

An Efficient Bit Allocation Algorithm for Multicarrier Modulation

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Abstract

In this paper we present an efficient bit allocation algorithm for multicarrier systems operating in frequency-selective environments. The proposed algorithm strives to maximize the overall throughput while guaranteeing that the mean bit error rate (BER) remains below a prescribed threshold. The algorithm is compared with several other algorithms found in literature in terms of the overall throughput, mean BER, and relative computational complexity. Furthermore, the algorithms are compared with an exhaustive search routine to determine the optimal bit allocation in terms of maximizing throughput given the constraint on error performance. No power allocation is performed by the algorithms. Results show the proposed algorithm has approximately the same throughput and mean BER as the optimal solution while possessing a significantly lower computational complexity relative to the other algorithms with similar performance. When compared to algorithms which employ approximations to waterfilling, the computational complexity is comparable while the overall throughput is closer to the optimum.

Keywords: Wireless Local Area Networks, Bit Loading, Wireless Multicarrier Transmission, Adaptive Modulation

1 Introduction

Several high-speed data transmission systems, such as wireless local area networks (WLAN) [1,2] and xDSL modems [3], employ multicarrier modulation at the core of their design. Multicarrier modulation operates by transmitting data in parallel subcarriers at a lower data rate, effectively transforming a frequency selective fading channel into a collection of flat fading subchannels. Thus, simple techniques can be employed at the receiver to reverse the effects of the channel.

Conventional wireless multicarrier systems use a fixed signal constellation across all subcarriers, thus the overall error probabilities are dominated by the subcarriers with the

worst performance. To improve the system error performance, adaptive bit allocation can be employed such that the information is redistributed across the subcarriers in order to minimize the overall error probability. This redistribution is achieved by varying the signal constellation size across the subcarriers according to the measured signal-to-noise ratio (SNR) values. In extreme cases, poorly performing subcarriers can be “turned off” or *nulled* [4].

Most bit allocation algorithms can be classified into three categories: incremental (i.e. “greedy”) allocation [5–8], channel capacity approximation-based allocation [9,10], and bit error probability expression-based allocation [11,12]. The first type of algorithm incrementally allocates an integer number of bits while the other two types use closed-form expressions of performance measures in order to determine a non-integer bit allocation and then round the results. On the other hand, bit allocation algorithms can also be classified according to the objective functions they are attempting to optimize. Common choices are the maximization of the overall throughput given a total power constraint, known as *rate-adaptive* loading [10], and the minimization of the energy given a fixed throughput, known as *margin-adaptive* loading [9]. Both cases also employ an error rate constraint. Although some algorithms may have certain advantages over others in terms of how close they come to the optimum allocation *or* how quickly they reach their final allocation, in this work, we present a rate-adaptive allocation algorithm that tries to balance these two criteria while attempting to maximize the throughput over a set of modulation schemes given that the mean bit error rate is below some prescribed threshold.

The paper is organized as follows. In Section 2, an overview of the three current types of bit allocation algorithms is presented. In Section 3, the proposed algorithm is described in detail. In Section 4, the simulation results are presented and comparisons between the proposed algorithm and four other algorithms are made with respect to throughput, mean BER, and execution time. Several concluding remarks are made in Section 5.

2 Previous Work

Before presenting the proposed loading algorithm, several bit allocation algorithms are briefly covered in order to

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define the current state-of-the-art for each of the three common types of allocation algorithms. The values of the subcarrier signal-to-noise ratios (SNR) are available to these algorithms, obtained via data-aided channel estimation at the receiver during the initialization phase of the system as well as during transmission, and sent to the transmitter using feedback.

2.1 Incremental Allocation

Most incremental allocation algorithms are *greedy* algorithms, where the algorithm allocates one bit at a time to the subcarrier that will do the most good for the current partial allocation. The algorithm is called greedy since it only maximizes the reduction in distortion for each step without regard to the global effects of its choice [13].

One example of a bit and power allocation algorithm for multicarrier systems was developed by Hughes-Hartog [5]. Starting from an all-zero allocation, this algorithm allocates an additional bit to the subcarrier requiring the smallest incremental energy until either the total power or aggregate bit error rate constraints are not violated. Another allocation algorithm that can be applied to bit allocation is by Fox [6], where bits are allocated incrementally to the subcarriers which maximize the ratio of the change in throughput to the change in bit error rate (BER). Finally, the bit allocation algorithm by Wyglinski, Kabal, and Labeau [7, 8] starts off with all the subcarriers allocated with the maximum number of bits, and then incrementally removes bits from the subcarriers with the worst BER values until the mean BER constraint is satisfied.

