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Performance of Noise-Shaping in Oversampled Filter Banks

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Abstract

The use of a noise-shaping system in oversampled filter banks has been shown to improve the effective resolution of subband coders. In this paper, a brief review of past work is given and a study of the effects of the characteristics of the employed filter banks is presented. It is shown that an increase in filter length and an increase in the degree of overlap between neighboring channels contribute independently to a better performance. Also, it is shown that near-perfect reconstruction filter banks are limited by their reconstruction error but yield good results at a low bitrate. This is supported by both theoretical and experimental results.

1 Introduction

The introduction of a noise-shaping (NS) system in oversampled filter banks (OFB) stems from the use of a such a system in oversampled analog to digital conversion. In fact, single-bit codewords obtained from artificially high sampling rates were achieved soon after delta modulation was introduced in 1946 [2].

At the output of the analysis filters of an OFB, a redundant representation of the input signal is obtained. Bölcskei and Hlawatsch drew an analogy with oversampled A/D conversion, suggesting the application of NS to the subband signals [1]. Due to the combination of the shape of the analysis filters and the oversampling of the signals, the information content of the subband signals is mostly contained within an interval of $2\pi/K$, where K is the oversampling factor. Consequently, if the noise were to be pushed outside of this interval, it would subsequently be attenuated by the reconstruction filters, leading to an overall reduction of the quantization error. This is indeed the purpose of the NS filters, and their effectiveness was shown through a significant improvement of the resolution of the quantizers [1].

The design of the NS filters themselves depends directly on the analysis and synthesis filters and will be discussed briefly in Section 2. Unfortunately, there has not yet been a study of the effect of varying parameters such as filter length, degree of overlap between subbands or PR (perfect-reconstruction) versus near-PR filter banks (FBs). Moreover, few experimental results have been reported, and the actual numerical stability of the procedure has yet to be investigated. This work aims to characterize these effects: Section 3 sets the background for the performed studies, while Section 4 shows the result.

2 Noise-Shaping in Oversampled Filter Banks

The system proposed in [1] is illustrated in Fig. 1: a MIMO (multiple-input-multiple-output) system denoted by the $N \times N$ matrix $\mathbf{G}(z)$ is inserted between the N-subband analysis and synthesis filter banks.

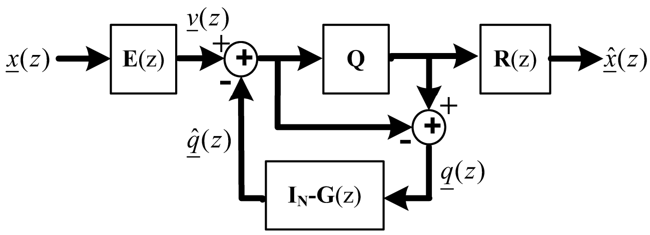

Fig. 1 OFB with Noise-Shaping

In this figure, $\mathbf{E}(z)$ and $\mathbf{R}(z)$ are, respectively, the $N \times M$ analysis and $M \times N$ synthesis polyphase domain representations of the FBs, where M is the downsampling factor [1]. It should be noted that $\underline{x}(z)$ and $\hat{\underline{x}}(z)$ are M-dimensional vectors and $\underline{v}(z)$, $\underline{q}(z)$ and $\hat{\underline{q}}(z)$ are N-dimensional vectors.

As discussed in [1], the ideal noise-shaper is not causal and so cannot operate in a feedback loop. Consequently, the MIMO noise-shaper is constrained to be FIR (finite impulse response), causal and of the form:

$$\mathbf{G}(z) = \mathbf{I_N} - \sum_{l=1}^{L} \mathbf{G}_l z^{-l}$$

where L is the order of the NS system.

