,C_2) satisfying $C_1+C_2=C$. We are specially interested in the case that $N_1=N_2=N/2$. Each cartesian product element in (1) is called a *cluster*. The C_1/C_2 are in some sense the state variables of the system. To reduce the complexity, we are interested in merging the neighboring states (aggregating the shells into macro-shells).

For a given input vector $\mathbf{x}$, by decoding of a shell we mean the process of finding the element of the shell which has the minimum distance to $\mathbf{x}$. Using (1), we can decode a shell recursively. To do this, $\mathbf{x}$ is split into two parts $\mathbf{x}_1$ and $\mathbf{x}_2$ of lengths N_1 and N_2. Assume that the nearest vectors of $F_{N_1}(C_1)/F_{N_2}(C_2)$ to $\mathbf{x}_1/\mathbf{x}_2$ are equal to $\hat{\mathbf{x}}_1/\hat{\mathbf{x}}_2$ with the minimum distances d_1/d_2. The nearest vector of $F_{N_1}(C_1)\otimes F_{N_2}(C_2)$ to $\mathbf{x}$ is equal to $(\hat{\mathbf{x}}_1,\hat{\mathbf{x}}_2)$ with the minimum distance d_1+d_2. The minimum distance of a shell is equal to the lowest of the minimum distances of its clusters. For $N_1=N_2=N/2$, if we know the minimum distance and the nearest vector for all the shells of the $N/2$-D subspaces, we can decode all the N-D shells. We can also use the recursive structure of the shells to develop an algorithmic addressing/reconstruction procedure. The basic idea is that the addressing within each cluster can be achieved independently along its lower dimensional components. It remains to select a single cluster within a shell. This is achieved by arranging the clusters in a preselected order and assuming that the points in a higher order cluster have a larger label. Based on this ordering, a cluster is selected according to the range of the index and the corresponding residue with respect to the start of the range is used for the addressing within the cluster. In the following, we introduce two methods for the recursive aggregation of shells.

In the first method, the one-D symbols are aggregated into K information macro-shells with a fixed spacing (increment in the self-information) Δ. The probabilities of the points in the ith macro-shell satisfy $0<-\log_2 p\le c_0$ for $i=0$ and $c_0+(i-1)\Delta<-\log_2 p\le c_0+i\Delta$, for $i=1,\ldots,K-1$. Obviously, some of the one-D shells may remain empty. The higher-dimensional macro-shells are considered as the set of the high dimensional symbols with a fixed sum of the indices. This results in a recursive merging rule for the states. The final subset is selected as the union of the N-D macro-shells with the sum of the indices less than a given value $L_{\max}$. This results in $\min[2^iK, L_{\max}]$ states in the ith step of our hierarchy.

In the second method, we have a sequential aggregation of the macro-shells in the 2^i-D subspaces, $i=0,\ldots,u-1$. In other words, the merging of the states is achieved gradually at different stages of our hierarchy. The subspaces involved at each step of the hierarchy are partitioned into a number of macro-shells of increasing average self information and identical cardinalities. The key point is to approximate the self information of the points within a given macro-shell at each stage by their average value. In other words, all the points within a macro-shell are assumed to have the same self information as their average value. To facilitate the addressing, the macro-shells are restricted to have an integral bit rate. The corresponding addressing is decomposed into a hierarchy of addressing steps where each step selects a macro-shell among a relatively *small* number of similar macro-shells. As the subsets involved in each addressing step are of *integral, equal bit rates*, the corresponding addressing has a trivial complexity. A similar addressing scheme is discussed in detail in [1].

Method	N	R	Memory	Computation	SNR (dB)
SMS	16	1.5	1.25 k	54(33)	7.43
L-F	16	1.5	7.9 k	670	7.47
SMS	16	2.5	2.5 k	220(97)	12.91
L-F	16	2.5	21.0 k	2240	13.00
SMS	32	3.5	14.3 k	1060(290)	18.7
L-F	32	3.5	307 k	12500	18.8†

Table 1: Comparison between our method based on the sequential merging of shells (denoted by SMS) with the scheme of [2] (denoted by L-F) in conjunction with a memoryless Gaussian source (N is the dimensionality and R is the rate per dimension.). The memory size is in byte (8 bits) per N dimensions and the computational complexity is the number of additions/comparisons per dimension. The values inside parenthesis are the computational complexities of our method in the case of allowing for the parallel processing. (The value denoted by † is obtained using interpolation.).

Table 1 presents a comparison between the present method and the scheme of [2] in terms of performance and complexity. Other advantages of our method are as follows: (i) it can be more easily used in conjunction with a quantization lattice, (ii) it allows for the parallel processing.

References

[1] A. K. Khandani and P. Kabal, "Shaping multi-dimensional signal spaces—Part II: shell-addressed constellations," To appear in the *IEEE Trans. Inform. Theory*,.

[2] R. Laroia and N. Farvardin, "A structured fixed-rate vector quantizer derived from variable-length encoded scalar quantizers," *Twenty-Fourth Annual Conference on Information Sciences and Systems*," pp. 796–801, Princeton, NJ, March 1990.

[†]INRS-Telecomm, 16 Place du Commerce, Verdun, Canada, H3E-1H6
[‡]McGill University, 3480 University, Montreal, Canada, H3A-2A7
This work was supported by Natural Sciences and Engineering Research Council of Canada (NSERC).