Although this type of algorithm may approach the optimal allocation in terms of maximizing the throughput given a BER constraint, they can have a high computational complexity, requiring numerous complex iterations before reaching a final allocation.

2.2 Channel Capacity Approximation-Based Allocation

A solution to the complexity problem is to perform bit allocation based on closed-form expressions of some error performance criteria. One approach uses an approximation of the Shannon capacity expression to determine the number of bits to be allocated per subcarrier.

For instance, to find the number of bits, b_i , for subcarrier i , the allocation algorithm of Chow, Cioffi, and Bingham [9] uses the expression

$$b_i = \log_2 \left(1 + \frac{\gamma_i}{\Gamma} \right), \quad (1)$$

where γ_i is the SNR of subcarrier i , and Γ is the difference in SNR values corresponding to the maximum number of bits the system can sustain, given a target probability of error P_T , and the capacity normalized by the signal bandwidth. The parameter Γ is also known as the *SNR Gap*. Assuming equal energy across all used subcarriers, Γ is adjusted until

the target bit rate is exceeded. Since the number of bits is non-integer, the allocation is rounded to the nearest integer value. After the bit allocation, the transmission power levels are then adjusted in order to achieve the same subcarrier bit error rate, P_i , per non-nulled subcarrier.

The allocation algorithm presented by Leke and Cioffi [10] assigns energy to different subcarriers in order to maximize the data rate for a given SNR margin. A sort and search is performed in order to find which subcarriers should be left on while others shut off. The bits are then allocated to each subcarrier using the SNR gap approximation of Eq. (1).

2.3 Probability of Error-Based Allocation

Bits can also be allocated using closed-form expressions for the probability of error, given the target probability of error and the SNR. For instance, the probability of error expression for M_i -QAM on subcarrier i is given by [14]

$$P_{M_i,i}(\gamma_i) = 4 \left(1 - \frac{1}{\sqrt{M_i}} \right) Q \left(\frac{3\gamma_i}{M_i - 1} \right) \cdot \left(1 - \left(1 - \frac{1}{\sqrt{M_i}} \right) Q \left(\frac{3\gamma_i}{M_i - 1} \right) \right) \quad (2)$$

where $\log_2(M_i)$ gives the (non-integer) number of bits to represent a signal constellation point, and $Q(\cdot)$ is the Q-function, defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt. \quad (3)$$

Making several simplifying approximations, M_i can be determined and discretized.

For instance, Fischer and Huber [11] distribute the bits and power across the subcarriers in order to minimize the error probability on each subcarrier. Using the union bound as an equality for the symbol error probabilities of QAM modulation, the algorithm iteratively distributes the bits and power until the probability of error on all subcarriers are equal. This algorithm is subjected to a total rate and total power constraint.

3 Proposed Algorithm

From the previous section, several allocation algorithms have been introduced. However, they are either too complex, such as the incremental algorithms, or do not approach the optimal allocation, such as the capacity approximation and probability of bit error-based algorithms. Due to the trade-off between the complexity of the bit allocation algorithm and its effectiveness at maximizing the throughput, what is needed is an algorithm which accurately maps the subcarrier SNR values to some final bit allocation in a low complexity fashion.

3.1 Algorithm Description

One solution to this problem is to limit the maximum BER, \hat{P} , allowed per subcarrier across all the N subcarri-

ers. Therefore, the modulation scheme with the largest signal constellation for which its BER is below \hat{P} is chosen for each subcarrier. Given the probability of bit error and number of bits for subcarrier i , P_i and b_i , the mean of the subcarrier BER values, \bar{P} , defined as

$$\bar{P} = \frac{\sum_{i=1}^N b_i P_i}{\sum_{i=1}^N b_i}, \quad (4)$$

is then computed and compared against the BER threshold, P_T . In this work, closed-form expressions of the probability of bit error are used, namely [14]

$$P_{2,i}(\gamma_i) = Q\left(\sqrt{2\gamma_i}\right) \quad (5)$$

for BPSK and Eq. (2) for QPSK, rectangular 16-QAM, and rectangular 64-QAM. Although these closed form expressions were derived for the AWGN channel case, they can be employed on a subcarrier basis since the subbands are narrow enough that the channel is spectrally flat across each subband.