2.1 Complete Interchannel Noise-Shaping System

As shown in [1], the reconstruction error variance σ_e^2 —under the assumption that the quantization noise σ_q^2 is white, uncorrelated and equal in all channels— can be expressed as a function of the NS coefficients $\mathbf{G}_i$ and $\mathbf{\Gamma}_l$. The $\mathbf{\Gamma}_l$ are defined as: $\mathbf{\Gamma}_l = \sum_{m=-\infty}^{\infty} \mathbf{R}_m^T \mathbf{R}_{m+l}$, where the $\mathbf{R}_m$ are $\mathbf{R}(z) = \sum_{m=-\infty}^{\infty} \mathbf{R}_m z^{-m}$. Further it is noted that the $\mathbf{\Gamma}_l$ satisfy $\mathbf{\Gamma}_{-l}^T = \mathbf{\Gamma}_l$, while in addition $\mathbf{\Gamma}_l = \mathbf{0}$ for $|l| > J$ for causal filters of finite length $L_h = JM$ ($J \in \mathcal{N}$).

By setting the partial derivatives $\partial\sigma_e^2/\partial\mathbf{G}_i = \mathbf{0}$ for $i = 1, 2, \ldots, L$ the linear set of equations

$$\begin{bmatrix} \mathbf{\Gamma}_0 & \cdots & \mathbf{\Gamma}_{-(L-1)} \\ \mathbf{\Gamma}_1 & \cdots & \mathbf{\Gamma}_{-(L-2)} \\ \vdots & \ddots & \vdots \\ \mathbf{\Gamma}_{(L-1)} & \cdots & \mathbf{\Gamma}_0 \end{bmatrix} \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \\ \vdots \\ \mathbf{G}_L \end{bmatrix} = \begin{bmatrix} \mathbf{\Gamma}_1 \\ \mathbf{\Gamma}_2 \\ \vdots \\ \mathbf{\Gamma}_L \end{bmatrix} \quad (1)$$

are obtained and yield the optimal complete interchannel NS coefficients ($\mathbf{G}_{i,\text{opt}}$) when solved. Finally, the minimum reconstruction error variance is given by:

$$\sigma^2_{e,\min} = \frac{\sigma^2_q}{M}\mathrm{Tr}\left\{\mathbf{\Gamma}_0 - \sum_{l-=1}^{L}\mathbf{\Gamma}_l^T\mathbf{G}_{l,\mathrm{opt}}\right\}. \qquad (2)$$

2.2 Local Interchannel NS System

In order to reduce the computational complexity of the system, the number of considered interchannel dependencies can be reduced. Here, the use of dependencies on only the adjacent channels is explored. In this case, the same method as that for the complete interchannel noise-shaper is used, but only the main and off-diagonal elements of the $\mathbf{\Gamma}_l$ are non-zero. Again the minimum reconstruction error variance is given by substituting the obtained coefficients into Eq. (2).

2.3 Intrachannel Noise-Shaping System

Because the solution to Eq. (1) involves the inversion of a matrix whose dimensions grow linearly with the order of the noise-shaper and the length L_h of the synthesis filters, performance is once more traded for reduced complexity and (as will be seen in section 3) better conditioning of the matrices. Here, the NS system is constrained to take advantage of *intra*channel dependencies only, meaning that there are separate NS systems in each channel, and thus a diagonal $\mathbf{G}(z)$. The determination of the $\mathbf{G}_{i,\mathrm{opt}}$ then involves only the inversion of N $L \times L$ matrices, as opposed to the inversion of one large $NL \times NL$ matrix, as in the previous two cases [1]. This time, σ^2_e is expressed as function of the diagonal elements of $\mathbf{\Gamma}_l$ and $\mathbf{G}_i$:

$$\sigma^2_{e,\min} = \frac{\sigma^2_q}{M}\sum_{i=0}^{N-1}\left[\gamma^{(0)}_{i,i} - \sum_{l=1}^{L}\gamma^{(l)}_{i,i}g^{(l)}_{i,i;\mathrm{opt}}\right], \qquad (3)$$

where $g^{(l)}_{i,i} = [\mathbf{G}_l]_{i,i}$ and $\gamma^{(l)}_{i,i} = [\mathbf{\Gamma}_l]_{i,i}$. [1].

3 Investigation Setup

In order to gauge the performance of various filters, a filter design algorithm based on [3] and [6] was developed. The algorithm generates cosine modulated (CM) FBs. A prototype filter was first designed based on minimizing the stop-band energy while satisfying the PR constraint, i.e. such that the overall response $\mathbf{P}(z)$ of the cascade of the analysis and synthesis polyphase matrices $\mathbf{R}(z)\mathbf{E}(z)$ has the form $\mathbf{P}(z) = cz^{-d}\mathbf{I}_N$ where c is an arbitrary constant and d is the delay through the system. The satisfaction of the PR constraint was achieved by using a lattice structure. The lattice coefficients were then iteratively optimized using a recursive algorithm to minimize the stop-band energy ϕ of the prototype filter. When the algorithm converged to a solution, the resulting prototype filter was modulated to obtain the member filters of the FB. In addition, the development of the algorithm allowed for the selection of an arbitrary number of subbands and of the prototype filter length. The FBs generated using this algorithm will be referred to as $\mathrm{CM}_{\mathrm{PR}}$ FBs.

Next, Eqs. (2) and (3) were used to predict the reconstruction error variance, and then the system in Fig. 1 was implemented to confirm the results. The quantizers used had equal step sizes in all the subbands and an infinite dynamic range. The input signal was a randomly generated AR(1) process, with $\rho = 0.9$ and signal variance $\sigma^2_s \approx 7.21$ dB.

In this section, the numerical stability of the solution for the optimal NS coefficients is first discussed, followed by a comparison of the performances of the three NS shaping systems.

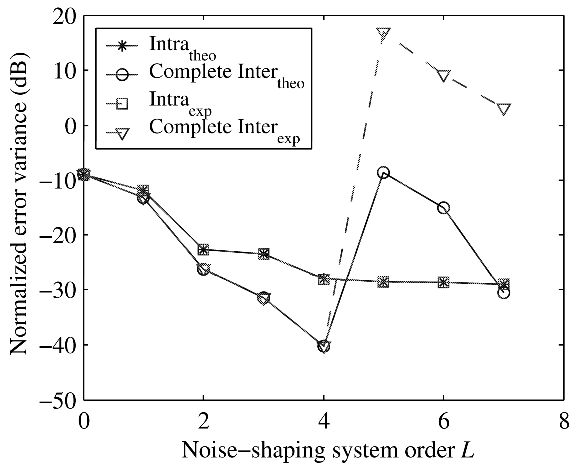

Fig. 2 Comparison of theoretical and experimental performances of both the complete interchannel and intrachannel NS systems, demonstrating the effect of ill-conditioned matrices ($N = 16$, $L_h = 64$ and $K = 8$).

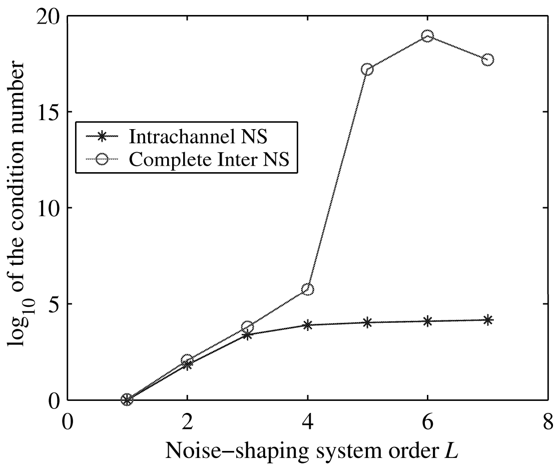

Fig. 3 Corresponding logarithm of the condition numbers of the matrices used in the solution for the NS coefficients, for both the optimal and intrachannel NS systems.