If \bar{P} is below P_T , the target probability of error, \hat{P} is increased by an amount δ . On the other hand, if \bar{P} is above P_T , \hat{P} is reduced by an amount δ . This continues until the algorithm crosses over P_T , in which case the δ is reduced and the algorithm converges fast to a final allocation.

The complete operation of the proposed algorithm is described as follows:

1. Given γ_i , $i = 1, \dots, N$, compute the P_i values for all the subcarriers.
2. Calculate \bar{P} for the case when all subcarriers employ the largest signal constellation.
3. If the resulting \bar{P} is below P_T , set the final allocation to the largest signal constellation for all subcarriers and end the algorithm.

The previous step provides for a quick exit from the algorithm when the subcarrier SNR values are large enough.

4. Calculate \bar{P} for the case when the subcarrier with the largest γ_i employs the smallest signal constellation and the other subcarriers are nulled.
5. If the resulting \bar{P} is above P_T , turn off all subcarriers and end the algorithm.

The previous step provides a quick exit from the algorithm if the subcarrier SNR values are too low.

5. Find the largest signal constellation for subcarrier i for which P_i is below \hat{P} .
6. Compute the current value of \bar{P} .
7. If the current and previous values of \bar{P} are either both above or both below P_T , go to Step 8, else go to Step 9.

8. If both current and previous \bar{P} values are above P_T , reduce \hat{P} by a factor δ and go to Step 5, else increase \hat{P} by a factor δ and go to Step 5.
9. If the previous and current allocations differ by one signal constellation level, make the allocation with \bar{P} below P_T the final allocation and end the algorithm, else go to Step 10.
10. Reduce δ .
11. If the current allocation gives a \bar{P} that is above P_T , reduce \hat{P} by a factor δ and go to Step 5, else increase \hat{P} by a factor δ and go to Step 5.

In the case that the previous and current \bar{P} values straddle P_T , as in Step 9, the allocations are compared in order to see if they differ by one signal constellation. If they do, it is obvious that the additional bit(s) is/are the cause of the violation of the mean BER constraint. Otherwise, we reduce δ until the case of one differing signal constellation is achieved.

3.2 Channel Characterization for Initial Peak Bit Error Rate Threshold

The speed at which the algorithm in Section 3.1 reaches its final allocation depends on the choice of the initial \hat{P} and δ it uses. Therefore, it is desirable to estimate the initial values for \hat{P} and δ before starting the iterations using the available information, i.e. subcarrier SNR values.

One approach to this problem is to determine how much any given subcarrier can individually exceed P_T while \bar{P} remains below it. Given that a subcarrier can support one of five possible modulation schemes, resulting in five possible values for P_i , we define the largest P_i value that is below P_T as β_i ; while the smallest value of P_i above P_T as α_i . Using this leeway, ΔP , which is the difference between P_T and the sum of β_i , it can be determined how many subcarriers can have a P_i which exceeds P_T on an individual basis while still maintaining a \bar{P} below it.

The algorithm for finding the peak BER estimate is as follows:

1. Given the subcarrier SNR values, γ_i , calculate P_i for all the different modulation schemes which could potentially be employed in the system.
2. Find β_i , the largest P_i that does not exceed P_T .
3. Find α_i , the smallest P_i that exceeds P_T .
4. Find all values of β_i that are within an order of magnitude of its largest β_i and assign their indices to a set \mathcal{S} (β_i not within an order of magnitude can be neglected).
5. Given β_i , $i \in \mathcal{S}$, we need to determine ΔP to have several subcarriers exceed P_T on an individual basis while

having \bar{P} below P_T . In this case, we have

$$P_T = \frac{\sum_{i \in S} b_i \beta_i + \Delta P}{\sum_{i \in S} b_i} \quad (6)$$

or equivalently

$$\Delta P = P_T \cdot \sum_{i \in S} b_i - \sum_{i \in S} b_i \beta_i. \quad (7)$$

6. Add up the values of α_i , from smallest to largest, until the sum is greater than ΔP . Once exceeded, the last value of α_i added to the sum is chosen as the initial \hat{P} for the algorithm described in Section 3.1.