3.1 Numerical stability

The system simulation results (denoted by '$_{\mathrm{exp}}$') agreed with the predicted values (denoted by '$_{\mathrm{theo}}$'), except when the matrices used in the derivation of the NS coefficients were ill-conditioned. Indeed, this is exemplified in Figs. 2 and 3, which show, respectively, the normalized error variance in dB ($10\log_{10}(\sigma^2_{e,\min}/\sigma^2_q)$) and the logarithm of the condition number of the matrices to be inverted in the solution of the NS coefficients, as a function of the order L of the NS system. The condition number is the ratio of the largest singular value to the smallest of the matrix; thus, a large condition number indicates a nearly singular matrix which in turn leads to inaccurate results.

Upon closer inspection of these figures, it is clear that the performance of the complete interchannel noise shaper for both the theoretical and experimental cases deviates from the expected results: the performance actually *deteriorates* beyond a system order of $L = 4$. This corresponds to a substantial increase in the condition number (Fig. 3) at $L = 4$ for the complete interchannel NS case. On the other hand, the intrachannel NS system performs consistently at higher system orders; in fact, the results of the theoretical calculations and the experimental results coincide almost perfectly (Fig. 2). Unfortunately, this at a cost of approximately 10 dB (at $L = 4$).

Finally, in all subsequent figures, where the matrices were ill-conditioned, the performance of the corresponding noise-shaper was omitted.

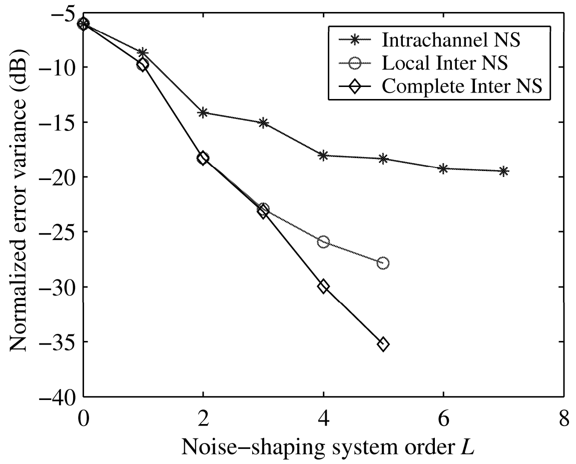

Fig. 4 Comparison of intrachannel, local interchannel and complete interchannel NS systems ($N = 16$, $L_h = 64$ and $K = 4$).

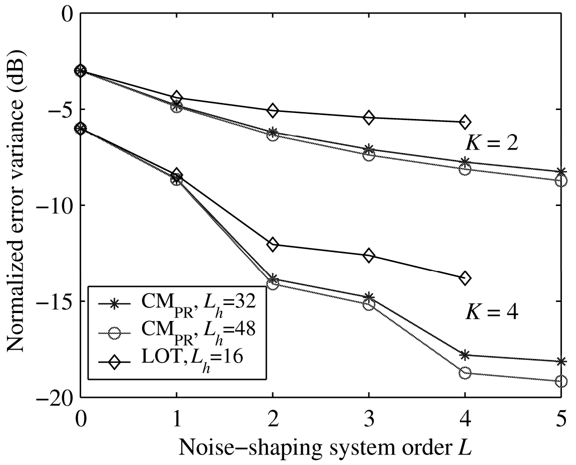

Fig. 5 Performance of the NS system with intrachannel NS filters, using an LOT FB and the generated $\mathrm{CM_{PR}}$ FBs, ($N = 8$ and $K = 2, 4$ and $L_h = 16, 32, 48$).

3.2 Comparison of Intrachannel, Local and Complete Interchannel Noise-Shaping Systems

Referring to Fig. 4, it is clear that while the intrachannel NS system has the poorest performance, the local interchannel NS system performance coincides with that of the complete interchannel NS system up to an order of $L = 3$. The good performance of the local interchannel NS system is not surprising: most of the overlap of filters in a FB occur between adjacent channels. It follows that most of the useful interchannel dependencies would be inherent to the neighboring channels.