The initial value of δ is directly proportional to the average SNR of the system, $\bar{\gamma}$. As a result, using empirical measurements, the values for δ as a function of $\bar{\gamma}$ was determined. Using these values of δ in conjunction with the initial \hat{P} algorithm, the number of iterations required to find the final \hat{P} can be reduced by as much as half when compared to a scheme using an initialization that is independent from the channel.

4 Simulations

4.1 System Configuration

In this work, we refer to the IEEE Std. 802.11a [1], a wireless local area network (WLAN) standard employing conventional multicarrier modulation, in order to obtain realistic system parameters, such as the number of subcarriers, the frequency band of operation, and available modulation schemes. The signal constellations used are BPSK, QPSK, rectangular 16-QAM, and rectangular 64-QAM. The subcarrier can also be nulled, depending on subcarrier SNR values. Unlike the standard, where the same modulation scheme is employed across all subcarriers, the allocation algorithms can use a different modulation scheme per subcarrier. As in other studies, we consider only uncoded systems for the sake of straightforward comparison. However, the introduction of coding would improve the performance relative to an uncoded system and can be accounted for by a nonlinear modification of the SNR value, in relationship with the coding gain.

In this work we evaluated the proposed algorithm alongside with the algorithms which solely perform bit allocation, namely, Fox [6], Wyglinski, Kabal, & Labeau [7, 8], and Leke & Cioffi [10], where the multicarrier system employs 52 subcarriers (as in IEEE Std. 802.11a) and has P_T values of 10^{-3} and 10^{-5} . Furthermore, an exhaustive search algorithm was also employed with a reduced number of subcarriers over a portion of the band, to keep the complexity manageable, in order to determine how close the various methods were to the optimal solution.

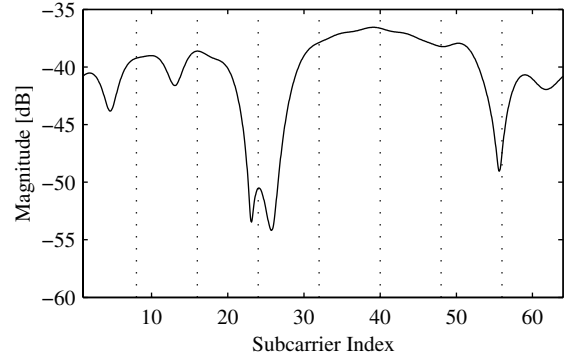


Fig. 1 Frequency response of an indoor channel environment in the 5.15–5.25 GHz U-NII band with transmitter/receiver distance of 50 m. Note that for the case of 8 subcarriers, only a portion of the channel (the boundaries of which are indicated by the dotted lines) is used.

4.2 Channel Model

Since one of the target applications of these algorithms is WLAN, a channel impulse response that adequately modelled an indoor environment was required. The statistical indoor propagation modelling technique devised by Saleh and Valenzuela [15] is used.

In these experiments, WLAN systems, such as IEEE Std. 802.11a [1] or HiperLAN/2 [2], operate at around 5 GHz, such as in the lower portion of the unlicensed national information infrastructure (UNII) band at 5.15–5.25 GHz [16] for IEEE Std. 802.11a. The transmitter-receiver separation was varied between between 1 m and 60 m and the signal, which is composed of 52 subcarriers, was transmitted over a 16.6 MHz bandwidth. Furthermore, there was no line-of-sight and the channel was assumed to be quasi-stationary, thus time-invariant during the adaptation phase of the allocation algorithm. Finally, only a single pair of antennas were employed. An example of a typical channel frequency response is shown in Fig. 1. It can be observed that the channel experiences frequency selective fading, with nulls as deep as 20 dB.

For each channel realization, the algorithms were operating at 70 different TSNR values ranging from 70 dB to 140 dB. The TSNR is defined here as the nominal transmitted power divided by the noise power in the signal bandwidth. When measured this way, the TSNR values tend to be large relative to SNR values measured at the receiver due to the channel attenuation. For instance, the channel attenuation is approximately 80 dB across a 50 m distance at 5 GHz. The trials were repeated for 10000 different channel realizations and the results averaged. Furthermore, the change in TSNR corresponds to the change in transmitter/receiver separation distance.