4 Results on Variations of FB Characteristics

4.1 Length and Degree of Overlap

The first characteristics that were studied were the effect of the length of the filters L_h and the degree of overlap between the subbands on the performance of the NS system. For this purpose, FBs with 8 subbands were used as a test bed. An FB using Lapped Orthogonal Transform (LOT) filters with $L_h = 16$ [4] and two FBs generated using the aforementioned algorithm ($\mathrm{CM_{PR}}$) with lengths $L_h = 32$ and $L_h = 48$ were used. Thus, while the $\mathrm{CM_{PR}}$ filters are longer, the transition bandwidth of the LOT filters is wider than that of the $\mathrm{CM_{PR}}$ filters.

Referring to Figure 5, the effect of increasing the length of the filters is clearly visible: longer filters result in an improved performance. This is explained by the fact that longer filters induce longer-term dependencies, yielding a greater gain.

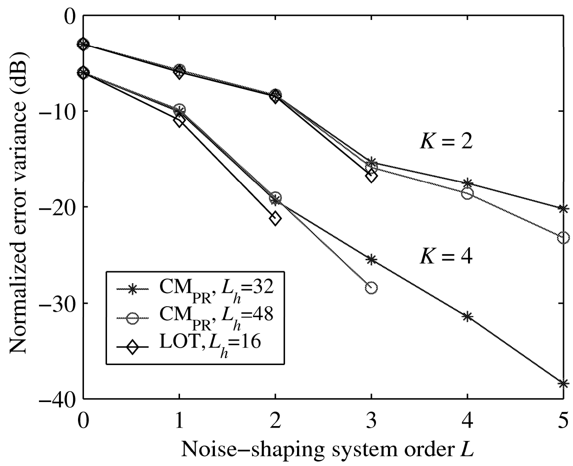

Fig. 6 Performance of the NS system with complete interchannel NS filters, using an LOT FB and the generated $\mathrm{CM_{PR}}$ FBs, ($N = 8$ and $K = 2, 4$ and $L_h = 16, 32, 48$).

On the other hand, Figure 6 shows that when using the complete interchannel NS coefficients, the LOT filters outperform the $\mathrm{CM_{PR}}$ filters. This is due to the shape of the frequency response of the LOT filters: the larger degree of overlap between neighbouring subbands means that the use of interchannel dependencies in the calculation of the NS coefficients $\mathbf{G}_{i,\mathrm{opt}}$ will yield a better improvement in performance. However, it was observed that the use of the LOT filters resulted in the matrices becoming ill-conditioned at a lower system order.

Furthermore, it is noted that increasing the length of the $\mathrm{CM_{PR}}$ still leads to an improved performance. In fact, as the length of the filters in the FB increases, its corresponding performance approaches that of the LOT filters. Also this FB has the advantage of better numerical stability: a gain of approximately 10 dB with respect to the LOT's performance at $L = 3$ can be achieved by increasing the order of the noise-shaper to $L = 5$.

4.2 PR vs. Near-PR FBs

Attention is now turned to the performance of near-PR cosine modulated FBs (denoted by CM FB). It was observed that the CM FB outperforms the $\mathrm{CM_{PR}}$ FB – because the filters of the CM FB are longer – and that the corresponding matrices remain well-behaved at higher system orders. However, the analysis becomes more interesting when the theoretical performance is compared with experimental results. Fig. 7 displays the results for both the CM and the CM_{PR} FBs, with varying stepsize s. These correspond to quantization error variances σ_q^2 of approximately -31 dB, -23 dB and -11 dB, respectively (recall that the variance of the signal was $\sigma_s^2 \approx 7.21$ dB).