4.3 Results and Discussion

In Fig. 2, the overall throughput of the five bit allocation algorithms are presented for the case of 8 subcarriers. The algorithm of Leke and Cioffi does not reach the same throughput as the other algorithms until high TSNR values of 130 dB. As for the other methods, the difference in throughput between them is small. The largest throughput is produced by the exhaustive search algorithm, followed by both Fox's and Wyglinski, Kabal, & Labeau's algorithms, and finally by the proposed algorithm. Since the objective function is not concave and the constraint function is not strictly convex, there is no guarantee that Fox's algorithm would reach the optimal allocation [6].

The \bar{P} values corresponding to the throughputs in Fig. 2 are shown in Fig. 3. It can be observed that all the algorithms, except for Leke and Cioffi, have approximately the same values as the exhaustive search algorithm. The algorithm by Leke & Cioffi possesses values of \bar{P} that are significantly lower than the other algorithms at the expense of lower throughput. Since the algorithm of Leke and Cioffi does not check if the bit allocation exceeds P_T , there is a possibility that P_T may be violated. In such cases, the results of that allocation were not considered. Table 1 shows the number of violations as a percentage of the total number of channel realizations per TSNR value.

The results are similar when 52 subcarriers are employed, as shown in Fig. 4. All the algorithms, except for Leke and Cioffi, achieve nearly the same throughput with some small differences. The throughput of the algorithm of Leke and Cioffi is substantially less than that of the other methods, only reaching the other algorithms at high TSNR values. Note how at low TSNR values, the algorithm of Leke and Cioffi goes to zero. This is mostly due to the algorithm producing allocations that exceed P_T . Table 1 shows the number of violations. The corresponding \bar{P} values are shown in Fig. 5. As in the 8 subcarrier case, except for Leke & Cioffi, all the algorithms have approximately the same values.

As seen in Fig. 2, the throughput of the proposed algorithm is very close to that of the optimal algorithm. In Fig. 4, the proposed algorithm also has one of the largest throughput values. However, as for the execution times, the proposed algorithm executes much more quickly relative to either Fox or Wyglinski, Kabal, & Labeau. Although Leke & Cioffi may execute at the same speed as the proposed algorithm, the latter achieves far greater throughput. A summary of mean and worst-case computation times for a 52 subcarrier system with a P_T of 10^{-5} is shown in Table 2 for several TSNR values. For a fair comparison, all algorithms were programmed in C and executed on the same workstation (Intel Pentium IV 2 GHz processor).

Table 1 Number of P_T Violations by the bit allocation algorithm of Leke & Cioffi.

TSNR	81 dB	86 dB	91 dB	106 dB	116 dB
$N = 8, P_T = 10^{-3}$	8.23%	3.53%	0.66%	0.00%	0.00%
$N = 52, P_T = 10^{-5}$	54.95%	96.84%	99.94%	19.62%	1.99%

Table 2 Mean (Worst) computation times in milliseconds at different TSNR values, 52 subcarriers, $P_T = 10^{-5}$

Algorithm	91 dB	106 dB	121 dB	136 dB
Fox	1.13 (3.23)	1.48 (5.01)	1.41 (5.00)	1.37 (4.40)
Leke & Cioffi	0.94 (2.78)	0.96 (4.98)	0.93 (4.24)	0.90 (4.66)
Wyglinski <i>et al.</i>	1.09 (2.86)	0.91 (4.10)	0.84 (2.09)	0.80 (2.62)
Proposed	0.91 (2.96)	0.91 (2.71)	0.86 (3.98)	0.82 (4.54)

5 Conclusion

An efficient bit allocation scheme was presented which achieves a throughput that is close to the optimal solution while possessing a relatively small computation time. The algorithm uses a peak BER threshold which the subcarriers cannot exceed. The mean BER and overall throughput are computed and the peak BER is adjusted iteratively until the throughput is maximized while guaranteeing that the mean bit error rate is less than some specified threshold. The proposed algorithm is compared with three practical bit allocation algorithms as well as an exhaustive search algorithm. The results show that the proposed algorithm approaches the optimal solution while achieving a low computational complexity.