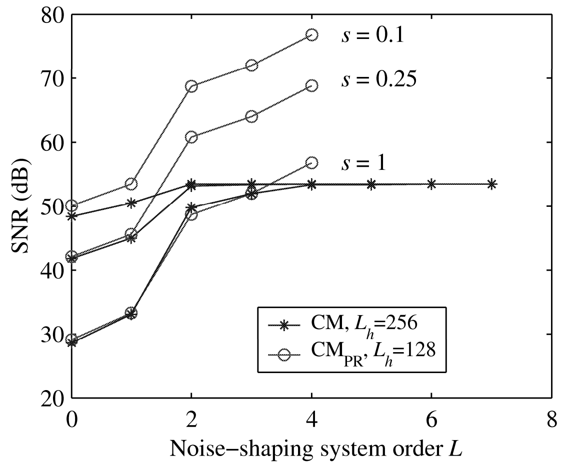

Fig. 7 SNR of the overall system with complete interchannel NS filters, using a PR FB (CM_{PR}) and a near-PR FB (CM) with $N = 32$, $K = 8$ and varying quantizer stepsize $s = 0.1, 0.25, 1$.

Figure 7 shows that the reconstruction error of the CM without quantization (53.4 dB) limits the performance of the system. However, as the quantizer step size s increases (leading to a larger σ_q^2), a significant gain in SNR can be achieved using the NS system.

4.3 Rate-distortion Characteristic

It was shown in [1] that although the rate-distortion (RD) characteristic of the subband coder was improved by the inclusion of the NS system, it was still poorer when compared to the RD characteristic at a lower K. These results were confirmed, however it was further observed that the length of the FB improved the RD characteristic, at high bitrates. The rate was calculated as the sum of the entropies of the subband signals, normalized by M, such that it corresponded to the minimum achievable number of bits per input sample.

Indeed, Figure 8 shows that increasing the length L_h of the filters in the FB resulted in a higher SNR when using the NS system above 10bps. This came at no cost to the overall rate.

5 Conclusion

In this paper, the numerical stability of the solution for the NS coefficients was first investigated; it was shown that the ill-conditioning of matrices limited the performance and the practical implementation of the NS system to a certain system order, depending on the employed FB. Next, it was seen that the local interchannel NS system provided a good approximation to the complete interchannel NS system, up to a certain system order, with the advantage of a reduced number of computations.

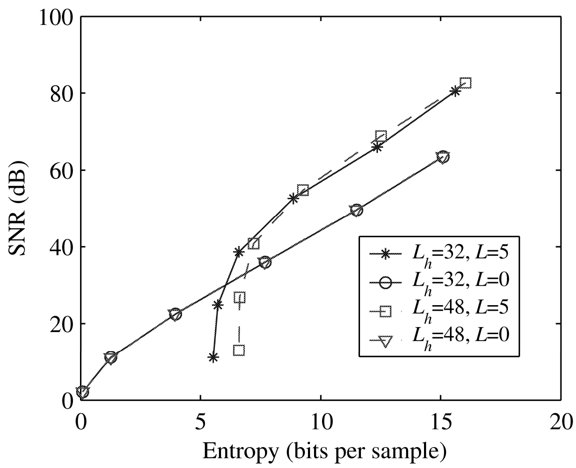

Fig. 8 Rate-distortion characteristic of the NS system with complete interchannel NS filters, using the generated CM_{PR} FB with $N = 8$, $K = 2$ and $L_h = 32, 48$.

Subsequently, the susceptibility of the performance of the NS system to variations in filter specifications was studied. It was concluded that an increase in the filter length lead to a larger decrease in the reconstruction error variance, while the LOT filters demonstrated that a significant gain can be achieved when using interchannel dependencies. It was also demonstrated that although near-PR filter banks may show an impressive theoretical performance, they are limited by their reconstruction error; however, a gain can be achieved using the NS system for large quantization noise variance, leading to a lower bitrate. Finally, it was observed that increasing the length of the FB at higher bitrates improves the RD characteristic of the NS system.

References

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