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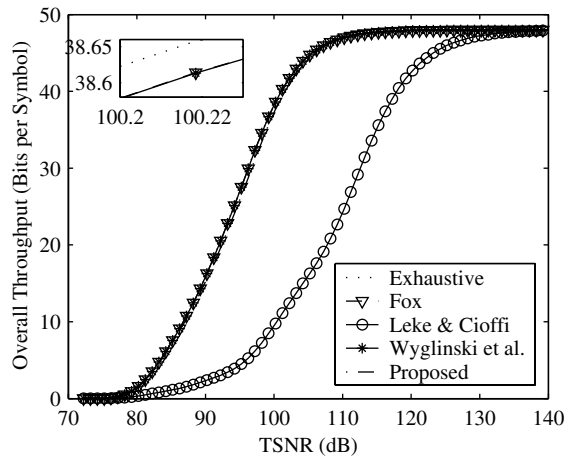


Fig. 2 Throughput of systems with 8 subcarriers satisfying a P_T of 10^{-3} . The range of points plotted corresponds to transmitter/receiver separation distances of 1 m to 60 m.

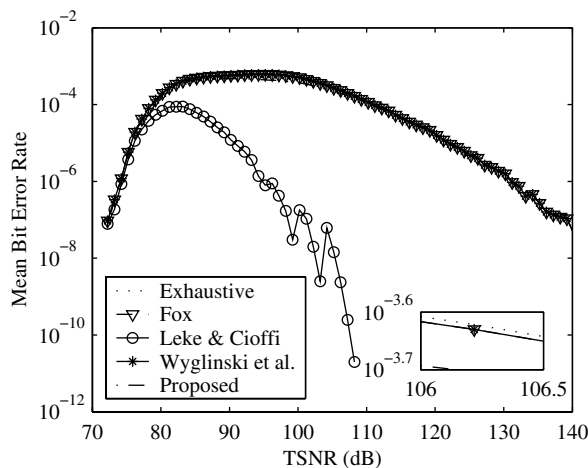


Fig. 3 Mean BER of systems with 8 subcarriers satisfying a P_T of 10^{-3} . Except for the curve corresponding to Leke & Cioffi, all the curves are superimposed.

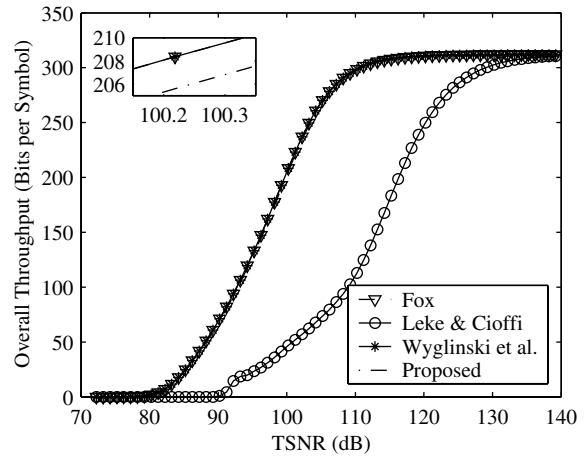


Fig. 4 Throughput of systems with 52 subcarriers satisfying a P_T of 10^{-5} . The range of points plotted corresponds to transmitter/receiver separation distances of 1 m to 60 m.

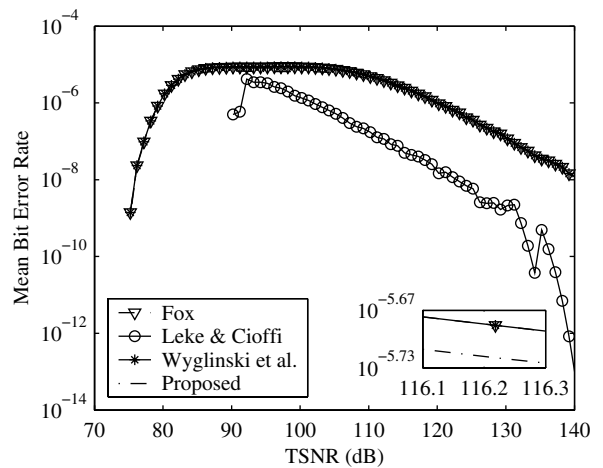


Fig. 5 Mean BER of systems with 52 subcarriers satisfying a P_T of 10^{-5} . Except for the curve corresponding to Leke & Cioffi, all the curves are superimposed.